Superconductivity and Ginzburg-Landau theory

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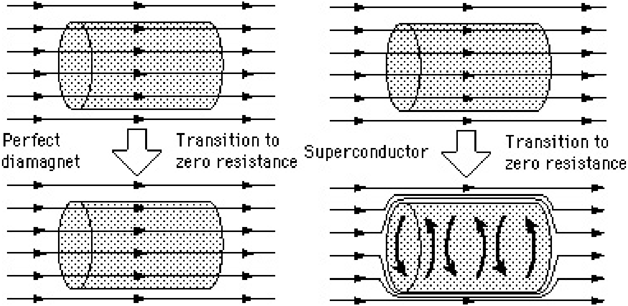
Structure

Meissner-Effect

Superconductivity is the consequence of an **electron-phonon interaction**.

One of the most striking features of a superconductor is the **Meissner effect**.



Meissner-Effect and London-Equation

For the equation of motion for an electron a filed **E** is:

$$m\frac{d}{dt}\mathbf{v} = e\frac{d}{dt}\mathbf{E}$$

$$\frac{d}{dt}\mathbf{j} \underbrace{=}_{\mathbf{j}=ne\mathbf{v}} \frac{ne^2}{m}\frac{d}{dt}\mathbf{E}$$

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Because of:

$$\mathbf{E} = \frac{-1}{c}\frac{d}{dt}\mathbf{A} - \nabla\Phi$$

We obtain the London-ansatz for **j** ($\Phi = 0$):

London-ansatz

$$\mathbf{j} = -\frac{n_s e^2}{mc}\mathbf{A}$$

with n_s the density of superconducting electrons.

Meissner-Effect and London-Equation

Take the rotation:

$$\partial x \mathbf{j} = -\frac{n_s e^2}{mc} \partial x \mathbf{A} = -\frac{n_s e^2}{mc} \mathbf{B}$$

Meissner-Effect and London-Equation

Take the rotation:

$$\partial x \mathbf{j} = -\frac{n_s e^2}{mc} \partial x \mathbf{A} = -\frac{n_s e^2}{mc} \mathbf{B}$$

Because of Maxwell equation $\partial x \mathbf{B} = \frac{4\pi}{c} \mathbf{j}$ and $\partial \mathbf{B} = 0$:

$$\begin{aligned} \partial x \partial x \mathbf{B} &= \frac{4\pi}{c} \partial x \mathbf{j} = \frac{4\pi n_s e^2}{mc^2} \mathbf{B} \\ &= -\Delta \mathbf{B} + \partial(\partial \mathbf{B}) = -\Delta \mathbf{B} \end{aligned}$$

it follows:

$$\Delta \mathbf{B} = \frac{4\pi n_s e^2}{mc^2} \mathbf{B} \qquad \Delta \mathbf{j} = \frac{4\pi n_s e^2}{mc^2} \mathbf{j}$$

Meissner-Effect and London-Equation

Inspect this equation in the case of:

- No superconductor at z<0
- A superconductor at z>0

$$\frac{\partial^2}{\partial z^2}\mathbf{B}(z) = \frac{4\pi n_s e^2}{mc^2}\mathbf{B}(z)$$

Meissner-Effect and London-Equation

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- No superconductor at z<0
- A superconductor at z>0

$$\frac{\partial^2}{\partial z^2}\mathbf{B}(z) = \frac{4\pi n_s e^2}{mc^2}\mathbf{B}(z)$$

Solution

$$\mathbf{B}(z) = \mathbf{B}_o e^{-\frac{z}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{me^2}{4\pi n_s e^2}}$$

λ_L: London penetration depth.

Hubbard Stratonovich transformation

Hubbard Stratonovich transformation maps **interacting** fermion systems to **non-interacting** fermions moving in an **effective field**.
$\rightarrow$ Interacting has to contain fermion bilinears

$$H = H_o + H_I \qquad \text{with} \qquad H_I = -g \int d^3x A^+(x) A(x)$$

$$\text{examples} \qquad A(x) = \Psi_\downarrow(x)\Psi_\uparrow(x) \qquad \text{or} \qquad A(x) = S^-(x)$$

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Replacement:

$$-gaA^+(x)A(x) \rightarrow A^+(x)\Delta(x) + \bar{\Delta}A(x) + \frac{\Delta\bar{(x)}\Delta(x)}{g}$$

Like in mean field theories.

