Physikalisches Institut | WT2011/12
Universität Bonn | 24.10.2011
Theoretische Physik | Exercise 2

Exercises Advanced Theoretical Physics

Prof. Dr. Albrecht Klemm

1 The physical interpretation of F_1 and F_2

During the exercise class we will focus on the electron vertex function.

a) Argue that, just by use of Lorentz and gauge symmetry, the general correction to the electron vertex function must be given as

$$\Gamma^\mu(p', p) = \gamma^\mu F_1(q^2) + \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)$$

b) Show that in the non-relativistic limit F_1 is related to the electric charge and F_2 to the anomalous magnetic moment.

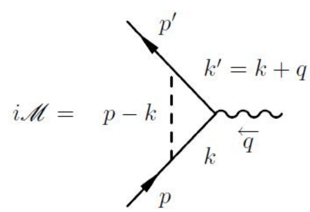
2 (homework - 8pts) - Higgs corrections to g-2

Any particle that couples to the electron can produce a correction to the anomalous magnetic moment $g-2$. In this exercise we want to calculate the contribution from a Higgs particle, a scalar h of mass m_h that couples to the electron via the interaction term

$$H_{\text{int}} = \int d^3x \frac{\lambda}{\sqrt{2}} h \bar{\psi} \psi \ .$$

a) Write down the Feynman rules for the new vertex and for the Higgs propagator.

b) Write down the amplitude for the 1-loop correction to the interection of the electron with an eletromagnetic source $A_\mu(q)$,



and obtain the amplitude $i\mathcal{M} = i\mathcal{M}^\mu A_\mu(q)$ with $\mathcal{M}^\mu$ written in the form

$$i\mathcal{M}^\mu = \frac{e\lambda^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{\mathcal{N}^\mu}{\mathcal{D}}. \tag{2.1}$$

For what follows we will only be interested in $\mathcal{M}^\mu$, the precise form of $A_\mu(q)$ will be irrelevant.

c) Show that, using the technique of Feynman parameters from last exercise sheet, the denominator can be written as

$$\frac{1}{\mathcal{D}} = \int dxdydz\delta(x+y+z-1)\frac{2}{[l^2-\Delta+i\epsilon]^3} \ ,$$

where l^ν is the shifted momentum and Δ is independent of k^ν,

$$l^\nu = k^\nu + yq^\nu - zp^\nu \ , \qquad \Delta = -xyq^2 + (1-z)^2m^2 + zm_h^2$$

with m the electron mass.

d) Show that the amplitude (2.1) in terms of the shifted momentum l can be written with the numerator in the form

$$\mathcal{N}^\mu = \bar{u}(p')\left[\gamma^\mu A + (p'^\mu + p^\mu)B + q^\mu C\right]u(p) \ .$$

Does the result you obtain satisfy the Ward identity?

e) Use the Gordon identity

$$2m\,\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left[(p'^\mu + p^\mu) + i\sigma^{\mu\nu}q_\nu\right]u(p)$$

to write the amplitude in the more convenient form

$$i\mathcal{M}^\mu = e\,\bar{u}(p')\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\mu}{2m}F_2(q^2)\right)u(p).$$

f) Recall that the anomalous magnetic moment of the electron $g-2$ is related to F_2 by

$$g-2 = 2F_2(0).$$

So we just need to calculate (that means, evaluate the integrals!) $F_2(q^2)$ in the limit $q^2 = 0$. Calculate it and plug in the values,

$$\lambda = 3\times 10^{-6} \ , \qquad m = 0.511MeV \ , \qquad m_h = 120GeV \ ,$$

to obtain $(g-2)/2$ for the electron. What happens if we condider the muon instead of the electron? Note that the Yukawa coupling is also different,

$$\lambda_{(\mu)} = 6\times 10^{-4} \ , \qquad m_{(\mu)} = 106MeV \ .$$

3 (homework - 2pts) Integrals you should know how to calculate

Prove the following equations, by Wick rotating on the l^0 plane

a)

$$\int \frac{d^4l}{(2\pi)^4}\frac{1}{[l^2-\Delta]^m} = \frac{i(-1)^m}{(4\pi)^2}\frac{1}{(m-1)(m-2)}\frac{1}{\Delta^{m-2}}, \qquad m>2$$

b)

$$\int \frac{d^4l}{(2\pi)^4}\frac{l^2}{[l^2-\Delta]^m} = \frac{i(-1)^{m-1}}{(4\pi)^2}\frac{1}{(m-1)(m-2)(m-3)}\frac{1}{\Delta^{m-3}}, \qquad m>3$$