Superconductivity and Ginzburg-Landau theory

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Structure

1. Meissner-Effect London-Equation
2. Hubbard Stratonovich transformation
3. Application to BCS-Hamiltonian
4. Ginzburg-Landau theory
5. Anderson-Higgs effect
Superconductivity is the consequence of an **electron-phonon interaction**.

One of the most striking features of a superconductor is the **Meissner effect**.
For the equation of motion for an electron a field $E$ is:

$$m \frac{d}{dt} \mathbf{v} = e \frac{d}{dt} E$$

$$\frac{d}{dt} j = \frac{ne^2}{m} \frac{d}{dt} E$$

in the London-ansatz for $\Phi = 0$: $j = ne \mathbf{v}$.
Meissner-Effect and London-Equation

For the equation of motion for an electron a filed $\mathbf{E}$ is:

\[
m \frac{d}{dt} \mathbf{v} = e \frac{d}{dt} \mathbf{E}
\]

And:

\[
\frac{d}{dt} \mathbf{j} = \frac{ne^2}{m} \frac{d}{dt} \mathbf{E}
\]

Because of:

\[
\mathbf{E} = -\frac{1}{c} \frac{d}{dt} \mathbf{A} - \nabla \Phi
\]

We obtain the London-ansatz for $\mathbf{j}$ ($\Phi = 0$):

London-ansatz

\[
\mathbf{j} = -\frac{n_s e^2}{mc} \mathbf{A}
\]

with $n_s$ the density of superconducting electrons.
Meissner-Effect and London-Equation

Take the rotation:

$$\partial x j = - \frac{n_s e^2}{mc} \partial x A = - \frac{n_s e^2}{mc} B$$
Meissner-Effect and London-Equation

Take the rotation:

$$\partial x j = -\frac{n_s e^2}{mc} \partial x A = -\frac{n_s e^2}{mc} B$$

Because of Maxwell equation $\partial x B = \frac{4\pi}{c} j$ and $\partial B = 0$:

$$\partial x \partial x B = \frac{4\pi}{c} \partial x j = \frac{4\pi n_s e^2}{mc^2} B$$

$$= -\Delta B + \partial (\partial B) = -\Delta B$$

it follows:

$$\Delta B = \frac{4\pi n_s e^2}{mc^2} B \quad \Delta j = \frac{4\pi n_s e^2}{mc^2} j$$
Meissner-Effect and London-Equation

Inspect this equation in the case of:

- No superconductor at \( z < 0 \)
- A superconductor at \( z > 0 \)

\[
\frac{\partial^2}{\partial z^2} B(z) = \frac{4\pi n_s e^2}{mc^2} B(z)
\]
Meissner-Effect and London-Equation

Inspect this equation in the case of:
- No superconductor at \( z < 0 \)
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\[
\frac{\partial^2}{\partial z^2} B(z) = \frac{4\pi n_s e^2}{mc^2} B(z)
\]

**Solution**

\[
B(z) = B_0 e^{-\frac{z}{\lambda_L}}
\]

\[
\lambda_L = \sqrt{\frac{me^2}{4\pi n_s e^2}}
\]

\( \lambda_L \): London penetration depth.
Hubbard Stratonovich transformation maps \textbf{interacting} fermion systems to \textbf{non-interacting} fermions moving in an \textbf{effective field}. 

$\rightarrow$ Interacting has to contain fermion bilinears

\[ H = H_o + H_I \quad \text{with} \quad H_I = -g \int d^3xA^+(x)A(x) \]

Examples

\[ A(x) = \psi_{\downarrow}(x)\psi_{\uparrow}(x) \quad \text{or} \quad A(x) = S^-(x) \]
Hubbard Stratonovich transformation maps interacting fermion systems to non-interacting fermions moving in an effective field. → Interacting has to contain fermion bilinears

\[ H = H_o + H_I \quad \text{with} \quad H_I = -g \int d^3x A^+(x) A(x) \]

