Physikalisches Institut
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Theoretische Physik

Exercise Sheet 8
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SS 16

Exercises General Relativity and Cosmology

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`http://www.th.physik.uni-bonn.de/klemm/grss16/`

–Homework–

1 Hyperbolic spaces (20 points)

In the lecture and on the previous sheets you have seen examples of flat spaces, e.g. $\mathbb{R}^n$ or Minkowski space, as well as positively curved spaces like the sphere. Here we consider maximally symmetric spaces with constant negative scalar curvature which are called *hyperbolic*.

1. Calculate the Ricci scalar R associated to the metric
$$ds^2 = a^2(dt^2 + \cosh^2 t\, d\phi^2) \tag{1}$$
and show that it takes the constant negative value $R = -2/a^2$. Use the technique introduced on sheet 6, exercise 2 to obtain the Christoffel symbols. **3 pts.**

2. Find a tensorial equation expressing the Ricci tensor in terms of the metric. **2 pts.**

3. Consider $\mathbb{M}^{1,2}$ whose metric is $ds^2 = -dx^2 + dy^2 + dz^2$ and embed into it the surface given by
$$x^2 - y^2 - z^2 = a^2\,. \tag{2}$$
Parametrize this surface and show that the induced metric equals (1). **3 pts.**

4. Calculate the Ricci scalar R associated to the metric
$$ds^2 = f(r)^2 dr^2 + r^2 d\theta^2 \tag{3}$$
in terms of the function f. Set $R = -2/a^2$ and solve the differential equation for f. This will introduce an integration constant. For which values of this constant can the metric (3) be brought into the form (1)? Show this explicitly by finding the appropriate coordinate transformation. **4 pts.**

5. Consider a geodesic $\mathcal{C}$ parametrized by the arc length τ. Show that the quantity
$$g\left(\partial_\phi, \frac{d\mathcal{C}}{d\tau}\right) = c \tag{4}$$
is constant along the geodesic for the metric (1). *Hint: There is no need to explicitly solve the geodesic equation.* **4 pts.**

6. Use (4) and the fact that τ is the arc length to find $d\phi/dt$. **2 pts.**

7. Integration of $d\phi/dt$ gives the geodesics. Do this for the case $c = a$. **2 pts.**