Exercises on Advanced Topics in String Theory

Prof. Dr. Albrecht Klemm, César Fierro-Cota, Rongvoram Nivesvivat

http://www.th.physik.uni-bonn.de/klemm/strings2_19/

PRESENCE EXERCISE

1 Boundary Conformal field Theory

In the following we consider a CFT for an open bosonic string. In this case the strip world-sheet Σ gets identified with the upper half plane $\mathbb{H}_+$ via the map $(\tau,\sigma)\mapsto z=e^{\pi(\tau-i\sigma)/\ell}$, where

$$\mathbb{H}_+ = \{z\in\mathbb{C} : \mathrm{Im} z > 0\}\,. \tag{1}$$

Recall that the mode expansions of opened bosonic string theory with Neumann-Neumann and Dirichlet-Dirichlet boundary conditions are respectively given by

$$X^\mu_{NN}(\tau,\sigma) \;=\; x^\mu + i\alpha' p^\mu \log|z|^2 + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n(z^{-n}+\bar z^{-n})\,, \tag{2}$$

$$X^\mu_{DD}(\tau,\sigma) \;=\; \frac{1}{2\pi i}(x^\mu_{end}-y^\mu_{end})\log\left|\frac{z}{\bar z}\right| + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n(z^{-n}-\bar z^{-n})\,. \tag{3}$$

Here x_{end} and y_{end} denote the fixed endpoints of the open DD string. WLOG we consider a single scalar field $X(z,\bar z)$ for both cases

1.1) Using the doubling trick, compute the following two-point functions on the upper-half plane

$$\langle X_{NN}(z,\bar z)X_{NN}(w,\bar w)\rangle_{\mathbb{H}_+} = -\frac{\alpha}{2}\Big(\log|z-w|^2+\log|z-\bar w|^2\Big)\,, \tag{4}$$

$$\langle X_{DD}(z,\bar z)X_{DD}(w,\bar w)\rangle_{\mathbb{H}_+} = -\frac{\alpha}{2}\Big(\log|z-w|^2-\log|z-\bar w|^2\Big)\,. \tag{5}$$

1.2) What happens to the above correlators at $z=\bar z = x, w=\bar w = y$.

In theories with D-branes, masless fields like gauge fields or matter fields are open string excitations which are localized on the D-brane world-volume. Hence at the boundary of Σ, i.e. the end points of an open string is attached to the D-brane world-volume and an open string vertex operetor is inserted boundary of $\mathbb{H}_+$. Note that the vertex operator for the open string tachyon and gauge boson are given by the following operators

$$V_\times(x) =: e^{ik\cdot X(x)}:\,,\quad V_b(x)=\epsilon_\mu : e^{ik\cdot X(x)}\partial_x X(x):\,. \tag{6}$$

It is straightforward to show that the physical condition of physical states, i.e. primary field of conformal weight $h=1$, implies $k^2=\frac{1}{\alpha'}$ for the former and. $k^2=0, k\cdot\epsilon = 0$ for the later. You can check the following link in case you need a guide for computing scattering amplitudes

`http://www.th.physik.uni-bonn.de/people/fierro/StringWS1718/Sheet11-1.pdf`