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Exercise 6
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Exercises String Theory

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1 The propagator of the free boson **2 points**

In two dimensions, the free boson has the following Euclidean action

$$S = \frac{1}{2} g \int d^2x \big(\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2\big). \tag{1.1}$$

1. Show that the propagator

$$K(x, y) =< \phi(x)\phi(y) > \tag{1.2}$$

obeys

$$g\big(-\partial_x^2 + m^2\big) K(x, y) = \delta(x - y). \tag{1.3}$$

2. Show that $K(x, y)$ only depends on $r = |x - y|$ and that

$$K(r) = -\frac{1}{2\pi g} \ln r. \tag{1.4}$$

Hint: Integrate (1.3) and derive an ordinary differential equation for $K(r)$.

3. Rewrite $K(r)$ in complex coordinates. Show that

$$\begin{aligned} < \partial_z \phi(z)\phi(w) > &= -\frac{1}{4\pi g} \frac{1}{z - w} \\ < \partial_z \phi(z)\partial_w \phi(w) > &= -\frac{1}{4\pi g} \frac{1}{(z - w)^2} \end{aligned} \tag{1.5}$$

2 The ghost system **4 points**

The action for the ghost system is given by

$$S = \frac{1}{2} \int d^2x b_{\mu\nu} \partial^\mu c^\nu. \tag{2.1}$$

Both, c and $b_{\mu\nu}$ are anti-commuting and $b_{\mu\nu}$ is a traceless tensor. The propagator is found to be

$$< b(z)c(w) >= \frac{1}{\pi g} \frac{1}{z - w}. \tag{2.2}$$

1. Determine the correlators

$$< b(z)\partial_w c(w) >, \quad < \partial_z b(z)c(w) > \quad \text{and} \quad < \partial_z b(z)\partial_w c(w) > . \tag{2.3}$$

The normal ordered energy-momentum tensor of the system is given by

$$T(z) = \pi g : (2\partial c b + c\partial b) : \tag{2.4}$$

2. Compute the OPE's $T(z)c(w)$ and $T(z)b(w)$ using Wicks's theorem and read of the corresponding conformal dimensions.

3. Compute the OPE of T with itself

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{hT(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)} \tag{2.5}$$

   and determine the central charge as well as the conformal weight.

3 Vertex operators **4 points**

Bosonic Vertex operators are defined by

$$V_\alpha(z,\bar{z}) =: e^{i\alpha\phi(z,\bar{z})} : \tag{3.1}$$

where ϕ is a free bosonic field.

1. Show that the OPE of $\partial\phi$ with V_α is

$$\partial\phi(z)V_\alpha(w,\bar{w}) = -\frac{i\alpha}{4\pi g}\frac{V_\alpha(w,\bar{w})}{z-w} + ... \tag{3.2}$$

2. Show that the OPE of V_α with T is given as

$$T(z)V_\alpha(w,\bar{w}) = \frac{\alpha^2}{8\pi g}\frac{V_\alpha(w,\bar{w})}{(z-w)^2} + \frac{\partial_w V_\alpha(w,\bar{w})}{z-w} + \dots \tag{3.3}$$

   What is the conformal dimension?

3. Show that for two free fields one has

$$: e^{a\phi_1} :: e^{b\phi_2} :=: e^{a\phi_1 + b\phi_2} : e^{ab<\phi_1\phi_2>}. \tag{3.4}$$

   Use this result to compute the OPE of two Vertex operators.