Exercises Quantum Field Theory II
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http://www.th.physik.uni-bonn.de/klemm/qft2ws1516/

−Homework−

1 The optical theorem in ϕ^4-theory (15 pts.)

In the lecture it was shown how to perturbatively check the optical theorem for $2 \to 2$ scattering to order λ^2 in ϕ^4-theory.

1. To check the validity of our analysis, evaluate all three one-loop diagrams, using the standard method of Feynman parameters. Check the validity of the optical theorem. In particular show that the t- and u- channel diagrams do not contribute. **10 pts.**

2. To arrive at the cutting rules we used

$$\lim_{\epsilon \to 0} \frac{1}{x + i\epsilon} = -i\pi\delta(x) + P\left(\frac{1}{x}\right). \tag{1}$$

   The formula is valid in a distributional sense and we implicitly assume that it is integrated against a test function. Derive this formula using the definition of the Cauchy principal value, i.e.

$$P\left[f(x)\right] = \lim_{\epsilon \to 0^+} \left[\int_a^{b-\epsilon} f(x)dx + \int_{b+\epsilon}^c f(x)dx \right] \tag{2}$$

   when $\int f(x)$ diverges iff b is inside the domain of integration. **5 pts.**