Examples \( A(x) = \psi_\downarrow(x) \psi_\uparrow(x) \) or \( A(x) = S^{-}(x) \)

Replacement:

\[ -gaA^+(x)A(x) \rightarrow A^+(x)\Delta(x) + \bar{\Delta}A(x) + \frac{\Delta(x)\Delta(x)}{g} \]

Like in mean field theories.


\[
\Delta = \Delta_1 + i \Delta_2 \quad \bar{\Delta} = \Delta_1 - i \Delta_2
\]

\[
\int d\Delta_1 d\Delta_2 e^{-\frac{(\Delta_1^2 + \Delta_2^2)}{g}} = \pi g
\]

\[
\Rightarrow \quad \int \frac{d\Delta d\bar{\Delta}}{2\pi i g} e^{\frac{\bar{\Delta}\Delta}{g}} = 1
\]
Hubbard Stratonovich transformation

\[ \Delta = \Delta_1 + i\Delta_2 \quad \bar{\Delta} = \Delta_1 - i\Delta_2 \]

\[ \int d\Delta_1 d\Delta_2 e^{-\frac{(\Delta_1^2 + \Delta_2^2)}{g}} = \pi g \]

\[ \Rightarrow \int \frac{d\Delta d\bar{\Delta}}{2\pi ig} e^{\frac{\bar{\Delta}\Delta}{g}} = 1 \]

Generalize to \( \Delta(x, t) \):

\[ \int \mathcal{D}[\Delta, \bar{\Delta}] \exp\left(-\int d^3 x \int_0^\beta d\tau \frac{\Delta(x, \tau)\Delta(x, \tau)}{g}\right) = 1 \]

\[ \mathcal{D}[\Delta, \bar{\Delta}] \equiv \prod_{\tau j} \frac{d\Delta(x_j, \tau) d\Delta(x_j, \tau)}{\mathcal{N}} \]
Hubbard Stratonovich transformation

\[ Z = \int \mathcal{D}[c, \bar{c}] e^{-\int_0^\beta d\tau [\bar{c}(\partial_\tau + h)c + H_I]} \quad h = \epsilon_a \delta_{ab} \]

By introducing a 1 we obtain:

\[ Z = \int \mathcal{D}[c, \bar{c}] \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-\int_0^\beta d\tau [\bar{c}(\partial_\tau + h)c + H'_I]} \]

\[ H'_I = \int d^3x \left[ \frac{\bar{\Delta} \Delta}{g} - g \bar{A}(x)A(x) \right] \]

We now shift the \( \Delta \)-field:

\[ \Delta(x) \rightarrow \Delta(x) + g A(x) \bar{\Delta}(x) \]

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\[ H'_I = \int d^3x \left[ \frac{\bar{\Delta} \Delta}{g} - g\bar{A}(x)A(x) \right] \]

We now shift the \( \Delta \)-field:

\[ \Delta(x) \rightarrow \Delta(x) + gA(x) \quad \bar{\Delta}(x) \rightarrow \bar{\Delta}(x) + g\bar{A}(x) \]

\[ \Rightarrow H'_I = \int d^3x \left[ \bar{A}(x)\Delta(x) + \bar{\Delta}A(x) + \frac{\bar{\Delta}(x)\Delta(x)}{g} \right] \]
First result

We have absorbed the interaction, replacing it by an **effective action** which couples to the fermion bilinear $A$. 
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\[
\mathcal{Z} = \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-\int d^3 x d\tau \frac{\bar{\Delta} \Delta}{g}} \int \mathcal{D}[c, \bar{c}] e^{-\tilde{S}}
\]

\[
\tilde{S} = \int_0^\beta d\tau \bar{c} \partial_\tau c + H_{eff}[\Delta, \bar{\Delta}]
\]

\[
H_{eff}[\Delta, \bar{\Delta}] = H_o + \int d^3 x [\bar{A}(x) \Delta(x) + \bar{\Delta} A(x)]
\]

