Tutorial Dualities in Field and String Theory — Sheet 11

1.) $N = 1$ **Seiberg duality and 't Hooft anomaly matching** 7 pt

a) For the $N = 1$ Seiberg dual $SU(N_f - N_c)$ Super-QCD theory ($N_f > N_c + 1$) with the discussed chiral spectrum D_f, $\tilde{D}^{\tilde{f}}$, $K^f_{\tilde{f}}$, $f, \tilde{f} = 1, \ldots, N_f$ determine the quantum numbers (representations) with respect to the global non-anomalous symmetries $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_{\tilde{R}}$.
Hint: Note that the superpotential $W = \sum_{f,\tilde{f}} K^f_{\tilde{f}} D_f D^{\tilde{f}}$ has $\tilde{R}$-charge 2.

b) Determine the $\tilde{R}$-charges of the fermions of the $N = 1$ Seiberg dual $SU(N_f - N_c)$ Super-QCD theory.

c) Calculate the anomaly coefficient for $SU(N_f)^3_{L/R}$, $SU(N_f)^2_{L/R}U(1)_B$, $SU(N_f)^2_{L/R}U(1)_{\tilde{R}}$, $U(1)^3_{\tilde{R}}$, $U(1)^2_B U(1)_{\tilde{R}}$ and $\mathrm{tr}\, U(1)_{\tilde{R}}$ in the $N = 1$ Seiberg dual $SU(N_f - N_c)$ Super-QCD theory.

d) Calculate the anomaly coefficient for $SU(N_f)^3_{L/R}$, $SU(N_f)^2_{L/R}U(1)_B$, $SU(N_f)^2_{L/R}U(1)_{\tilde{R}}$, $U(1)^3_{\tilde{R}}$, $U(1)^2_B U(1)_{\tilde{R}}$ and $\mathrm{tr}\, U(1)_{\tilde{R}}$ in the $N = 1$ $SU(N_c)$ Super-QCD theory with the discussed chiral spectrum Q^f and $\tilde{Q}_{\tilde{f}}$, $f, \tilde{f} = 1, \ldots, N_f$. Compare the anomaly coefficients of the two theories.

2.) **Extended supersymmetry** 3 pt

The extended supersymmetry algebra with $4N$ supercharges is given by

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}\, b}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu\, \delta^b_a \qquad a, b = 1, \ldots, N$$
$$\{Q^a_\alpha, Q^b_\beta\} = \{\bar{Q}_{\dot{\alpha}\, a}, \bar{Q}_{\dot{\beta}\, b}\} = 0$$

a) Construct anti-commuting raising and lowering operators for massless super multiplets of N-extended supersymmetry. Starting from a Clifford vacuum $|\Omega_\lambda\rangle$ of helicity λ, list and enumerate the possible states together with their helicities. What is the total number of bosonic and fermionic states?

b) Determine the maximal number of extended supersymmetries for (i) a theory with helicity states $|\lambda| \leq 1$ ("maximal globally supersymmetric field theories") and (ii) for a theory with helicity states $|\lambda| \leq 2$ ("maximal locally supersymmetric gravity theories").

c) For $N = 4$ extended supersymmetry determine all massless multiplets with helicity states $|\lambda| \leq 1$. Decompose the resulting $N = 4$ massless multiplets into massless $N = 2$ and massless $N = 1$ multiplets.

d) Consider an $N = 4$ extended supersymmetric gauge theory with gauge group G. What are the possible representations of matter fields with respect to the gauge symmetry G?
Hint: Assume that all massless vector fields in the spectrum give rise to gauge bosons in the usual way.