Physikalisches Institut
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Theoretische Physik

Exercise Sheet 6
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Exercises in Stringtheory

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`http://www.th.physik.uni-bonn.de/klemm/strings1617/`

1 The propagator of the free boson (2 points)

In two dimensions, the free boson has the following Euclidean action

$$S = \frac{1}{2} g \int d^2x \left(\partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 \right) . \tag{1}$$

1. Show that the propagator

$$K(x, y) = \langle \phi(x) \phi(y) \rangle , \tag{2}$$

obeys

$$g(-\partial_x^2 + m^2) K(x, y) = \delta(x - y) . \tag{3}$$

2. Show that $K(x, y)$ only depends on $r = |x - y|$ and that

$$K(r) = -\frac{1}{2\pi g} \ln(r) . \tag{4}$$

Hint: Integrate (3) and derive an ordinary differential equation for $K(r)$.

3. Rewrite $K(r)$ in complex coordinates. Show that

$$\begin{aligned} \langle \partial_z \phi(z) \phi(w) \rangle &= -\frac{1}{4\pi g} \frac{1}{z - w} , \\ \langle \partial_z \phi(z) \partial_w \phi(w) \rangle &= -\frac{1}{4\pi g} \frac{1}{(z - w)^2} . \end{aligned} \tag{5}$$

2 The ghost system (4 points)

The action for the ghost system is given by

$$S = \frac{1}{2} \int d^2x b_{\mu\nu} \partial^\mu c^\nu . \tag{6}$$

Both c and $b_{\mu\nu}$ are anti-commuting and $b_{\mu\nu}$ is a traceless tensor. The propagator is found to be

$$\langle b(z) c(w) \rangle = \frac{1}{\pi g} \frac{1}{z - w} . \tag{7}$$

1. Determine the correlators

$$\langle b(z)\partial_w c(w)\rangle\,, \quad \langle \partial_z b(z)c(w)\rangle \quad \text{and} \quad \langle \partial_z b(z)\partial_w c(w)\rangle\,. \tag{8}$$

The normal ordered energy-momentum tensor of the system is given by

$$T(z) = \pi g : (2\partial c b + c\partial b) : . \tag{9}$$

2. Compute the OPE's $T(z)c(w)$ and $T(z)b(w)$ using Wick's theorem and read off the corresponding conformal dimensions.

3. Compute the OPE of T with itself

$$T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{hT(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}\,, \tag{10}$$

and determine the central charge as well as the conformal weight of T.

3 Vertex operators (4 points)

Bosonic Vertex operators are defined by

$$V_\alpha(z,\bar{z}) =: e^{i\alpha\phi(z,\bar{z})} :, \tag{11}$$

where ϕ is a free bosonic field.

1. Show that the OPE of $\partial\phi$ with V_α is

$$\partial\phi(z)V_\alpha(w,\bar{w}) = -\frac{i\alpha}{4\pi g}\frac{V_\alpha(w,\bar{w})}{z-w}\,. \tag{12}$$

2. Show that the OPE of V_α with T is given as

$$T(z)V_\alpha(w,\bar{w}) = \frac{\alpha^2}{8\pi g}\frac{V_\alpha(w,\bar{w})}{(z-w)^2} + \frac{\partial V_\alpha(w,\bar{w})}{z-w}\,. \tag{13}$$

What is the conformal dimension?

3. Show that for two free fields one has

$$: e^{a\phi_1} :: e^{b\phi_2} :=: e^{a\phi_1 + b\phi_2} : e^{ab\langle\phi_1\phi_2\rangle}\,. \tag{14}$$

Use this result to compute the OPE of two Vertex operators.