Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 1
April 8, 2015
Summer term 2015

Exercises on General Relativity and Cosmology

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http://www.th.physik.uni-bonn.de/klemm/gr_15/

–Organisational remarks–

The lecture is accompanied by exercise sessions of two hours per week, which will start next week. The exercise sheets will usually be handed out during the Monday lecture and are due in the Monday lecture one week later. You may hand in the solutions in groups of two, provided these two are registered in the same tutorial session. For admission to the final exam you have to

1. achieve at least 50% of the credits from all excercise sheets
2. and present your solutions to the exercises in the tutorials at least twice.

Your mark will then be based upon performance in the final exam.

Current information will be posted on the webpage given above. Also, the exercise sheets can alternatively be downloaded from there.

Responsible for the organization of the exercises:

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Please feel free to contact me in case of any question.

–Home Exercises–

Due on April 13, 2015

H 1.1 A cosmic muon *(2 points)*

By cosmic radiation a muon is created at an altitute of 12 km, it moves with velocity $v = 0.998\,\mathrm{c}$ as measured from Earth. The mass of a muon is $m = 105.7\,\mathrm{MeV/c^2}$, its lifetime is $\tau_{1/2} = 2.2\,\mu\mathrm{s}$ and c is the speed of light.

a) Calculate the energy of the muon. (*1 point*)

b) How far does the muon travel in average before it decays? (*1 point*)

H 1.2 Moving conductors get charged? *(10 points)*

Consider a straight, infinitely long wire at rest in reference system A. It carries a current but is uncharged. We model the wire as a sequence of (equally spaced) ions, which have charge e and are at rest in A, and the corresponding (equally spaced) electrons of charge $-e$ that move with velocity $v_e > 0$ along the wire. Let the spacing between two ions be a_I.

a) What is the spacing between two electrons in A? (*1 point*)

b) Which kind of fields does an observer measure in system A? (*1 point*)

A second observer, whose restframe we denote as B, is moving with velocity $v_B > 0$ parallel to the wire. Due to relativistic length contraction, the charge and current density created by the wire in system B are different from those in A.

c) What are the velocities of ions and electrons in system B? Use the relativistic rule for addition of velocities. (*1 point*)

d) Calculate the spacing between two ions and between two electrons as measured in B. For the electrons, transform into their rest frame as an intermediate step and then into B. (*2 points*)

e) Deduce the charge and current density in B. (*2 points*)

f) Which kind of fields does the second observer measure? (*1 point*)

Let the wire have, in an arbitrary reference system, the charge (length-)density λ and current I. Denoting the minimal distance to the wire by r, the electric and magnetic fields created read (cylindrical coordinates, where the wire is along the z-axis and $v_B > 0$ means $\vec{v_B} = v_B \vec{e}_z$)

$$\vec{E}(r) = 2\frac{\lambda}{r}\vec{e}_r, \qquad \vec{B}(r) = 2\frac{I}{c \cdot r}\vec{e}_\phi. \tag{1}$$

g) Calculate the force on a charged particle at rest in system B, as measured in A ($\vec{F}_A$) and as measured in B ($\vec{F}_B$). Show $\vec{F}_B = \gamma(v_B)\vec{F}_A$. For this, use your result from item (e) and $I_e = \lambda_e \cdot v_e$, where I_e is the current in A and λ_e the charge density of electrons in A.