Tutorial Dualities in Field and String Theory — Sheet 8

1.) **Off-shell supersymmetry variations of chiral superfields** 3 pt

The component expansion of a chiral superfield Φ is given by

$$\Phi(y,\theta,\bar\theta) \;=\; \phi(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F(y) \ ,$$

in terms of $y^\mu \;=\; x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}$.

a) Expand $\Phi(y,\theta,\bar\theta)$ into a chiral superfield as a function of $(x,\theta,\bar\theta)$.
Hint: Carry out a taylor expansion of $\Phi(x^\mu + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha},\theta,\bar\theta)$ arround $(x^\mu,\theta,\bar\theta)$.

b) Determine the (off-shell) supersymmetry variations $\delta_\xi\phi$, $\delta_\xi\psi_\alpha$, and $\delta_\xi F$ of the components fields ϕ, ψ_α, and F of the chiral multiplet $\Phi(x,\theta,\bar\theta)$ by acting with

$$\delta_\xi \;=\; \xi^\alpha Q_\alpha + \bar\xi_{\dot\alpha}\bar Q^{\dot\alpha} \qquad \text{with} \qquad Q_\alpha \;=\; \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu$$

on the chiral superfield $\Phi(x,\theta,\bar\theta)$.

2.) $N=1$ **supersymmetric action** 4 pt

a) Expand the $N=1$ supersymmetric action

$$\mathcal{S}[\Phi] \;=\; \int d^4x\Big[\ \Phi(x,\theta,\bar\theta)\bar\Phi(x,\theta,\bar\theta)\big|_{\theta\theta\bar\theta\bar\theta} + W(\Phi)\big|_{\theta\theta} + \bar W(\bar\Phi)\big|_{\bar\theta\bar\theta}\Big]$$

of a single chiral superfield Φ into its components ϕ, ψ_α, and F.

b) Eliminate (integrate out) the auxiliary field F from the action. Then use integration by parts to obtain conventional kinetic terms for the complex scalar ϕ and the Weyl fermion ψ_α.

c) Determine the equations of motion for the component fields ϕ and ψ_α from the derived component action. Interpret the equation of motions for the superpotential

$$W(\Phi) \;=\; \frac{1}{2}\lambda_2\Phi^2 \ .$$

What is the physical meaning of the coupling λ_2?

3.) $N=1$ **supersymmetric action for several chiral superfields** 3 pts

The $N=1$ supersymmetric action for $i=1,\ldots,n$ chiral superfields Φ^i is given by

$$\mathcal{S}[\Phi^i] \;=\; \int d^4x\Big[\ K(\Phi^i,\bar\Phi^{\bar\jmath})\big|_{\theta\theta\bar\theta\bar\theta} + W(\Phi^i)\big|_{\theta\theta} + \bar W(\bar\Phi^{\bar\jmath})\big|_{\bar\theta\bar\theta}\Big] \ ,$$

in terms of the (real) Kähler potential $K(\Phi^i,\bar\Phi^{\bar\jmath})$.

a) Argue that the action $\mathcal{S}[\Phi^i]$ is invariant under super-Kähler transformations

$$K(\Phi^i,\bar\Phi^{\bar\jmath}) \to K(\Phi^i,\bar\Phi^{\bar\jmath}) + f(\Phi^i) + \bar f(\bar\Phi^{\bar\imath}) \ .$$

b) Determine the bosonic action of the supersymmetric action $\mathcal{S}[\Phi^i]$ — that is to say drop all terms that involve fermions ψ^i_α. Integrate out the auxiliary fields F^i in the bosonic action and use integration by parts to obtain a conventional non-linear σ-model kinetic term for the bosonic fields ϕ^i.