Tutorial Dualities in Field and String Theory Sheet 9

1.) **Nonrenormalization theorem for the holomorphic superpotential** 2 pt

Consider the Wilsonian effective action $\mathcal{S}_{\mu_0}[\Phi]$ of a single chiral superfield Φ

$$\mathcal{S}_{\mu_0}[\Phi] \;=\; \int d^4x\,\left(\Phi\bar{\Phi}\big|_{\theta\theta\bar{\theta}\bar{\theta}} + W(\Phi,\mu_0)|_{\theta\theta} + \overline{W}(\bar{\Phi},\mu_0)\big|_{\bar{\theta}\bar{\theta}}\right)$$

with

$$W(\Phi,\mu_0) \;=\; \mu_0^2\lambda_1\Phi + \mu_0\lambda_2\Phi^2 + \ldots + \mu_0^{3-r}\lambda_r\Phi^r$$

at a scale μ_0.

a) Assign consistent charges to the coupling constants λ_r, such that $\mathcal{S}_{\mu_0}[\Phi]$ has a global $U(1)$ symmetry and a $U(1)_R$ symmetry.

b) Use superselection rules and demand a smooth limit as all $\lambda_r \to 0$ in order to show that $W(\phi,\mu)$ is nonrenormalized at a scale μ (with $0 < \mu < \mu_0$).

2.) **Nonrenormalization theorem for a superpotential of 2 chiral superfields** 2 pt

Consider now the Wilsonian effective action $\mathcal{S}_{\mu_0}[\Phi_1,\Phi_2]$ of two chiral superfield Φ_1 and Φ_2

$$\mathcal{S}_{\mu_0}[\Phi_1,\Phi_2] \;=\; \int d^4x\left(\sum_{A=1}^{2}\Phi_A\bar{\Phi}_A\big|_{\theta\theta\bar{\theta}\bar{\theta}} + W(\Phi,\mu_0)|_{\theta\theta} + \overline{W}(\bar{\Phi},\mu_0)\big|_{\bar{\theta}\bar{\theta}}\right)$$

with

$$W(\Phi,\mu_0) \;=\; \sum_{A,B=1}^{2}\frac{1}{2}\mu_0\lambda_2^{AB}\Phi_A\Phi_B + \sum_{A,B,C=1}^{2}\frac{1}{3}\lambda_3^{ABC}\Phi_A\Phi_B\Phi_C$$

at a scale μ_0. Use again superselection rules and demand smooth limits as the couplings λ_2^{AB} and λ_3^{ABC} approach zero so as to show that $W(\Phi,\mu)$ is nonrenormalized at a scale μ (with $0 < \mu < \mu_0$).

3.) **Abelian $N = 1$ gauge theory coupled to a charged chiral super-multiplet** 6 pt

a) The components of a vector super-multiplet are encoded in a real superfield $V(x,\theta,\bar{\theta})$ with

$$V(x,\theta,\bar{\theta}) = \overline{V}(x,\theta,\bar{\theta}) \tag{1}$$

Determine the constraints on the component fields

$$\begin{aligned} V(x,\theta,\bar{\theta}) \;&= a(x) + \theta\rho(x) + \bar{\theta}\bar{\zeta}(x) + \theta\theta b(x) + \bar{\theta}\bar{\theta}\bar{c}(x) \\ &\quad - \theta\sigma^\mu\bar{\theta}A_\mu(x) + i(\theta\theta)\bar{\theta}\bar{\lambda} + i(\bar{\theta}\bar{\theta})\theta\psi + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \end{aligned}$$

arising from the reality condition (1).

b) A super-gauge transformation shifts a real superfield V with respect to a chiral superfield Λ according to

$$V \to V + i(\Lambda - \bar{\Lambda}) \ . \tag{2}$$

Show that a general real superfield $V(x,\theta,\bar{\theta})$ can be shifted to the *Wess-Zumino-gauge-fixed* vector superfield

$$V(x,\theta,\bar{\theta}) \ = \ -\theta\sigma^\mu\bar{\theta}A_\mu(x) + i(\theta\theta)\bar{\theta}\bar{\lambda} + i(\bar{\theta}\bar{\theta})\theta\psi + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x) \ ,$$

where A_μ is the gauge boson, λ_α is the gaugino and D is the auxiliary field of the Wess-Zumino gauge-fixed vector superfield.

c) Argue that the action

$$\mathcal{S}[V,\Phi] \ = \ \int d^4x \Big[\left. \Phi e^{qV}\bar{\Phi}\right|_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \left. \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}\right|_{\bar{\theta}\bar{\theta}} \right) \Big] \ , \tag{3}$$

with

$$W_\alpha \ = \ -\frac{1}{4}(\bar{D}\bar{D})D_\alpha V(x,\theta,\bar{\theta}) \qquad \overline{W}_{\dot{\alpha}} \ = \ -\frac{1}{4}(DD)\bar{D}_\alpha V(x,\theta,\bar{\theta})$$

is manifest supersymmetric and gauge invariant under super-gauge transformations, if the charged chiral superfield Φ transforms as

$$\Phi \to e^{-iq\Lambda}\Phi$$

with respect to (2).

d) Determine the bosonic terms of the supersymmetric action (3) — that is to say drop all terms that involve fermions. Integrate out the auxiliary fields D and F in the resulting bosonic action and use integration by parts to obtain a conventional Lagrangian of a charged complex boson, which is minimally coupled to the $U(1)$ gauge boson A_μ.
Hint: Derive the component expansion in Wess-Zumino gauge.