Tutorial Dualities in Field and String Theory — Sheet 10

1.) **Classical moduli spaces for $N = 1$ Super-QCD** `2 pt`

a) For $N_f = N_c$ the gauge invariant mesonic and baryonic operators are given by

$$
\begin{aligned}
M^f_{\tilde f} &= \sum_{A=1}^{N_c} Q^{A\,f} \tilde Q_{A\,\tilde f} \\
B &= \sum_{f_1,\ldots,f_{N_f}} \sum_{A_1,\ldots,A_{N_c}} \epsilon_{f_1\cdots f_{N_f}} \epsilon_{A_1\cdots A_{N_c}} Q^{A_1\,f_1} \ldots Q^{A_{N_c}\,f_{N_c}} \\
\widetilde B &= \sum_{\tilde f_1,\ldots,\tilde f_{N_f}} \sum_{A_1,\ldots,A_{N_c}} \epsilon^{\tilde f_1\cdots \tilde f_{N_f}} \epsilon^{A_1\cdots A_{N_c}} \tilde Q_{A_1\,\tilde f_1} \cdots \tilde Q_{A_{N_c}\,\tilde f_{N_c}}
\end{aligned}
$$

Show that these operators fulfill the classical constraint $\det M - \tilde B\, B = 0$.

b) For $N_f = N_c + 1$ construct gauge invariant mesonic and baryonic operators. Determine the classical constraints among these operators. Show that the resulting number of independent gauge invariant operators agrees with the dimension of the classical moduli space $\mathcal{M}_{cl}$ for $N_f = N_c + 1$.

2.) **Normalization of the Affleck-Dine-Seiberg superpotential** `4 pt`
Consider for $N = 1$ Super-QCD with $N_f < N_c$ the effective Affleck-Dine-Seiberg superpotential

$$
W^{eff}_{\text{ADS}} = c_{N_f,N_c} \left(\frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{\frac{1}{N_c-N_f}} .
$$

Using the normalization $c_{N_c-1,N_c} = 1$ for $N_f = N_c - 1$, we want to determine the normalization c_{N_f,N_c} for general $N_f \leq N_c - 1$.

a) Consider for $N_f = N_c - 1$ the effective superpotential

$$
W_{eff} = \text{tr}(mM) + \left(\frac{\Lambda^{2N_c+1}}{\det M} \right) \quad , \quad m = \begin{pmatrix} 0_{\hat N_f \times \hat N_f} & 0_{\hat N_f \times (N_f - \hat N_f)} \\ 0_{(N_f - \hat N_f)\times \hat N_f} & \hat m \end{pmatrix}
$$

for $0 \leq \hat N_f < N_f$ with the degenerate mass matrix m. Calculate the F-terms from the effective superpotential W_{eff} and eliminate in the effective superpotential those flavors that couple to the mass matrix $\hat m$.

b) Use the renormalization group matching condition to determine the effective Affleck-Dine-Seiberg superpotential together with the normalization constants $c_{\hat N_f,N_c}$.
Hint: Match the 1-loop dynamical couplings $\tau(\mu,\Lambda)$ at the mass scales $\hat m$.

3.) **'t Hooft anomaly matching for $N = 1$ Super-QCD with $N_c = N_f$** `4 pt`
Consider in the quantum deformed moduli space $\det M - \tilde B B = \Lambda^{2N_c}$ of $N = 1$ Super-QCD with $N_c = N_f$. For $M^f_{\tilde f} = 0$ and $B = -\tilde B = \Lambda^{N_c}$ compute the anomaly coefficients of the triangles $SU(N_f)^3_{L/R}$, $SU(N_f)^2_{L/R} \times U(1)_{\tilde R}$, and $U(1)^3_{\tilde R}$ both at the high and at the low energy scale. Check the ‘t Hooft anomaly matching conditions for your result.