Exercises on Conformal Field Theory

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Due on June 17th, 2016

H 8.1 Inner product of descendant states *(2 points)*

Show that the norm squared of the descendant state $L_{-1}^n |h\rangle$ of the (normalized) primary state $|h\rangle$ of conformal weight h is

$$\langle h| \left(L_{-1}^n\right)^\dagger L_{-1}^n |h\rangle \;=\; 2^n n! \prod_{\ell=1}^{n} \left(h + \frac{\ell - 1}{2}\right) \;.$$

H 8.2 Singular vectors *(4 points)*

We want to determine the singular vectors at level 2.

a) Show that a vector $|\chi\rangle$ is singular (i.e., $L_n |\chi\rangle = 0$ for all $n \geq 1$), if $L_1 |\chi\rangle = 0$ and $L_2 |\chi\rangle = 0$.

b) Determine the two singular vectors at level 2.

Hint: Start with a general ansatz for a descendant state at level 2 *of a primary state* $|h\rangle$*.*

H 8.3 Gram matrix at level 3 *(4 points)*

Let us consider a (normalized) primary state $|h\rangle$ in a conformal field theory with central charge c.

a) Determine a basis of descendant states at level 3 and calculate the Gram matrix $M^{(3)}(h, c)$.

b) Consider the Kac determinant at level 3, i.e., the determinant of $M^{(3)}(h, c)$. Check that the eigenvalue $h = 0$ appears with the expected multiplicity and — by taking into account the previous problem — speculate on the number of singular vectors at level 3.

Hint: To answer these questions, there is no need to explicitly calculate the determinant.