Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 5
9th May 2019
SS 2019

Exercises on Advanced Topics in String Theory

Prof. Dr. Albrecht Klemm, César Fierro-Cota, Rongvoram Nivesvivat

http://www.th.physik.uni-bonn.de/klemm/strings2_19/
PRESENCE EXERCISE

Open-closed duality & branes I

Consider two parallel Dp-branes at positions $X^\mu = 0$ and $X^\mu = Y^\mu$. These two objects feel each other's presence by exchanging clodsed strings as shown in figure 1.

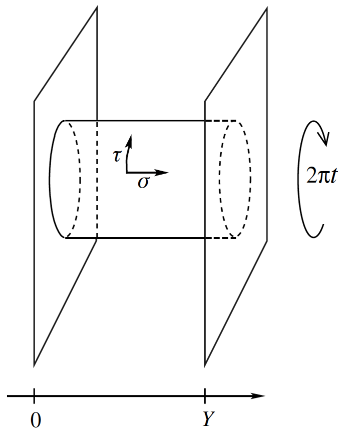

Figure 1: Exchange of a closed string between two D-branes. This is equivalent to a vacuum loop of an open string with one end on each D-brane.

Our task here will be obtaining the tension T_p of a Dp-brane. For simplicity, we consider such a computation for bosonic strings first. In section II we generalize this result for Type II strings. We take as a initial point the calculation of the one-loop amplitude $\mathcal{A}$ for an open string stretching between two Dp-branes. The theory is defined on the infinite strip with $\sigma \in [0, \pi]$ and $\tau \in \mathbb{R}$. For the definition of the one-loop partition function, we make the time coordinate τ periodic and consequently obtain the topology of a cylinder. Denote $t \in \mathbb{R}_{\geq 0}$ the modular parameter of the cylinder describing inequivalent cylinders given by $\{(\tau, \sigma) : 0 \leq t \leq \pi, 0 \leq \tau \leq 2\pi t\}$.

1. Show that

$$\mathcal{A} = V_{p+1} \int_0^\infty \frac{dt}{t} \frac{1}{(8\pi^2\alpha' t)^{-\frac{(p+1)}{2}}} \exp\Big(-\frac{t}{2\pi\alpha'} Y^2\Big) \frac{1}{\eta^{24}(it)} \,. \tag{1}$$

2. Relate the modulus of the cylinder due to the one loop open string vacuum $\mathcal{A}\big|_{\text{open}}$ with that of the tree level closed string amplitude $\mathcal{A}\big|_{\text{closed}}$. Using a modular transformation, obtain $\mathcal{A}\big|_{\text{closed}}$.

3. Show that
$$\mathcal{A}\Big|_{\text{massless}} = V_{p+1}\frac{3\pi}{2^7}(4\pi^2\alpha')^{11-p}G_{25-p}\Big(|Y|\Big)\,. \tag{2}$$

4. Making use of the following subsection, obtain the tension T_p of the corresponding Dp-brane.
$$T_p = \frac{\sqrt{\pi}}{16\kappa_0}(4\pi^2\alpha')^{\frac{(11-p)}{2}}\,. \tag{3}$$

A background field interlude (Homework)

In the following, we do a field theory calculation to work out the amplitude for the exchange of the graviton and dilaton between a pair of parallel Dp-branes. Consider the action for a D-brane as follow

$$S = S_{\text{bulk}} + S_p\,, \tag{4}$$

where

$$S_{\text{bulk}} = \frac{1}{2\kappa_0}\int_{M_D}\sqrt{-G}e^{-2\Phi}\Big(\mathcal{R} + 4\nabla_\mu\Phi\nabla^\mu\Phi\Big)\,, \tag{5}$$

and

$$S_p = -T_p\int_{\Sigma_{p+1}} d^{p+1}\xi e^{-\Phi}\det\Big(-X^*G_{ab} - X^*B_{ab} - 2\pi\alpha' X^*F_{ab}\Big)\,. \tag{6}$$

Here M_D denotes the D-dimensional target background spacetime and is endowed with a metric $G_{\mu\nu}$ and $\mathcal{R}$ denotes its Ricci scalar. Φ denotes the dilaton field. Σ_{p+1} is the world-volume of the Dp-brane with local coordiantes ξ^a. The target space coordinates $X^\mu(\xi^a)$ describe the embedding of the brane into space-time, i.e.

$$X : \Sigma_{p+1} \hookrightarrow M_D\,. \tag{7}$$

The action (6) is called the Dirac-Born-Infeld action and is written in terms of the pullbacks $X^*G_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$, similarly for the $B_{\mu\nu}$ antisymmetric field and the field-strength field $F_{\mu\nu}$.

1. Take the actions S_{bulk} and S_p into their Einstein frame form via the following redefinition

$$\tilde{G}_{\mu\nu}(X) = e^{-\frac{4\tilde{\Phi}}{D-2}}G_{\mu\nu}(X)\,,\quad \tilde{\Phi} = \Phi - \Phi_0\,. \tag{8}$$

2. Linearise S^E_{bulk} and S^E_p about a flat background by writing $G_{\mu\nu}(X) = \eta_{\mu\nu} + h_{\mu\nu}(X)$, and expanding up to second order in $h_{\mu\nu}$.

3. Work out the propagators in momentum space for the graviton $\Delta_G(k)$ and the dilation $\Delta_\Phi(k)$. The result should read

$$\begin{aligned}\Delta_G(k) &= -\frac{2i\kappa^2}{k^2}\Big(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}\Big)\,,\\ \Delta_\Phi(k) &= -\frac{i\kappa^2(D-2)}{4k^2}\,.\end{aligned} \tag{9}$$

Open-closed duality & Branes II

Now, we consider the same vacuum cylinder diagram for the open type II string. Due to the fermionic sectors, our trace must sum over the NS and R sectors. Moreover, we must include the GSO projection onto even fermion number. Such an amplitude reads

$$\mathcal{A} = \int_0^\infty \frac{dt}{2t}\text{Tr}_{NS+R}\left(\frac{1+(-1)^F}{2}q^{L_0-\frac{c}{24}}\right)\,. \tag{10}$$

1. Show that
$$\mathcal{A} = V_{p+1}\int_0^\infty \frac{dt}{2t}\frac{e^{-\frac{t}{2\pi\alpha'}Y^2}}{(8\pi^2\alpha' t)^{-\frac{(p+1)}{2}}}\frac{\theta_3^4(q) - \theta_2^4(q) - \theta_4^4(q)}{\eta^{12}(q)}\,. \tag{11}$$

2. Using the duality $\mathcal{A}\big|_{\text{open}} \longleftrightarrow \mathcal{A}\big|_{\text{closed}}$, can you say something about the physical meaning of (11) for $\mathcal{A}\big|_{\text{closed}}$?

3. Consider the massless modes coupling to the NS-NS sector in $\mathcal{A}\big|_{\text{closed}}$. Similarly as in first section, derive the tension T_p of the Dp-branes. Moreover, obtain the Dp-brane charge μ_p due to the R-R $(p+1)$-form potential by $\mu_p\int_{\Sigma_{p+1}} C_{p+1}$.