Exercises Advanced Theoretical Physics

Prof. Dr. Albrecht Klemm

1 Questions

1. What is the LSZ reduction formula?
2. What is the optical theorem? Give an intuitive reasoning ("hand-waving" argument).
3. What is a *one-particle irreducible* (1PI) diagram?
4. Draw the corrected photon-propagator as well as the corrected electron-propagator to first order in α'. What is their physical interpretation?
5. What is a branch-cut? How does it appear in the context of the propagator in an interacting field theory?

Homework

2 $F_1(0)$ - once again **(6 points)**

Go through chapters six and seven of Peskin & Schroeder and show that $F_1(q^2 = 0) = 1$, also if one includes the first corrections in α' to the propagator as well as to the vertex function (Why is this of physical importance?).
Hint: Of course you are not asked to copy the relevant sections. In the exercise we already calculated the corrections to the propagator as well as to the vertex function. You can start at this point. Formulae, e.g. the Feynman parameter integrals, that are already known can be used without any further derivation. On the other hand, fill in the gaps that are left in Peskin & Schroeder.

3 The optical theorem **(2 points)**

Demonstrate explicitly the optical theorem by considering Bhabha scattering $e^+e^- \to e^+e^-$. The squared modulus can schematically be written as

$$|M|^2 = |A|^2 + |B|^2 + \bar{A}B + \bar{B}A. \tag{3.1}$$

Which of these quantities can be calculated using the optical theorem? Draw the corresponding loop diagrams and confirm the optical theorem by explicit calculations.

4 Some mathematical background **(2 points)**

4.1 The delta distribution

Show that

$$\lim_{\epsilon \searrow 0} \frac{1}{x \pm i\epsilon} = \mp i\pi\delta + pv\frac{1}{x}. \tag{4.1}$$

Hint: Statements about the delta function are proved by considering the action on test functions.

4.2 Riemann surfaces and branch-cuts

In this exercise we want to construct the Riemann surface corresponding to the graph of the complex square-root.

1. Show that every complex number has two square-roots (counted with multiplicity). Give the "biggest" subset of the complex plane on which we can assign to every complex number a square-root with positive real part and one with negative real part. The complement is called the *branch-cut*. What happens to the function at its "starting point"?

2. Let now $z \in \mathbb{C}^*$. Show that we can find balls $U(w_1)$, $U(w_2)$ around its square-roots w_1, w_2 that do not contain zero and are disjoint. Show that this implies that we can locally define two square-root functions as follows. Take $\tilde{z}$ close to z and define a positive root by assigning that root to $\tilde{z}$ that is in $U(w_1)$ and a negative root by assiging that one that is contained in $U(w_2)$. The graphs of these two functions are called leaves and are so far only defined locally.

3. Suppose we start slightly above our branch-cut on the positive leave and walk on it once around the origin to the other "riverside" of the branch-cut. If we now proceed do we return to the point we started at? Try to draw a global picture of the two leaves. You obtain a non-trivial example of a Riemann surface.

   Comment: These issues already appear in scattering problems of non-relativistic quantum mechanics. In this situation one is naturally led to this Riemann surface by considering the Energy as a function of the squared momentum and encounters several branch-cuts. One is the dynamical branch-cut, whose position gives the range of the potential and the incontinuity gives the strength of the potential. If you want to read more about this you are referred to Florian Scheck, "Quantum Physics", Springer Verlag, 2007.