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Exercises Superstring Theory

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Dirac-Born-Infeld action

Born and Infeld proposed a nonlinear generalization of Maxwell theory in an attempt to eliminate the infinite classical self-energy of a charged point particle. Their proposed action reads

$$S_{\text{BI}} \sim \int \sqrt{-\det(\eta_{\alpha\beta} + kF_{\alpha\beta})}\, d^4\sigma, \tag{1}$$

where k is some constant.
In string theory a generalization of this action appears in the context of Dp-branes. The world-volume action of a Dp-brane includes the so-called DBI action

$$S_{\text{DBI}} = -T_{\text{D}p} \int d^{p+1}\sigma \sqrt{-\det(G_{\alpha\beta} + k\mathcal{F}_{\alpha\beta})}, \tag{2}$$

where $T_{\text{D}p}$ is the brane tension, $k = 2\pi\alpha'$, $G_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$ is the induced metric and $\mathcal{F}$ is the pullback of $F + B$ with $F = dA$ the Maxwell field strength and B the B-field. Note, that this is only the bosonic part of the action. Further, we will work with a vanishing B-field $B = 0$ and flat spacetime $g_{\mu\nu} = \eta_{\mu\nu}$.

Warm-Up

1. Recall, that the classical self-energy of an electrically charged point particle in Maxwell theory is infinite.

2. Show, that the Born-Infeld action (1) gives a finite classical self-energy for an electrically charged point particle. Hint: show that the solution of the equations of motion with a point particle of electrical charge e at the origin is given by $E_r = F_{rt} = \frac{e}{4\pi\sqrt{r^4+r_0^4}}$, $r_0^2 = ke$.

3. To see evidence that a DBI form is required by string theory consider the two-dimensional D1-brane case of (2) and suppose that the spatial direction is a circle of radius R. By T-duality deduce that the DBI form and $k = 2\pi\alpha'$ are indeed required to obtain the correct form of the D0-brane action.

The DBI action

1. Expand (2) up to fourth order in k and show that the quadratic term gives the Maxwell action. Use this to argue, that $T_{\text{D}p} \sim g_S^{-1}$, where g_S is the closed string coupling constant.

2. Consider the DBI action (2). Derive the equation of motion for the gauge field. Expand this equation in powers of k to obtain the leading correction to the usual Maxwell field equation of electrodynamics in the absence of sources.

3. Static gauge consists of using the diffeomorphism symmetry on the world-volume to identify $p+1$ of the spacetime coordinates X^μ with σ^α. The remaining X^μ are labelled as $2\pi\alpha'\Phi^i$. Write down the static gauge DBI action for a Dp-brane. What types of charged solutions is this theory expected to have? What are their physical interpretations?

4. Consider the static gauge DBI action. Find the action of a D3-brane in spherical coordinates (t, r, θ, φ) for the special case in which the only nonzero fields are $A_t(r)$ and one scalar $\Phi(r)$. Obtain the equations of motion for those fields. Find the solution of the equations of motion that corresponds to an electric charge at the origin and deduce the profile of the string that is attached to the D3-brane.

5. Repeat above analysis with nonvanishing field $A_\varphi(r)$ instead of $A_t(r)$.