

## Exercises String Theory

Prof. Dr. Albrecht Klemm

### 1 State degeneracy of the open string

3 Points

The mass operator in open bosonic string theory is given by

$$M^2 = -p_\mu p^\mu = 2p^+ p^- - \sum_{i=1}^{24} p_i^2 = 2 \frac{(N-1)}{l_s^2}, \quad N = \sum_{i=1}^{24} \alpha_{-n}^i \alpha_n^i. \quad (1.1)$$

1. Show that the states corresponding to  $N$  have the form

$$\alpha_{-n_1}^{i_1} \alpha_{-n_2}^{i_2} \dots \alpha_{-n_k}^{i_k} |0, k\rangle, \quad \sum_{l=1}^k n_l = N. \quad (1.2)$$

2. Show that the number of states  $d_N$  corresponding to  $N$ , is given by the coefficient of  $q^N$  in

$$\prod_{n=1}^{\infty} (1 - q^n)^{-24}. \quad (1.3)$$

Hint: First show that

$$\prod_{n=1}^{\infty} (1 - q^n)^{-1}. \quad (1.4)$$

describes the degeneracy of one oscillator at level  $N$ .

### 2 The Hagedorn temperature

1 Points

For large  $N$ ,  $d_N$  can be approximated by

$$d_N \simeq \frac{1}{\sqrt{2}} N^{-\frac{24}{4}} \exp(4\pi\sqrt{N}) \quad (2.1)$$

The entropy is given by

$$S(E) = k \log(d_N) \quad (2.2)$$

Compute the Hagedorn temperature using

$$\frac{1}{kT_H} = \frac{1}{k} \frac{\partial S}{\partial E} \quad (2.3)$$

and interpret the result physically, i.e. why does the temperature stay constant if we increase the energy?

### 3 Unoriented open strings

6 Points

We recall the expansion of the open string

$$\begin{aligned}
 X^I(\tau, \sigma) &= x_0^I + \sqrt{2\alpha'} \alpha_0^I \tau + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^I \cos(n\sigma) e^{-in\tau} \\
 X^+(\tau, \sigma) &= 2\alpha' p^+ \tau \\
 X^-(\tau, \sigma) &= x_0^- + \sqrt{2\alpha'} \alpha_0^- + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^- e^{-in\tau} \cos(n\sigma) \\
 \sqrt{2\alpha'} \alpha_n^- &= \frac{1}{2p^+} \sum_{p \in \mathbb{Z}} \alpha_{n-p}^I \alpha_p^I.
 \end{aligned} \tag{3.1}$$

1. Draw a picture of the open string  $X^\mu(\tau, \sigma)$  for  $\tau$  fixed. Consider the open string  $X^\mu(\tau, \pi - \sigma)$  for the same  $\tau$  and draw a picture as well. How are the orientations related?
2. We introduce the orientation twist operator  $\Omega$  such that

$$\Omega X^I(\tau, \sigma) \Omega^{-1} = X^I(\tau, \pi - \sigma), \quad \Omega x_0^- \Omega^{-1} = x_0^-, \quad \Omega p^+ \Omega^{-1} = p^+. \tag{3.2}$$

How is  $\Omega$  related to the world-sheet parity operator?

3. Use the open string expansion to calculate

$$\Omega x_0^I \Omega^{-1}, \quad \Omega \alpha_0^I \Omega^{-1}, \quad \Omega \alpha_n^I \Omega^{-1} \quad (n \neq 0). \tag{3.3}$$

4. Show that

$$\Omega X^-(\tau, \sigma) \Omega^{-1} = X^-(\tau, \pi - \sigma). \tag{3.4}$$

In addition show that the open string Hamiltonian

$$H = L_0 - 1 \tag{3.5}$$

is invariant under  $\Omega$ .

5. Assume that the ground states are twist invariant

$$\Omega |p^+, p\rangle = \Omega^{-1} |p^+, p\rangle = |p^+, p\rangle. \tag{3.6}$$

6. List the open string states for  $N \leq 3$  and determine the twist eigenvalues. Show that

$$\Omega = (-1)^N. \tag{3.7}$$

7. A state is called unoriented, if it is invariant under  $\Omega$ . Which states do appear in the open, unoriented spectrum? What about the tachyon in particular? Can you in analogy to (1.4) give a function that counts the number of states of the unoriented open string?