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Exercise 2
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Exercises on Advanced Topics in String Theory

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http://www.th.physik.uni-bonn.de/klemm/strings2_19/
PRESENCE EXERCISE

The fermionic string spectrum from LCQ

Let us first recall the physical state conditions are

$$(L_n - a_{\mathrm{NS,R}}\delta_{n,0})|\psi\rangle = 0 \quad \text{for} \quad n \geq 0, \tag{1}$$

and

$$G_r|\psi\rangle = 0, \tag{2}$$

where $r \in \mathbb{Z} \geq 0$ for the Ramond boundary condition and $r \in \mathbb{Z} + \frac{1}{2} > 0$ for the Neveu-Schwarz sector.

1. As you have already seen from the lecture, one can argue that $a_{\mathrm{R}} = 0$ from $L_0 \sim G_0^2$, using the super-Virasoro algebra. Obtain the same result from computing the normal ordering constant of L_0 in the R sector.

2. Consider the uniqueness of the ground state in the NS and R sector. Explain why there is a degeneracy in the Ramond vacuum.

3. From the first level excitation in the NS sector, show that

$$a_{\mathrm{NS}} = \frac{1}{2} \quad \text{and} \quad d = 10, \tag{3}$$

   where d is the dimension of the target space.

4. Compute the mass of the second excitation level of the NS sector and show that they can be embedded in the tensor representation of $SO(9)$.

5. Let us define the operator:

$$G = \begin{cases} (-1)^{F+1} & \text{for The NS sector} \\ 16\Gamma_2 \dots \Gamma_9 (-1)^F & \text{for The R sector} \end{cases} \tag{4}$$

   where $\Gamma_2, \dots \Gamma_9$ are the Dirac matrices in the transverse directions and F is the worldsheet-fermion number operator given by

$$F = \begin{cases} \sum_{r=\frac{1}{2}} b_{-r} b_r & \text{for The NS sector} \\ \sum_{r=1} b_{-r} b_r & \text{for The R sector} \end{cases} \tag{5}$$

   Compute the mass of the lowest level of NS$\pm$ and R$\pm$ sector, where $\pm$ denotes the eigenvalue of G.

6. Which one of NS$\pm$ and R$\pm$ cannot be combined with the others to build a closed string theory? *Hint: Recall the matching condition*

Type IIA/B spectrum

To obtain a consistent closed string theory, one then requires that the eigenvalue of G on every state in the Hilbert space to be + for the NS sector and $\pm$ for the R sector. This is called the Gliozzi-Scherk-Olive (GSO) projection.

1. Write down the two inequivalent closed string theories after applying the GSO projection to the Hilbert space. These are the so-called type IIA/B superstring theory.

2. Construct the massless spectrum of the type IIA/B superstring theory. The following might be useful

$$
\begin{aligned}
8_V \otimes 8_V &= 1 \oplus 28 \oplus 35_V \\
8_V \otimes 8_S &= 8_C \oplus 56_C \\
8_V \otimes 8_C &= 8_S \oplus 56_S \\
8_C \otimes 8_S &= 8_V \oplus 56_V \\
8_C \otimes 8_C &= 1 \oplus 28 \oplus 35_V,
\end{aligned}
\tag{6}
$$

   where V, S, and C denote the vector, spinor, and co-spinor of $SO(8)$, respectively.