Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise sheet 9
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# Exercises Quantum Field Theory II

Prof. Dr. Albrecht Klemm, Thorsten Schimannek

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`http://www.th.physik.uni-bonn.de/klemm/qft2ws1516/`

–Homework–

## 1 Fixing the non-Abelian gauge (with ghosts) (20 pts.)

In the lecture you applied the Faddeev-Popov trick to fix the gauge in the path integral of the electromagnetic field and derive the propagator. While this elaborate machinery might seemed a little unproportional in the case of an abelian gauge theory it directly generalises to non-Abelian gauge theories. Then it leads to new Feynman rules of unphysical *ghost particles* propagating as intermediate states which render the theory unitary.

1. Most of the steps directly apply to the non-Abelian case as well and we obtain

$$\int \mathcal{D}A \, \exp\left[-i\int d^4x \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}\right]$$
$$= \left(\int \mathcal{D}\alpha\right)\int \mathcal{D}A \, \exp\left[i\int d^4x\left(-\frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu} - \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2\right)\right]\det\left(\frac{\delta G(A^{a,\alpha})}{\delta\alpha}\right). \quad (1)$$

   Show that for an $SU(N)$ Yang-Mills theory

$$\frac{\delta G(A^{a,\alpha})}{\delta\alpha} = \frac{1}{g}\Box + f^{abc}\partial^\mu A^b_\mu. \quad (2)$$

   What is the propagator? **2 pts.**

2. We can not factor out the Faddeev-Popov determinant as we did in the electromagnetic case because it depends on $A_\mu$ for a general non-Abelian theory. We can solve this problem by using the Gaussian path integral for Grassman fields and write

$$\det\left(\frac{1}{g}\Box + f^{abc}\partial^\mu A^b_\mu\right) = \int \mathcal{D}\bar{c}\mathcal{D}c \, \exp\left[-i\int d^4x \, \bar{c}\left(\frac{1}{g}\Box + f^{abc}\partial^\mu A^b_\mu\right)c\right]. \quad (3)$$

   The field $c$ is scalar in spite of being anti commuting. This violation of the spin-statistics theorem is allowed because the field does only propagate in intermediate loops. What are the ghost Feynman rules? Use the propagator from the last exercise to write down your generating functional of gauge fixed Yang-Mills with ghosts and in particular carefully derive the Feynman rule for the ghost-gauge vertex. *Hint: Use Fourier transforms to deal with the derivative in the interaction.* **5 pts.**

3. Write down the complete Lagrangian of Yang-Mills theory coupled to a fermion in the fundamental representation. In particular express $F^a_{\mu\nu}$ in terms of the potential and don't forget the gauge fixing or the ghost terms. **3 pts.**

4. Write down all the (non-ghost) Feynman rules for this theory. *Hint: In particular will have a three- and four-gluon vertex each with six terms (not carrying out the color sums).* **10 pts.**