1 Gravitational radiation of a spinning rod (10 pts.)

Consider a conducting metal rod of length $L$ and mass density $\rho$, spinning with frequency $\omega$.

1. Calculate the time-dependent part of the quadrupole moment $I_{ij}$ and the luminosity

\[ L = -\frac{1}{5} \left\langle \frac{d^3 J_{ij}}{dt^3} \frac{d^3 J^{ij}}{dt^3} \right\rangle. \]  

5 pts.

2. Calculate the charge induced in the rod due to the centrifugal force. An order of magnitude approximation is sufficient. Will the rotation generate electromagnetic dipole radiation?

3 pts.

3. Calculate the luminosity of the electromagnetic quadrupole radiation. What is the ratio between the power of electromagnetic and gravitational radiation for $\rho = 10 \text{ g/cm}^3$ and $\omega = 1 \text{ kHz}$? 2 pts.

2 Detection of gravitational waves (10 pts)

Gravitational waves can be detected by monitoring the distance between two free flying masses. If one of the masses is equipped with a laser and an accurate clock, and the other with a good mirror, the distance between the masses can be measured by timing how long it takes for a pulse of laser light to make the round-trip journey. How would you want your detector to be oriented to register the largest response from a plane wave of the form

\[ ds^2 = -dt^2 + [1 + A \cos(\omega(t - z))] dx^2 + [1 - A \cos(\omega(t - z))] dy^2 + dz^2. \]  

If the masses have a mean separation $L$, what is the largest change in the arrival time of the pulse caused by the wave? What frequencies $\omega$ would go undetected?