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Exercise Sheet 10
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Exercises in Stringtheory

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http://www.th.physik.uni-bonn.de/klemm/strings1617/

1 String theory on the orbifold $S^1/\mathbb{Z}_2$ (10 pts.)

We again look at bosonic string theory compactified on a circle, but now we truncate the theory by dividing out the $\mathbb{Z}_2$ symmetry that acts as

$$R_{25}: \, X^{25} \to -X^{25}\,. \tag{1}$$

This procedure is called orbifolding and applications in string theory are ubiquitous. In this exercise you will see how a truncation of the spectrum and the addition of twisted sectors magically lead to a partition function that is again modular invariant.

1. Draw a picture to show which points are identified under this symmetry and how the resulting geometry looks like. How many fixed points are there?
2. How does R_{25} act on the modes of the expansion of the string theory on S^1?
3. Which massless states do survive a projection onto the invariant states? Does the Tachyon survive?

Analogously to the winding states that arose after compactifying on a circle, the orbifold projection allows for a second type of boundary condition which gives rise to the twisted states

$$X^{25}(\sigma + 2\pi, \tau) = -X^{25}(\sigma, \tau) + 2\pi w R\,. \tag{2}$$

4. Draw a picture of such a twisted string configuration for different winding numbers.
5. Solve the Laplace equation

$$\left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2}\right) X^{25}(\tau, \sigma) = 0\,, \tag{3}$$

   respecting the boundary condition (2).

Both the truncation of the spectrum to states invariant under R_{25}, as well as the inclusion of twisted states affect the partition function. The total orbifold partition function is given by

$$Z_{\text{orb}} = \text{Tr}_{\text{untwisted}}\left(\frac{1+R_{25}}{2} q^{L_0 - \frac{1}{24}} \bar{q}^{\bar{L}_0 - \frac{1}{24}}\right) + \text{Tr}_{\text{twisted}}\left(\frac{1+R_{25}}{2} q^{L_0 - \frac{1}{24}} \bar{q}^{\bar{L}_0 - \frac{1}{24}}\right)\,. \tag{4}$$

6. Evaluate the partition function in the untwisted sector. Is it modular invariant? What do you conclude?

7. How is the trace over the twisted sector evaluated in principle? (You are not asked to do this explicitly.)

8. The whole partition function is given by

$$Z_{\text{orb}}(R,q) = \frac{1}{2}\left(Z_{S^1}(R,q) + 2\left|\frac{\eta(q)}{\theta_{10}(0,q)}\right|^2 + 2\left|\frac{\eta(q)}{\theta_{01}(0,q)}\right|^2 + 2\left|\frac{\eta(q)}{\theta_{00}(0,q)}\right|^2 \right). \tag{5}$$

9. Check that this is modular invariant. What is the interpretation of the factor 2?