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## Exercises General Relativity and Cosmology

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<http://www.th.physik.uni-bonn.de/klemm/grss16/>

–HOMEWORK–

### 1 Falling freely into a black hole (15 pts.)

Consider a particle that has crossed the event horizon of a Schwarzschild black hole at  $r = R_S$  and is then falling freely within the event horizon. Work with the standard Schwarzschild coordinates  $(t, r, \theta, \phi)$  and denote the particle's proper time by  $\tau$ .

1. Show that

$$\left| \frac{dr}{d\tau} \right| \geq \sqrt{\frac{R_S}{r} - 1}. \quad (1)$$

When is this bound saturated? (7 pts.)

2. Calculate the maximum lifetime of a particle falling freely from  $r = R_S$  to  $r = 0$ . Plug in numbers to express this in seconds for a black hole with one solar mass,  $M = 2 \cdot 10^{30}$  kg. (4 pts.)
3. Can an observer that also travels from  $r = R_S$  to  $r = 0$  but not on a geodesic have a longer lifetime than the one calculated in item b)? Justify your answer. (4 pts.)