Physikalisches Institut
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Theoretische Physik

Exercise sheet 10
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Exercises Quantum Field Theory II

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http://www.th.physik.uni-bonn.de/klemm/qft2ws1516/

–Homework–

1 Renormalization of ϕ^3-theory in 6d (25 pts.)

Consider the Lagrangian of a real scalar field in 6 spacetime dimensions

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - c\phi - \frac{m^2}{2}\phi^2 - \frac{g}{3!}\phi^3. \tag{1}$$

We assume that the coefficients are choosen so that the theory has a metastable vacuum state at the origin.

1. What are the dimensions of the coupling constants c and g? Work in $6-2\epsilon$ dimensions to regularize the loop-diagrams. In particular substitute $g \to \mu^x g$ to make the coupling constant of the 3-scalar vertex massless. What do you need to insert for x? **3 pts.**
2. Write down the Feynman rules and derive a formula for the superficial degree of divergence D of a diagram in terms of the number of propagators and vertices. **6 pts.**
3. Draw all 1-PI one-loop diagrams that are superficially divergent and give their superficial degree of divergence and symmetry factor. *Hint: You should end up with four diagrams.* **5 pts.**

To renormalize the theory you will work in the so-called MS scheme (minimal substraction). In this scheme the renormalization condition is that the counterterms only substract the poles. This makes your life particularly easy because you can expand diagrams in external momenta.

4. A superficial look at D suggests that after Taylor-expanding the amplitudes in external momenta there are altogether six divergent terms. Write down the amplitudes and carry out the expansion explicitly to show that there are only four. **2 pts.**
5. Wick-rotate and calculate the four divergent integrals in $6-2\epsilon$ dimensions using

$$\int d^n k \frac{1}{(k^2+a^2)^r} = \pi^{\frac{n}{2}} \left(a^2\right)^{\frac{n}{2}-r} \frac{\Gamma\left(r-\frac{n}{2}\right)}{\Gamma(r)}, \quad \text{for } r \geq 1 \tag{2}$$

and

$$\text{Res}(\Gamma, -n) = (-1)^n/n!\,. \tag{3}$$

Determine the coefficients of the poles. **2 pts.**

6. What are the four counterterms needed in order to renormalize the one-loop diagrams of this theory? Write down the Lagrangian including counterterms and their value in the MS scheme. **6 pts.**

7. Why is the linear term in the lagrangian necessary? **1 pt.**