Physikalisches Institut
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Exercise 4
06.11.2012
WT 12/13

Exercises String Theory

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1 State degeneracy of the open string — 3 Points

The mass operator in open bosonic string theory is given by

$$M^2 = -p_\mu p^\mu = 2p^+p^- - \sum_{i=1}^{24} p_i^2 = 2\frac{(N-1)}{l_s^2}, \quad N = \sum_{i=1}^{24} \alpha^i_{-n}\alpha^i_n. \tag{1.1}$$

1. Show that the states corresponding to N have the form

$$\alpha^{i_1}_{-n_1}\alpha^{i_2}_{-n_2}\dots\alpha^{i_k}_{-n_k}|0,k\rangle, \quad \sum_{l=1}^{k} n_l = N. \tag{1.2}$$

2. Show that the number of states d_N corresponding to N, is given by the coefficient of q^N in

$$\prod_{n=1}^{\infty}(1-q^n)^{-24}. \tag{1.3}$$

   Hint: First show that

$$\prod_{n=1}^{\infty}(1-q^n)^{-1}. \tag{1.4}$$

   describes the degeneracy of one oscillator at level N.

2 The Hagedorn temperature — 1 Points

For large N, d_N can be approximated by

$$d_N \simeq \frac{1}{\sqrt{2}} N^{-\frac{24}{4}} \exp(4\pi\sqrt{N}) \tag{2.1}$$

The entropy is given by

$$S(E) = k\log(d_N) \tag{2.2}$$

Compute the Hagedorn temperature using

$$\frac{1}{kT_H} = \frac{1}{k}\frac{\partial S}{\partial E} \tag{2.3}$$

and interpret the result physically, i.e. why does the temperature stay constant if we increase the energy?

3 Unoriented open strings **6 Points**

We recall the expansion of the open string

$$\begin{aligned}
X^I(\tau,\sigma) &= x_0^I + \sqrt{2\alpha'}\alpha_0^I\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^I\cos(n\sigma)e^{-in\tau} \\
X^+(\tau,\sigma) &= 2\alpha' p^+\tau \\
X^-(\tau,\sigma) &= x_0^- + \sqrt{2\alpha'}\alpha_0^- + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^- e^{-in\tau}\cos(n\sigma) \\
\sqrt{2\alpha'}\alpha_n^- &= \frac{1}{2p^+}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I. \qquad (3.1)
\end{aligned}$$

1. Draw a picture of the open string $X^\mu(\tau,\sigma)$ for τ fixed. Consider the open string $X^\mu(\tau,\pi-\sigma)$ for the same τ and draw a picture as well. How are the orientations related?

2. We introduce the orientation twist operator Ω such that

$$\Omega X^I(\tau,\sigma)\Omega^{-1} = X^I(\tau,\pi-\sigma), \quad \Omega x_0^-\Omega^{-1} = x_0^-, \quad \Omega p^+\Omega^{-1} = p^+. \qquad (3.2)$$

   How is Ω related to the world-sheet parity operator?

3. Use the open string expansion to calculate

$$\Omega x_0^I\Omega^{-1}, \quad \Omega\alpha_0^I\Omega^{-1}, \quad \Omega\alpha_n^I\Omega^{-1} \quad (n\neq 0). \qquad (3.3)$$

4. Show that

$$\Omega X^-(\tau,\sigma)\Omega^{-1} = X^-(\tau,\pi-\sigma). \qquad (3.4)$$

   In addition show that the open string Hamiltonian

$$H = L_0 - 1 \qquad (3.5)$$

   is invariant under Ω.

5. Assume that the ground states are twist invariant

$$\Omega|p^+,p\rangle = \Omega^{-1}|p^+,p\rangle = |p^+,p\rangle. \qquad (3.6)$$

6. List the open string states for $N\leq 3$ and determine the twist eigenvalues. Show that

$$\Omega = (-1)^N. \qquad (3.7)$$

7. A state is called unoriented, if it is invariant under Ω. Which states do appear in the open, unoriented spectrum? What about the tachyon in particular? Can you in analogy to (1.4) give a function that counts the number of states of the unoriented open string?