Please note that $\tilde{S}$ is quadratic in the fermion operators.
Hubbard Stratonovich transformation

\[\int \mathcal{D}[c, \bar{c}] e^{-\tilde{S}} = \det[\partial_\tau + h_{\text{eff}}[\Delta, \bar{\Delta}]]\]

with \(h_{\text{eff}}\) the matrix representation of \(H_{\text{eff}}\).
Hubbard Stratonovich transformation

\[
\int \mathcal{D}[c, \bar{c}] e^{-\tilde{S}} = \det[\partial_{\tau} + h_{\text{eff}}[\Delta, \bar{\Delta}]]
\]

with \( h_{\text{eff}} \) the matrix representation of \( H_{\text{eff}} \).

This leads to:

\[
Z = \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-S_{\text{eff}}[\Delta, \bar{\Delta}]}
\]

\[
S_{\text{eff}}[\Delta, \bar{\Delta}] = \int d^3x d\tau \frac{\bar{\Delta} \Delta}{g} - \ln \det[\partial_{\tau} + h_{\text{eff}}[\Delta, \bar{\Delta}]]
\]

\[
= \int d^3x d\tau \frac{\bar{\Delta} \Delta}{g} - \text{Tr} \ln[\partial_{\tau} + h_{\text{eff}}[\Delta, \bar{\Delta}]]
\]
BCS-Hamiltonian

\[ H = \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^+ c_{k\sigma} - \frac{g_0}{V} A^+ A \]

\[ A = \sum_{k, |\epsilon_k| < \omega_D} c_{-k\downarrow} c_{k\uparrow} \quad A^+ = \sum_{k, |\epsilon_k| < \omega_D} c_{-k\uparrow}^+ c_{-k\downarrow}^+ \]
BCS-Hamiltonian

\[ H = \sum_{k,\sigma} \epsilon_{k\sigma} c_{k\sigma}^+ c_{k\sigma} - \frac{g_o}{\sqrt{V}} A^+ A \]

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By using the results of the previous section we obtain:

\[ \mathcal{Z} = \int D[\Delta, \bar{\Delta}, c, \bar{c}] e^{-S} \]

\[ S = \int_0^\beta d\tau \left[ \sum_{k,\sigma} \bar{c}_{k\sigma} (\partial_{\tau} + \epsilon_k) c_{k\sigma} + \bar{\Delta} A + \bar{A} \Delta + \frac{\bar{\Delta} \Delta}{g} \right] \]
Application to BCS-Hamiltonian

Introduce a Nambu notation:

\[
S = \int_0^\beta d\tau \left[ \sum_k \bar{\psi}_k (\partial_{\tau} + h_k) \psi_k + \frac{\bar{\Delta} \Delta}{g} \right]
\]

\[
\psi_k = \begin{pmatrix} c_{k\uparrow} \\ \bar{c}_{-k\downarrow} \end{pmatrix}
\]

\[
h_k = \begin{pmatrix} \epsilon_k & \Delta(\tau) \\ \bar{\Delta}(\tau) & -\epsilon_k \end{pmatrix}
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Introduce a Nambu notation:

\[ S = \int_0^\beta d\tau \left[ \sum_k \bar{\Psi}_k (\partial_\tau + h_k) \Psi_k + \frac{\Delta \Delta}{g} \right] \]

\[ \Psi_k = \begin{pmatrix} c_{k\uparrow} \\ \bar{c}_{-k\downarrow} \end{pmatrix} \]

\[ h_k = \begin{pmatrix} \epsilon_k & \Delta(\tau) \\ \bar{\Delta}(\tau) & -\epsilon_k \end{pmatrix} \]

Again we can integrate out the fermionic contribution:

\[ Z = \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-S_{\text{eff}}[\Delta, \bar{\Delta}]} \]

\[ S_{\text{eff}}[\Delta, \bar{\Delta}] = \int_0^\beta d\tau \frac{\Delta \Delta}{g} + \sum_k Tr \ln(\partial_\tau + h_k) \]
To proceed we must invoke some approximation, we expect that fluctuations will be small.

