Physikalisches Institut
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Theoretische Physik

Exercise Sheet 4
10/24/2018
WS 2018/19

Group Theory

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http://www.th.physik.uni-bonn.de/klemm/grouptheory/index.php

Due date: 10/31/2018

4.1 Remarks on the Definition of a Normal Subgroup

In the lecture we introduced a normal subgroup by two different properties which we so far have not shown to be equivalent. Therefore, show the following.
Let H be a subgroup of G then the two statements are equivalent:

i) $gH = Hg$ for all $g \in G$

ii) $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$.

(2 pts)

4.2 Normal Subgroups and Quotient groups

Let H be a subgroup and N be a normal subgroup of G. Then show:

i) HN is a subgroup of G, where $HN := \{hn | h \in H, n \in N\}$.

ii) The intersection $H \cap N$ is a normal subgroup of H and N is a normal subgroup of HN.

iii) There is an isomorphism of quotient groups $HN/N \cong H/(H \cap N)$.

(5 pts)

4.3 Properties of Finite Groups

Prove that the following statements for a finite group G are equivalent:

i) $|G|$ is prime.

ii) $G \neq \langle e \rangle$ and G has no proper subgroups.
Recall that a subgroup of G is proper if it is neither $\langle e \rangle$ nor G.

iii) $G \cong \mathbb{Z}_p$ for some prime p.

(5 pts)

4.4 Subgroups of the Dihedral Group D_4

Find subgroups H and K of D_4 such that H is normal in K and K is normal in D_4, but H is <u>not</u> normal in D_4.

(4 pts)

4.5 Semidirect Product

Let G be the set of matrices of the form:

$$G = \left\{ \begin{pmatrix} a & 0 & b \\ 0 & a & c \\ 0 & 0 & d \end{pmatrix} ,\ ad \neq 0 \right\} \subset \mathrm{GL}(3, \mathbb{R}) \ .$$

Check that G is a subgroup of $\mathrm{GL}(3, \mathbb{R})$. Moreover, prove that it is a semidirect product of $(\mathbb{R}^2, +)$ by $(\mathbb{R}^* \times \mathbb{R}^*, *)$, i.e. $G = \mathbb{R}^2 \rtimes (\mathbb{R}^* \times \mathbb{R}^*)$.
Recall that $\mathbb{R}^* := \mathbb{R} \backslash \{0\}$.

(4 pts)