Exercises Advanced Topics in String Theory
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1 Basics of Lie algebra technology

1.1 Questions

- What is a Lie algebra?

- What is a Cartan generator? How many are there?

- What is the Killing form?

- What is a simple respectively a semi-simple algebra?

- What is a weight?

- What is a root? What is a positive and what a simple root?

- What is a highest weight?

- What is the "master formula"?

- What are fundamental weights and fundamental representations?

- What is the Cartan matrix and how is it related to the Dynkin diagram?

1.2 SU(3)

The simple roots of $SU(3)$ are given as

$$\alpha^1 = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right), \quad \alpha^2 = \left(\tfrac{1}{2}, -\tfrac{\sqrt{3}}{2}\right) \tag{1.1}$$

Construct the fundamental as well as the anti-fundamental representation from this.

2 Symmetry-Enhancement on the torus

We consider the compactification of the string on a D-dimensional torus, that is specified as

$$T^D = \mathbb{R}^D/2\pi\Lambda_D, \quad \Lambda_D = \bigoplus_{i=1}^{D} \mathbb{Z}e_i \tag{2.1}$$

Here the e_i denote the basis vectors of the lattice. The mass formula is given as

$$\alpha' m_{L,R}^2 = 2(N_L + N_R - 2) + \frac{1}{2\alpha'}n^T(g - bg^{-1}b)n + n^T bg^{-1}m \pm n^T m + 2(N_{L,R} - 1) \tag{2.2}$$

- Explain the quantities that appear in the mass formula. How are they defined in terms of lattice quantities and background fields?

- What is the spectrum obtained for generic values of the moduli?

- Consider the case of a two-dimensional lattice given by the root lattice of $SU(3)$. Which gauge symmetry enhancement do you encounter if you correctly tune the moduli?