→ integral will be dominated by minimal value of $S_{\text{eff}}$

→ **saddlepoint approximation** $Z = e^{-S_{\text{eff}}[\Delta_o,\bar{\Delta}_o]}$

Expect that $\Delta_o$ is independent of $\tau$ because of translational invariance.
To proceed we must invoke some approximation, we expect that fluctuations will be small → integral will be dominated by minimal value of $S_{\text{eff}}$
→ **saddlepoint approximation** $\mathcal{Z} = e^{-S_{\text{eff}}[\Delta_o,\bar{\Delta}_o]}$

Expect that $\Delta_o$ is independent of $\tau$ because of translational invariance.

$$\Psi_k(\tau) = \frac{1}{\sqrt{\beta}} \sum_n \Psi_k e^{-i\omega_n \tau}$$

with the Matsubara frequencies $\omega_n = (2n + 1) \frac{\pi}{\beta}$

$$\det[\partial_{\tau} + h_k] = \prod_n \det[-i\omega_n + h_k] = \prod_n \left[\omega_n^2 + \epsilon^2_k + |\Delta|^2\right]$$
Application to BCS-Hamiltonian

Inserting this into $S_{\text{eff}}$ yields:

$$\frac{S_{\text{eff}}}{\beta} = -T \sum_{kn} \ln[\omega_n^2 + \epsilon_k^2 + |\Delta|^2] + \frac{\Delta^2}{g} = F_{\text{eff}}$$
Application to BCS-Hamiltonian

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$$

Minimizing wrt $\Delta$:

$$
\frac{\partial F_{\text{eff}}}{\partial \Delta} = - T \sum_{kn} \frac{\Delta}{\omega_n^2 + E_k^2} + \frac{\Delta}{g} = 0
$$
Application to BCS-Hamiltonian

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BCS Gap equation

$$\frac{1}{g} = \frac{1}{\beta} \sum_{kn} \frac{1}{\omega_n^2 + E_k^2}$$

$$E_k = \sqrt{\epsilon_k^2 + |\Delta|^2}$$
From this follows (after some work):

**Equations for \( \Delta \) and \( T_c \)**

\[
\Delta = 2 \omega_D e^{-\frac{1}{gN(0)}}
\]

\[
T_c = \frac{e^{-\psi\left(\frac{1}{2}\right)}}{2\pi} \omega_D e^{-\frac{1}{gN(0)}}
\]

With \( N \): density of states and \( \psi(z) = \frac{d}{dz} \ln \Gamma(z) \) the digamma function.
Since transition is continuous close to $T_c$, expand $S_{\text{eff}}$ for small $\Delta$.

$$S_{\text{eff}}[\Delta, \bar{\Delta}] = \int_{0}^{\beta} d\tau \frac{\bar{\Delta} \Delta}{g} + \sum_{k} Tr \ln(\partial_{\tau} + h_k)$$

with

$$\partial_{\tau} + h_k = \begin{pmatrix} \partial_{\tau} + \epsilon_k & \Delta(\tau) \\ \bar{\Delta}(\tau) & \partial_{\tau} - \epsilon_k \end{pmatrix} = G^{-1}$$

$$G^{-1} := \begin{pmatrix} [G^p_o]^{-1} & \frac{\Delta}{\bar{\Delta}} \\ \Delta & [G^h_o]^{-1} \end{pmatrix} = G^{-1}_o \left[ 1 + G_o \begin{pmatrix} 0 & \Delta \\ \bar{\Delta} & 0 \end{pmatrix} \right]$$

$$Tr \ln(G^{-1}) = Tr \ln(G^{-1}_o) - \frac{1}{2} Tr [G_o \begin{pmatrix} 0 & \Delta \\ \bar{\Delta} & 0 \end{pmatrix}]^2 + ...$$
Ginzburg-Landau theory

\[ \text{Tr}(G^p_o \Delta G^h_o \Delta) = \sum_{kk'} G^p_o < k|\Delta|k' > G^h_o < k'|\bar{\Delta}|k > \]

\[ \equiv \sum_{q=k-k'} \Delta_q \bar{\Delta}_{-q} \frac{1}{\beta L^d} \sum_k G^p_o(k) G^h_o(k+q) \]

\[ \Pi(q) \text{pairing susceptibility} \]
Ginzburg-Landau theory

\[ \text{Tr}(G_o^p \Delta G_o^h \Delta) = \sum_{kk'} G_o^p < k|\Delta|k' > G_o^h < k'|\bar{\Delta}|k > \]

