Physikalisches Institut
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Theoretische Physik

Exercise Sheet 5
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Exercises in Stringtheory

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http://www.th.physik.uni-bonn.de/klemm/strings1617/

1 Two-point function (2 points)

We consider the two-point function

$$G(z_1, z_2) = \langle \Phi_1(z_1)\Phi_2(z_2)\rangle \,, \tag{1}$$

of two primary fields Φ_i with conformal weight Δ_i, $i = 1, 2$ in a two-dimensional conformal field theory. Recall that under infinitesimal transformations

$$z \to z + \epsilon \,, \tag{2}$$

the fields transform as

$$\delta_\epsilon \Phi_i(z) = \left[\epsilon(z)\partial + \Delta_i \partial\epsilon(z)\right] \Phi_i \,, \quad i = 1, 2 \,. \tag{3}$$

1. Show that conformal invariance implies
$$\left[\epsilon(z_1)\partial_1 + \Delta_1\partial\epsilon(z_1) + \epsilon(z_2)\partial_2 + \Delta_2\partial\epsilon(z_2)\right] G(z_1, z_2) = 0 \,. \tag{4}$$
2. By setting $\epsilon = 1$, show that $G(z_1, z_2)$ is a function of $x = z_1 - z_2$ only.
3. By setting $\epsilon = z$, show that $G(z_1, z_2)$ takes the form
$$G(x) = \frac{C}{x^{\Delta_1+\Delta_2}} \,, \tag{5}$$
where C is a constant.
4. By setting $\epsilon = z^2$, show that $G(z_1, z_2)$ vanishes unless $\Delta_1 = \Delta_2$.

2 Global conformal transformations (2 points)

As is known from the lecture, the invariance under global conformal transformations can be encoded in the following differential equations imposed on the correlators of primary fields:

$$\begin{aligned}\sum_i \partial_{w_i}\langle\phi_1(w_1)\cdots\phi_n(w_n)\rangle &=0\\ \sum_i (w_i\partial_{w_i} + \Delta_i)\langle\phi_1(w_1)\cdots\phi_n(w_n)\rangle &=0\\ \sum_i (w_i^2 + w_i\Delta_i)\langle\phi_1(w_1)\cdots\phi_n(w_n)\rangle &=0\end{aligned} \tag{6}$$

Show explicitly for the two- and the three-point function that these relations are indeed satisfied.

3 Operator product expansion (2 points)

Starting with

$$\delta_{\epsilon,\bar{\epsilon}}\phi(z,\bar{z}) = [Q_\epsilon + Q_{\bar{\epsilon}}, \phi(z,\bar{z})] \,, \quad Q_\epsilon = \oint_{C_0} \frac{dz}{2\pi i}\epsilon(z)T(z)\,, \tag{7}$$

and

$$\delta_{\epsilon,\bar{\epsilon}}\phi(z,\bar{z}) = (\Delta\partial\epsilon + \bar{\Delta}\bar{\partial}\epsilon + \epsilon\partial + \bar{\epsilon}\bar{\partial})\phi(z,\bar{z})\,, \tag{8}$$

show that

$$T(z)\phi(w,\bar{w}) = \frac{\Delta}{(z-w)^2}\phi(w,\bar{w}) + \frac{1}{z-w}\partial_w\phi(w,\bar{w}) + \text{reg.} \tag{9}$$

In the last expression, radial ordering is understood. Hint: Use the operator product expansion

$$\phi_i(z,\bar{z})\phi_j(w,\bar{w}) = \sum_k c_{ijk}(z-w)^{\Delta_k-\Delta_i-\Delta_j}(\bar{z}-\bar{w})^{\bar{\Delta}_k-\bar{\Delta}_i-\bar{\Delta}_j}\,, \tag{10}$$

and the Cauchy formula

$$\oint_{C_w} \frac{dz}{2\pi i}\frac{f(z)}{(z-w)^n} = \frac{1}{(n-1)!}f^{(n-1)}(w)\,. \tag{11}$$