Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 6
6^{th} December 2010
WS 10/11

Exercises Superstring Theory

Prof. Dr. Albrecht Klemm, Priv.-Doz. Dr. Stefan Förste

1 Strings with space-time supersymmetry

There are currently three main formulations of superstrings: The Ramond-Neveu-Schwarz (RNS), the Green-Schwarz (GS) and the pure spinor formalism. In the RNS formalism one studies maps from a two-dimensional supersymmetric worldsheet to a bosonic space-time. In the GS and the pure spinor formalism one considers maps from a two-dimensional bosonic worldsheet to a supersymmetric space-time. In the following we are concerned with the GS formulation of superstrings.

1.1 The D0-brane action

We consider the fields

$$X^\mu(\tau), \qquad \theta^A(\tau),$$

where $A = 1, \dots, N$ and N is the number of supersymmetries. SUSY acts according to

$$\delta\theta^A = \varepsilon^A, \qquad \delta X^\mu = \bar{\varepsilon}^A \Gamma^\mu \theta^A.$$

Introduce the supersymmetric combination

$$\Pi_0^\mu = \dot{X}^\mu - \bar{\theta}\Gamma^\mu\dot{\theta}.$$

Hence, a natural supersymmetric extension of the massive point particle action in flat Minkowski space-time is given by

$$S_1 = -m \int \sqrt{-\Pi_0 \cdot \Pi_0}\, d\tau.$$

1. How many entries has a Dirac spinor in D dimensions, D even? What is a Majorana spinor?
2. Given two Majorana spinors θ_1 and θ_2 prove that
$$\bar{\theta}_1 \Gamma_\mu \theta_2 = -\bar{\theta}_2 \Gamma_\mu \theta_1.$$
3. Check that the commutator of two supersymmetry transformations gives
$$[\delta_1, \delta_2]\theta^A = 0, \qquad [\delta_1, \delta_2] X^\mu = -2\bar{\varepsilon}_1^A \Gamma^\mu \varepsilon_2^A.$$
4. Show that Π_0^μ is invariant under the supersymmetry transformations.
5. Derive the equations of motion for X^μ and θ^A obtained from the action S_1.
6. Upon adding
$$S_2 = -m \int \bar{\theta}\Gamma_{11}\dot{\theta}\, d\tau$$
what is the equation of motion for θ. Notice, that $\Gamma_{11} = \Gamma_0\Gamma_1 \cdots \Gamma_9$. Further show, that
$$(P \cdot \Gamma + m\Gamma_{11})^2 = 0.$$

1.2 Kappa-symmetry

By adding the action S_2 to the point particle action, one gains a new symmetry, called κ-symmetry, which is a local fermionic symmetry. Without this symmetry there would be the wrong number of propagating fermionic degrees of freedom.

1. Show, that the action $S = S_1 + S_2$ is invariant under diffeomorphisms of the worldline.
2. Define $P_\pm = \frac{1}{2}(1 \pm \gamma)$ with $\gamma = \frac{\Gamma \cdot \Pi_0}{\sqrt{-\Pi_0^2}} \Gamma_{11}$. Show, that $P_\pm$ are projection operators.
3. Show, that the D0-brane action S is invariant under the transformations

$$\delta\bar{\Theta} = \bar{\kappa} P_-, \qquad \delta X^\mu = -\bar{\kappa} P_- \Gamma^\mu \Theta,$$

   where $\kappa(\tau)$ is an arbitrary Majorana spinor.
4. In order to have an appropriate massless limit of S, introduce an auxiliary field $e(\tau)$, similar as one does in the Polyakov action. Re-express the massive D0-brane action S with the auxiliary field $e(\tau)$ and find the massless limit. Verify the κ-symmetry of this massless D0-brane action.