= \sum_{q=k-k'} \Delta_q \bar{\Delta}_{-q} \frac{1}{\beta L^d} \sum_k G_o^p(k)G_o^h(k+q) \Pi(q) \text{pairing susceptibility} \]

Apart from the term \( \text{Tr} \ln G_o^{-1} \) \( S_{\text{eff}} \) contains:

\[ S_{\text{eff}} \supset \sum_{\omega_nq} \left[ \frac{1}{g} + \Pi(\omega_n, q) \right] |\Delta_{\omega_nq}|^2 + \phi(\Delta^4) \]
Approximate $\Pi$

$$\Pi(\omega_n, q) = \Pi(0, 0) + \frac{q^2}{2} \partial_q^2 \Pi(0, 0) + \phi(\omega_n, q^4)$$
Ginzburg-Landau theory

Approximate $\Pi$

$$\Pi(\omega_n, q) = \Pi(0, 0) + \frac{q^2}{2} \partial_q^2 \Pi(0, 0) + \phi(\omega_n, q^4)$$

Transform to position representation (and include a term of order $\Delta^4$):

$$S_{\text{eff}} \supset \beta \int d^d r \left[ \frac{t}{2} |\Delta|^2 + \frac{K}{2} |\partial \Delta|^2 + u |\Delta|^4 \right]$$

$$\frac{t}{2} = \frac{1}{g} + \Pi(0, 0) \quad K = \partial_q \Pi(0, 0) > 0 \quad u > 0$$
Ginzburg-Landau theory

Approximate $\Pi$

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$$\frac{t}{2} = \frac{1}{g} + \Pi(0, 0) \quad K = \partial_q \Pi(0, 0) > 0 \quad u > 0$$

We again assume that $Z$ will be dominated by the minimal action:

$$\partial \Delta = 0$$

$$\frac{\delta S_{\text{eff}}}{\delta |\Delta|} \Rightarrow 0 = \frac{\delta}{\delta |\Delta|} \left( \frac{t}{2} |\Delta|^2 + u |\Delta|^4 \right)$$
Ginzburg-Landau theory

\[ |\Delta| (t + 4u|\Delta|^2) = 0 \]

\[ |\Delta| = \begin{cases} 
0 & t > 0 \\
\sqrt{\frac{t}{4u}} & t < 0 
\end{cases} \]

For \( t < 0 \) \( U(1) \) symmetry is spontaneously broken.
\( \Delta = |\Delta|e^\Phi \rightarrow \Phi \) field remains massless/gapless.
Ginzburg-Landau theory

\[ |\Delta|(t + 4u|\Delta|^2) = 0 \quad |\Delta| = \begin{cases} 0 & t > 0 \\ \sqrt{\frac{t}{4u}} & t < 0 \end{cases} \]

For \( t < 0 \) \( U(1) \) symmetry is spontaneously broken. 
\( \Delta = |\Delta|e^\Phi \rightarrow \Phi \) field remains massless/gapless.

