Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise Sheet 1
16.10.2018
WS 18/19

Exercises in Superstring Theory

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Sheets & Organization: César Fierro-Cota

The sheets will be collected during the first lecture of the week, unless differently specified. There you will be provided with a new sheet.

The rooms and tutors for the exercise classes are as follows:

Thursday,	14-16	Serminarroom I,	BCTP	Rongvoram Nivesvivat
Friday,	16-18	Serminarroom II,	BCTP	Reza Safari

Tutorials start on wednesday, 18.10.2018. In the first week you will solve a presence exercise.

If you have any questions feel free to contact me under fierro@th.physik.uni-bonn.de. You can find the addresses of the tutors on the course website:

`http://www.th.physik.uni-bonn.de/klemm/strings1819/`

–Presence exercises–

1 The relativistic point particle

We consider the action for a relativistic point particle

$$S_{pp} = -m \int ds = -m \int \sqrt{-\eta_{\mu\nu}\dot{X}^\mu \dot{X}^\nu} d\tau \,, \tag{1}$$

where τ parameterizes the worldline of the particle and $\eta_{\mu\nu}$ is the D-dimensional Minkowski metric.

1. Show that the action is invariant under Poincaré transformations.
2. Show that the action is invariant under reparametrizations $\tau \to \tau'(\tau)$.
3. Show that

$$p^\mu = \frac{m\dot{X}^\mu}{\sqrt{-\eta_{\nu\rho}\dot{X}^\nu \dot{X}^\rho}} \,, \tag{2}$$

   is a conserved quantity. Do this once by evaluating the Euler Lagrange equations and once by exploiting the symmetry $X^\mu \to X^\mu + b^\mu$. Hint: For the latter, assume that b^μ depends on τ to do a partial integration and only then restrict to constant b^μ.
4. Why is this action inappropriate to describe massless particles?

5. Show that

$$S_e = -\frac{1}{2}\int d\tau e\left(-\frac{1}{e^2}\dot{X}^\mu\dot{X}^\nu\eta_{\mu\nu} + m^2\right) , \quad (3)$$

is equivalent to the action (1). Hint: Integrate out e.

6. Explain the statement "We have coupled the particle to worldline gravity". What kind of field is e?

7. Show the invariance of the new action (3) under reparametrizations of τ. How does e transform?

2 The Nambu-Goto action versus the Polyakov action

The Nambu-Goto action for a string is given by

$$S_{NG} = -T\int d^2\sigma\sqrt{-\det\left(\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\right)} . \quad (4)$$

Here σ^α, $\alpha = 0, 1$ label the worldsheet time τ and space σ.

1. Write down explicitly the action (4), i.e. without referring to σ^α, but to τ and σ instead.

2. What is the geometric interpretation of this action?

3. Show that the Polyakov action

$$S_P = -\frac{T}{2}\int d^2\sigma\sqrt{-h}\left(h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X^\nu\eta_{\mu\nu}\right) , \quad (5)$$

is equivalent to the Nambu-Goto action.