Physikalisches Institut
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Theoretische Physik

Exercise 1
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Exercises on Advanced Topics in String Theory

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http://www.th.physik.uni-bonn.de/klemm/strings2_19/
PRESENCE EXERCISE

1 Mini-supergravity

The purpose of this exercise is to consider a simple theory, where **supersymmetry is local**, i.e. where the infinitesimal Grassmann ϵ parameter depends on spacetime. For simplicity, we borrow the (0+1) worldline gravity theory, which describes a particle propagating on a D-dimensional spacetime. Here $x : I \to x(I) \simeq \mathbb{R}^{1,D-1}$, with supersymmetric partner given by ψ, e is an auxilary field (einbein) and its fermionic partner we denote it by χ. The action takes the form

$$S_p|_{SUGRA} = \int_I d\tau \left(\frac{1}{2e}\dot{x}^\mu \dot{x}_\mu + \frac{i}{e}\dot{x}^\mu \psi_\mu \chi - i\psi^\mu \dot{\psi}_\mu \right). \qquad (1)$$

(a) Show that (1) is invariant under reparametrizations, i.e. under the following infinitesimal transformations with parameter $\xi(\tau)$ given by

$$\delta x^\mu = \xi \dot{x}^\mu \,, \quad \delta\psi^\mu = \xi\dot{\psi}^\mu \,, \quad \delta e = \frac{d}{d\tau}(\xi e)\,, \quad \delta\chi = \frac{d}{d\tau}(\xi\chi)\,. \qquad (2)$$

(b) Show that (1) is invariant under local supersymmetry transformations, which read

$$\delta x^\mu = i\epsilon\psi^\mu \,, \quad \delta\psi^\mu = \frac{1}{2e}\Big(\dot{x}^\mu - i\chi\psi^\mu\Big)\epsilon \,, \quad \delta e = -i\chi\epsilon \,, \quad \delta\chi = \dot{\epsilon}\,. \qquad (3)$$

(c) Show that in the gauge $e = 1, \chi = 0$, one obtains the following action

$$S_p|_{SUSY} = \int d\tau \left(\frac{1}{2}\dot{x}^\mu \dot{x}_\mu - i\psi^\mu \dot{\psi}_\mu \right). \qquad (4)$$

Moreover show that the action is invariant under global supersymmetry transformations

$$\delta x^\mu = i\epsilon\psi^\mu \,, \quad \delta\psi^\mu = \frac{1}{2}\epsilon\dot{x}^\mu \,. \qquad (5)$$

Here ϵ is now an infinitesimal real constant Grassmann parameter.

(d) Derive the constraint equation for the gauge-fixed theory, i.e. the e.o.m. for e and χ.