Goldstone theorem

Every time a continuous global symmetry gets spontaneously broken there exists a gapless excitation \( \rightarrow \) Goldstone mode.
By calculating $\Pi(0,0)$ we can relate $t$ with $T$:

\[
T_c = \pi \omega_D \exp(-\frac{1}{N(0)g})
\]

\[
\frac{t}{2} \approx N(0) \frac{T - T_c}{T_c}
\]
By calculating $\Pi(0,0)$ we can relate $t$ with $T$:

$$T_c = \pi \omega_D \exp(-\frac{1}{N(0)g})$$

$$\frac{t}{2} \approx N(0) \frac{T - T_c}{T_c}$$

The Ginzburg-Landau theory of superconductors was known (1956) before BCS-theory (1957) and predicted the right results. Parameters are unknown if you start with a GL theory but meaningful predictions are possible.
Inclusion of em fields via minimal coupling:

\[ \partial \rightarrow \partial + ieA \]

\[ \mathcal{L}_{em} = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \]

\[ \rightarrow Z = \int D\mathbf{A} \int D[\Delta, \bar{\Delta}] e^{-S_{eff}} \]

Where \( S_{eff} \) gets modified:

\[ S_{eff} = \beta \int d\mathbf{r} \left[ \frac{t}{2} |\Delta|^2 + \frac{K}{2} (\partial + i2eA)|\Delta|^2 + u|\Delta|^4 + \frac{1}{2} (\partial \times A)^2 \right] \]

where we used \( A_0 = \Phi = 0 \)
Anderson-Higgs effect

All terms in this Lagrangian are gauge invariant under local gauge transformations:

\[ A \mapsto A - \partial \Phi(r) \]
\[ \Delta \mapsto e^{-2i\Phi(r)}\Delta \]
Anderson-Higgs effect

All terms in this Lagrangian are gauge invariant under local gauge transformations:

\[ A \mapsto A - \partial \Phi(r) \]
\[ \Delta \mapsto e^{-2i\Phi(r)} \Delta \]

Write:

\[ \Delta(r) = |\Delta(r)| e^{-2i\Phi(r)} \]

and choose a gauge (unitary gauge):

\[ A \mapsto A - \partial \Phi(r) \quad \Delta \mapsto |\Delta| \]
Anderson-Higgs effect

this results in the action:

\[ S_{\text{eff}} = \beta \int dr \left[ \frac{t}{2} |\Delta|^2 + \frac{K}{2} (\partial |\Delta|)^2 - \frac{4e^2 K |\Delta|^2}{2} A^2 + u |\Delta|^4 + \frac{1}{2} (\partial \times A)^2 \right] \]
this results in the action:

\[ S_{\text{eff}} = \beta \int d\mathbf{r} \left[ \frac{t}{2} |\Delta|^2 + \frac{K}{2} (\partial|\Delta|)^2 - \frac{4e^2 K |\Delta|^2}{2} \mathbf{A}^2 + u|\Delta|^4 + \frac{1}{2} (\partial \times \mathbf{A})^2 \right] \]

below \( T_c \) \( \Delta \neq 0 \Rightarrow m_A^2 = 4e^2 K |\Delta|^2 \neq 0. \)

The goldstone mode gets eaten by the gauge field which acquires a mass.
The $m_A^2$-term is responsible for London equation, because minimisation wrt $A$:

$$\partial_x \left( \partial_x A \right) + m_A^2 A = 0$$
The $m_A^2$-term is responsible for London equation, because minimisation wrt A:

$$\partial_x (\partial_x A) + m_A^2 A = 0$$

With the help of Maxwell equation $\partial_x B = \frac{4\pi}{c} j$ we obtain:

$$j = -\frac{cm_A^2}{4\pi} A$$

This was the basis for the **London equation**.
Conclusions

- We saw how a interacting fermionic system can be rewritten as a system noninteracting fermions in a bosonic field.
- We could compute several important quantities like $T_c$ or the gap $\Delta$.
- A Ginzburg-Landau ansatz with in orderparameter $\langle \Psi_\downarrow \Psi_\uparrow \rangle$ showed how symmetry breaking occurs.
- We saw an example of Goldstones theorem.
- By gauging the symmetry we obtained an example of Anderson-Higgs mechanism which is also important in high energy physics.
literature

- P. Coleman 04, chapter 12
- Czycholl, Theoretische Festkörperphysik