Physikalisches Institut
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Theoretische Physik

Exercise sheet 6
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Exercises Quantum Field Theory II

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`http://www.th.physik.uni-bonn.de/klemm/qft2ws1516/`

–HOMEWORK–

1 Grassmann numbers and the fermionic path integral (15 pts.)

The path integral quantization of Dirac fields works along the same lines as that of scalar fields. However, the fact that fermionic quantum-fields anticommute makes it necessary to treat the classical Dirac field, that is integrated over in the path integral, not as an ordinary complex number but as an anticommuting *Grassmann* number. In this exercise you will learn how these numbers behave and use your knowledge about the scalar field path integral to derive the generating functional for the Dirac field.

1. For now just think about Grassmann numbers as *"numbers"* that anticommute, i.e.

$$\theta\eta = -\eta\theta. \tag{1}$$

Argue that the taylor expansion of a function in Grassmann numbers terminates after two terms. Then show that

$$\int d\theta f(\theta) = c, \tag{2}$$

where $f(\theta)$ is an arbitrary function and $c \in \mathbb{C}$. Here the integral can be thought of as going over the full range of values, analogous to $\int\limits_{-\infty}^{\infty} dx$. *Hint: Impose invariance under shifts $\theta \to \theta + \eta$ to show the claim.* **2 pts.**

From now on we choose the normalization

$$\int d\theta\, \theta = 1. \tag{3}$$

Note that Grassmann integration and derivation are equivalent.

2. Show that integrals over complex Grassmann numbers are invariant under unitary transformations $\theta'_i = U_{ij}\theta_j$. *Hint: Use $\prod_i \theta'_i = \frac{1}{n!}\epsilon^{ij\ldots l}\theta'_i\theta'_j\ldots\theta'_l$.* **3 pts.**

3. Show that

$$\left(\prod_i \int \bar{\theta}_i \theta_i\right) e^{-\bar{\theta}_i B_{ij} \theta_j} = \det B. \tag{4}$$

and

$$\left(\prod_i \int \bar{\theta}_i \theta_i\right) \theta_k \bar{\theta}_l e^{-\bar{\theta}_i B_{ij} \theta_j} = \det B \left(B^{-1}\right)_{kl}. \tag{5}$$

3 pts.

4. Modify the derivation for the scalar field to obtain the Dirac field generating functional

$$\begin{aligned} Z[\bar{\eta}, \eta] =& \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[i \int d^4x \left[\bar{\psi}(i\partial\!\!\!/ - m)\psi + \bar{\eta}\psi + \bar{\psi}\eta\right]\right] \\ =& Z_0 \cdot \exp\left[-\int d^4x\, d^4\, y \bar{\eta}(x) S_F(x-y)\eta(y)\right], \end{aligned} \tag{6}$$

where $S_F(x-y)$ is the usual Feynman propagator for the Dirac field. **8 pts.**

5. Although you saw many examples of regularization and also some renormalization it is hard to get the big picture and how all of these nuts and bolts fit together. Without checking up on it i ask you to read the following text, which sketches the essence of renormalization without the additional complications due to quantum fields: *http://arxiv.org/pdf/hep-th/0212049v3.pdf*