Group Theory

Dr. Hans Jockers

http://www.th.physik.uni-bonn.de/klemm/grouptheory/index.php

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7.1 Irreducible Representations of S_4

We want to work out all irreducible representations of the permutation group S_4.

i) Determine the number of conjugacy classes of S_4 and the number of elements they contain. *Hint*: Use the result of exercise 6.4.

ii) Compute the characters for the trivial $\mathbf{1}_T$ and alternating $\mathbf{1}_A$ representations.

iii) Compute the character for the natural representation $\mathbf{4}_N$. Show that the natural representation is reducible and decompose it into its irreducible pieces.

iv) Show that apart form the irreducible representations in ii) - iii) there are two additional irreducible representations. Calculate their dimensions.

v) Show that the tensor product $\mathbf{3}_S \otimes \mathbf{1}_A$ yields one of the remaining irreducible representation ρ. Compute the character of ρ.

vi) Compute the character of the last irreducible representation ρ' using the orthonormality relations of the characters.

vii) Summarize all irreducible representations in a character table.

viii) Decompose the tensor product $\rho \otimes \rho'$ into irreducible representations.

(*10 pts*)

7.2 Irreducible Representations of A_4

Similar to the first exercise we want to find all irreducible representations of the alternating group A_4.

i) Show that the four conjugacy classes can be represented by $\{[(1)], [(12)(34)], [(123)], [(132)]\}$. Determine their number of elements.

ii) Determine all irreducible representations of $\mathbb{Z}_n$. *Hint*: There are n distinct one-dimensional irreducible representations of $\mathbb{Z}_n$.

iii) Use from exercise 5.2 that $A_4/N \cong \mathbb{Z}_3$ with $N = \{(1), (12)(34), (13)(24), (14)(23)\}$ to show that all irreducible representations of $\mathbb{Z}_3$ lift to irreducible representations of A_4. Compute their characters.

iv) Show that there is one more irreducible representation of A_4. Determine its dimension and character.

(10 pts)