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Theoretcal Physics

Exercise 8
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Exercises Superstring theory

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1 Warm-Ups

1. Explain the bosonics construction of the heterotic string?
2. What are the allowed gauge groups for the 16 extra bosons?

2 Basics of heterotic orbifolds

In this exercise we want to study orbifolds of the heterotic string. Orbifolds provide an interesting playground for string phenomenology as we want to discuss in this chapter for the case of $T^6/\mathbb{Z}_3$. However, as orbifolds are quite an intense topic and we can only cover some aspects we refer to the literature for further details. Most of the time we will just collect some facts about the construction.

1. What are the massless left and right movers of the heterotic string? Therefore discuss the spectrum of the heterotic string!
2. In order to make contact with the four-dimensional world, we compactify the heterotic string on a T^6. By counting the number of gravitini, determine the amount of supersymmetry.

   Therefore in order to make contact with phenomenology one needs compactification backgrounds in which the number of supersymmetry is reduced to $\mathcal{N} = 1$ supersymmetry. This is exactly achieved by an orbifold.

3. Now an orbifold of this compactification on T^6 consists out of a discrete symmetry group P, called the point group s.t. we compactify on T^6/P. Its elements are referred to as twists. Can you think of any restrictions that P has to satisfy? Taking into account the shifts, that generate the T^6, then one has

   $$T^6/P = \mathbb{R}^6/S,$$

   where S is the so called space group.

4. Focusing on the case of abelian groups, what are the two possibilities and what are their elements?
5. Modular invariance forces an embedding of the space group S into the gauge degrees of freedom $S \hookrightarrow G$, G is referred to as the gauge twisting group and can be realized by shifts V of the $E_8 \times E_8$ lattice. For the two cases discussed above, how does the embedding look

like? What ist the action of $g \in G$ on X^I, with $I = 1, \ldots, 16$.

Now a heterotic orbifold is given by: $\mathcal{O} = \mathbb{R}^6/S \otimes T_{E_8 \times E_8}/G$ and $S \otimes G$ is called the orbifold group.

6. What are fixed points of the T^6 under the action of the orbifold group? Consider the case of $T^6/\mathbb{Z}_3$. We denote the basis vectors by

$$e_1^a = \sqrt{2}, \quad e_2^a = \sqrt{2}e^{2\pi i/3}, \qquad a = 1, 2, 3$$

How many fix points does this orbifold have and determine them! For this you may introduce complex coordinates $Z^1 = X^1 + iX^2$ etc. such that the point group acts as

$$\vartheta^k : Z^a \mapsto e^{2\pi i k v^a} Z^a, \quad a = 1, 2, 3$$

where $v = \{v^a\}$ is the so called twist vector. Show that the fixed points are given by

$$\begin{aligned} Z_f^a &= 0, \\ Z_f^a &= \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{6}}, \\ Z_f^a &= \sqrt{\frac{2}{3}} i. \end{aligned} \tag{2.1}$$

7. Discuss in class what are possible further conditions on the orbifold group and discuss the constraints on the states of the orbifold. Especially discuss why, the twist vector has to satisfy

$$v^1 + v^2 + v^3 = 0$$

for $\mathcal{N} = 1$ supersymmetry in four dimensions. What happens in the twisted sector? For details about the construction of invariant states in the twisted vector see [3].

8. (homework) Discuss the orbifold $T^6/\mathbb{Z}_3$ in detail. Choose the so called standard embedding, i.e. $V = (v^1, v^2, v^3, 0^5)(0^8)$. Check that it satisfies the conditions discussed above. Show that the gauge group $E_8 \times E_8$ is broken to $E_6 \times SU(3) \times E_8$.

References

[1] L. J. Dixon, J. A. Harvey, C. Vafa *et al.*, "Strings on Orbifolds," Nucl. Phys. **B261** (1985) 678-686.

[2] L. J. Dixon, J. A. Harvey, C. Vafa *et al.*, "Strings on Orbifolds. 2.," Nucl. Phys. **B274** (1986) 285-314.

[3] P. K. S. Vaudrevange, "Geometrical Aspects of Heterotic Orbifolds", Diploma thesis, Bonn 2004, available under *http://www.th.physik.uni-bonn.de/nilles/db/thesis/vaudrevange.ps*