Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 5
May 16th, 2019
SS 2019

Exercises on Advanced Topics in String Theory

Prof. Dr. Albrecht Klemm, César Fierro-Cota, Rongvoram Nivesvivat

http://www.th.physik.uni-bonn.de/klemm/strings2_19/
PRESENCE EXERCISE

1 Unoriented theories

Recall that the mode expansions of opened bosonic string theory with Neumann-Neumann boundary condition are given by [1]

$$\begin{aligned} X^i(\tau,\sigma) &= x^i + \sqrt{2\alpha'}p^i\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i\cos(n\sigma)e^{-in\tau} && (1)\\ X^+(\tau,\sigma) &= 2\alpha' p^+\tau && (2)\\ X^-(\tau,\sigma) &= x^- + \sqrt{2\alpha'}p^-\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^-\cos(n\sigma)e^{-in\tau} && (3)\end{aligned}$$

1.1) We define the worldsheet-parity operator Ω as

$$\Omega X^i(\tau,\sigma)\Omega^{-1} = X^i(\tau,\pi-\sigma) \quad , \quad \Omega x^-\Omega^{-1} = x_0^- \quad \text{and} \quad \Omega p^+\Omega^{-1} = p^+ \tag{4}$$

Compute $\Omega x^i\Omega^{-1}$, $\Omega p^i\Omega^{-1}$, and $\Omega\alpha_n^i\Omega^{-1}$. Also show that

$$\Omega X^-(\tau,\sigma)\Omega^{-1} = X^-(\tau,\pi-\sigma). \tag{5}$$

Homework: Repeat this question again for Neumann-Dirichlet b.c. and Dirichlet-Dirichlet b.c.

1.2) The action of Ω on the ground state is given by

$$\Omega|0;k\rangle = \Omega^{-1}|0;k\rangle = |0;k\rangle. \tag{6}$$

Construct the open strings spectrum for $N \geq 3$ and write down their twist eigenvalue. The states which are twisted invariant are called unoriented.

1.3) Let us take a quick look at a closed bosonic string theory. The expansion is given by

$$X^\mu(\tau,\sigma) = x^\mu + \alpha' p^\mu\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^\mu e^{2ni(\tau-\sigma)} + \bar\alpha_n^\mu e^{2ni(\tau+\sigma)}\right). \tag{7}$$

Thus, under Ω, we have $\Omega X^\mu(\tau,\sigma)\Omega^{-1} = X^\mu(\tau,2\pi-\sigma)$. Compute $\Omega\alpha_n^\mu\Omega^{-1}$, $\Omega\bar\alpha_n^\mu\Omega^{-1}$, $\Omega x^\mu\Omega^{-1}$, and $\Omega p^\mu\Omega^{-1}$.

1.4) Can you see which states in closed bosonic string theory are invariant under Ω

[1] In this exercise, we set $l = \pi$.

Type-I superstring theory spectrum

The mode expansions of the fermionic strings are

$$\psi_+^i = \sum_r \bar{b}_r^i e^{2ir(\tau+\sigma)} \quad \text{and} \quad \psi_-^i = \sum_r b_r^i e^{2ir(\tau-\sigma)}, \tag{8}$$

where r is (half)-integral for (Neveu-Schwarz) Ramond sector.

1.5) Again the action of Ω is defined as

$$\Omega\psi_\pm^i(\tau,\sigma)\Omega^{-1} = \psi_\mp^i(\tau,\pi-\sigma). \tag{9}$$

Compute $\Omega b_r^i \Omega^{-1}$ and $\Omega \bar{b}_r^i \Omega^{-1}$.

1.6) Using the results from the previous question, can you deduce how the ground states in closed superstring theory should transform under Ω?

1.7) Recall the massless spectrum of type-II theories and show how Ω acts on the states. The spectrum which is invariant under Ω leads to the so-called type-I superstring theory. Can you tell whether if type-I superstring is tachyonic-free?

2 The Chan-Paton factors

We discuss how to include the gauge interaction to open string theories by assigning Chan-Paton factors to the string states. The goal of this section is to motivate that the relevant gauge interaction for the oriented theories is $U(N)$, while in the unoriented case we have $SO(N)$ and $Sp(2N)$.

2.1) Given a state $|n,k\rangle$ with occupation number n and momentum k associated to an open string. Let us introduce new degrees of freedom at both ends i.e. i and j running from 1 to N,

$$|n,k;i,j\rangle. \tag{10}$$

i and j are called the Chan-Paton factors. For simplicity, consider the case where (10) is a state in oriented open bosonic string theory. How many tachyons and massless vector boson are there? Can you deduce that they should form the representations of which group?

2.2) The worldsheet-parity operator now acts on the states as

$$\Omega|n,k;i,j\rangle = (-1)^n \gamma_{ii'}|n,k;i',j'\rangle\gamma_{jj'}^{-1}. \tag{11}$$

The states in the unoriented theory can be constructed by requiring $\Omega|n,k;i,j\rangle = n,k;i,j\rangle$. Therefore, we restrict that $\Omega^2 = 1$. Can you tell whether γ_{ij} has to be symmetric or antisymmetric, or neither of them?

2.3) From the previous question, which group can be the gauge group for unoriented theory?

Homework: Oriented theory and $U(N)$ gauge interaction[2]

Here you shall see how the Chan-Paton factors in oriented bosonic strings can lead to a $U(N)$ gauge interaction. Compute the disc amplitude of three tachyons from

$$\langle : g_0 e^{k_1\cdot X} :: g_0 e^{k_2\cdot X} :: g_0 e^{k_3\cdot X} : \rangle. \tag{12}$$

Show that, at the first order in k, the result from (12) agrees with $\langle \phi\phi\phi \rangle$ from

$$S = \frac{1}{g_0'} \int \mathrm{d}^{26}x \left(\mathrm{Tr}(D_\mu \phi D_\mu \phi) + \frac{1}{2\alpha'} \mathrm{Tr}(\phi^2) + \frac{2^{1/2}}{3\alpha'^{1/2}} \mathrm{Tr}(\phi^3) - \frac{1}{4} \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}) \right), \tag{13}$$

where $\phi = \phi^a \lambda^a$ representing a scalar field, $A_\mu = A_\mu^a \lambda^a$, $D_\mu = \partial_\mu - i[A_\mu, \cdot]$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$. Therefore (13) is the action for $U(n)$ gauge field coupled to a scalar field. If you have time, you should also check that the amplitudes of 3 gauge bosons, 2 tachyons and 1 gauge boson ... computed from (13) agrees with the result using vertex operators at the first order in k. Hence, the Chan-Paton factor leads to a theory with gauge interaction.

[2]For more detail, see Polchinski Vol.1