
Exercises General Relativity and Cosmology

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<http://www.th.physik.uni-bonn.de/klemm/grss16/>

–HOMEWORK–

1 Gravitational radiation of a spinning rod (10 pts.)

Consider a conducting metal rod of length L and mass density ρ , spinning with frequency ω .

1. Calculate the time-dependent part of the quadrupole moment I_{ij} and the luminosity

$$L = -\frac{1}{5} \left\langle \frac{d^3 J_{ij}}{dt^3} \frac{d^3 J^{ij}}{dt^3} \right\rangle. \quad (1)$$

5 pts.

2. Calculate the charge induced in the rod due to the centrifugal force. An order of magnitude approximation is sufficient. Will the rotation generate electromagnetic dipole radiation?
3 pts.
3. Calculate the luminosity of the electromagnetic quadrupole radiation. What is the ratio between the power of electromagnetic and gravitational radiation for $\rho = 10 \text{ g/cm}^3$ and $\omega = 1 \text{ kHz}$? **2 pts.**

2 Detection of gravitational waves (10 pts)

Gravitational waves can be detected by monitoring the distance between two free flying masses. If one of the masses is equipped with a laser and an accurate clock, and the other with a good mirror, the distance between the masses can be measured by timing how long it takes for a pulse of laser light to make the round-trip journey. How would you want your detector to be oriented to register the largest response from a plane wave of the form

$$ds^2 = -dt^2 + [1 + A \cos(\omega(t - z))] dx^2 + [1 - A \cos(\omega(t - z))] dy^2 + dz^2? \quad (2)$$

If the masses have a mean separation L , what is the largest change in the arrival time of the pulse caused by the wave? What frequencies ω would go undetected?