Physikalisches Institut | Exercise Sheet 11
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Theoretische Physik | SS 15

Exercises Quantum Field Theory I

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http://www.th.physik.uni-bonn.de/klemm/qft1ss15/

–Homework–

1 Muon pair production (8 pts.)

Calculate the spin summed and averaged tree-level cross section for the production of a muon/anti-muon pair by annihilation of an electron with a positron.

1. Denote the momenta of the ingoing particles by p_1 and p_2 while the outgoing momenta are k_1 and k_2. Draw the only contributing diagram, write down the invariant amplitude and calculate

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}(e^+e^- \to \mu^+\mu^-)|^2. \tag{1}$$

Hint: You can neglect the mass of the electrons but keep the mass of the muons. **4 pts.**

2. Calculate the cross section

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{CM} = \frac{1}{E_{cm}^2}\frac{|k_1|}{16\pi E_{cm}} \cdot \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2. \tag{2}$$

2 pts.

3. Calculate the scattering of a muon and an electron. What substitutions do you have to make in (1)? What changes for the kinematics? **2 pts.**

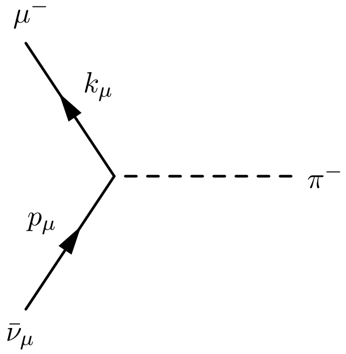
2 Decay of the charged pion (7 pts.)

Quantum fields cannot only be used to describe what we call elementary particles. In fact most people believe that also the standard model should be thought of as an effective description of more fundamental degrees of freedom. In this spirit you can treat the negatively charged pion π^- as a complex scalar field and describe its coupling to muons and the muon antineutrino with the lagrangian

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi - m_\pi^2\phi^*\phi + \bar{\psi}(i\partial\!\!\!/ - m_\mu)\psi + \bar{\chi}i\partial\!\!\!/\chi + \frac{G}{\sqrt{2}}f_\pi[(\partial_\mu\phi)\bar{\psi}\gamma^\mu(1-\gamma^5)\chi + h.c.], \tag{3}$$

where we introduced the coupling constants G, f_π and $h.c.$ stands for "hermitian conjugate". Note that the dirac field ψ corresponds to the muon and χ to the neutrino, which we assume to be massless. Note that the insertion of $1-\gamma^5$ projects the neutrino spinor on the left-handed component and arises due to the parity violating nature of the weak interaction.

1. Derive the momentum space Feynman rule for the $\pi^- \mu^- \bar{\nu}_\mu$-vertex,



$$= -\frac{G}{\sqrt{2}} f_\pi (p_\mu + k_\mu) \gamma^\mu (1 - \gamma^5).$$

In particular think about how to interpret the two nonlinear terms in the lagrangian, which are complex conjugate to each other. *Hint: How could you contract them?* Also think about how the momentum occurs in the solution. **3 pts.**

2. Write down the spin summed square of the invariant amplitude for a negative pion decaying into a muon and a muon antineutrino. How does expression (1) change? Use the trace identities introduced on the last sheet and

$$\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5) = -4i\epsilon^{\mu\nu\rho\sigma}, \tag{4}$$

to simplify the expression. **3 pts.**

3. A look into the particle data booklet[1] tells you that the branching ratio of negative pions into muons is $(99.98770 \pm 0.00004)\,\%$. Assuming that it is $100\,\%$, calculate the lifetime of π^- using the formula

$$\frac{d\Gamma}{d\Omega} = \frac{1}{m_\pi^2} \frac{|\vec{p}|}{32\pi^2} |\mathcal{M}(m_\pi \to p_\mu k_\nu)|^2, \tag{5}$$

for the differential decay rate. **1 pts.**

[1] http://pdg.lbl.gov/2014/download/rpp-2014-booklet.pdf