Hubbard Stratonovich transformation

$$\Delta = \Delta_1 + i\Delta_2 \qquad \bar{\Delta} = \Delta_1 - i\Delta_2$$

$$\int d\Delta_1 d\Delta_2 e^{-\frac{(\Delta_1^2+\Delta_2^2)}{g}} = \pi g$$

$$\Rightarrow \qquad \int \frac{d\Delta d\bar{\Delta}}{2\pi i g} e^{\frac{\bar{\Delta}\Delta}{g}} = 1$$

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Generalize to $\Delta(x,t)$:

$$\int \mathcal{D}[\Delta,\bar{\Delta}] \exp(-\int d^3x \int_0^\beta d\tau \frac{\bar{\Delta(x,\tau)}\Delta(x,\tau)}{g}) = 1$$

$$\mathcal{D}[\Delta,\bar{\Delta}] \equiv \prod_{\tau \mathbf{j}} \frac{d\bar{\Delta(x_j,\tau)} d\Delta(x_j,\tau)}{\mathcal{N}}$$

Hubbard Stratonovich transformation

$$\mathcal{Z} = \int \mathcal{D}[c,\bar{c}] e^{-\int_0^\beta d\tau [\bar{c}(\partial_\tau + h)c + H_I]} \qquad h = \epsilon_a \delta_{ab}$$

By introducing a 1 we obtain:

$$\mathcal{Z} = \int \mathcal{D}[c,\bar{c}] \int \mathcal{D}[\Delta,\bar{\Delta}] e^{-\int_0^\beta d\tau [\bar{c}(\partial_\tau + h)c + H_I']}$$

$$H_I' = \int d^3x [\frac{\bar{\Delta}\Delta}{g} - g\bar{A}(x)A(x)]$$

We now shift the Δ-field:

Hubbard Stratonovich transformation

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We now shift the Δ-field:

$$\Delta(x) \rightarrow \Delta(x) + gA(x) \qquad \bar{\Delta}(x) \rightarrow \bar{\Delta}(x) + g\bar{A}(x)$$

$$\Rightarrow H_I^{'} = \int d^3x [\bar{A}(x)\Delta(x) + \bar{\Delta}A(x) + \frac{\bar{\Delta}(x)\Delta(x)}{g}]$$

Hubbard Stratonovich transformation

First result

We have absorbed the interaction, replacing it by an **effective action** which couples to the fermion bilinear A.

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$$\mathcal{Z} = \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-\int d^3x d\tau \frac{\bar{\Delta}\Delta}{g}} \int \mathcal{D}[c, \bar{c}] e^{-\widetilde{S}}$$

$$\widetilde{S} = \int_0^\beta d\tau \bar{c} \partial_\tau c + H_{eff}[\Delta, \bar{\Delta}]$$

$$H_{eff}[\Delta, \bar{\Delta}] = H_o + \int d^3x [\bar{A}(x)\Delta(x) + \bar{\Delta} A(x)]$$

Please note that $\widetilde{S}$ is quadratic in the fermion operators.

Hubbard Stratonovich transformation

$$\int \mathcal{D}[c,\bar{c}]e^{-\widetilde{S}} = \det[\partial_\tau + h_{eff}[\Delta,\bar{\Delta}]]$$

with h_{eff} the matrix representation of H_{eff}.

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This leads to:

$$\mathcal{Z} = \int \mathcal{D}[\Delta,\bar{\Delta}]e^{-S_{eff}[\Delta,\bar{\Delta}]}$$

$$S_{eff}[\Delta,\bar{\Delta}] = \int d^3x d\tau \frac{\bar{\Delta}\Delta}{g} - \ln\det[\partial_\tau + h_{eff}[\Delta,\bar{\Delta}]]$$

$$= \int d^3x d\tau \frac{\bar{\Delta}\Delta}{g} - Tr\ln[\partial_\tau + h_{eff}[\Delta,\bar{\Delta}]]$$

BCS-Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}\sigma} c^+_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \frac{g_o}{V} A^+ A$$

$$A = \sum_{\mathbf{k}, |\epsilon_{\mathbf{k}}| < \omega_D} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \qquad A^+ = \sum_{\mathbf{k}, |\epsilon_{\mathbf{k}}| < \omega_D} c^+_{-\mathbf{k}\uparrow} c^+_{-\mathbf{k}\downarrow}$$

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By using the results of the previous section wse obtain:

$$\mathcal{Z} = \int \mathcal{D}[\Delta, \bar{\Delta}, c, \bar{c}] e^{-S}$$

$$S = \int_0^\beta d\tau \, [\sum_{\mathbf{k}\sigma} \bar{c}_{\mathbf{k}\sigma} (\partial_\tau + \epsilon_{\mathbf{k}}) c_{\mathbf{k}\sigma} + \bar{\Delta} A + \bar{A} \Delta + \frac{\bar{\Delta}\Delta}{g}]$$

Application to BCS-Hamiltonian

Introduce a Nambu notation:

$$S = \int_0^\beta d\tau \, [\sum_{\mathbf{k}} \bar{\Psi}_{\mathbf{k}}(\partial_\tau + h_{\mathbf{k}})\Psi_{\mathbf{k}} + \frac{\bar{\Delta}\Delta}{g}]$$

$$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ \bar{c}_{-\mathbf{k}\downarrow} \end{pmatrix}$$

$$h_{\mathbf{k}} = \begin{pmatrix} \epsilon_{\mathbf{k}} & \Delta(\tau) \\ \bar{\Delta}(\tau) & -\epsilon_{\mathbf{k}} \end{pmatrix}$$

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Again we can integrate out the fermionic contribution:

$$\mathcal{Z} = \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-S_{eff}[\Delta, \bar{\Delta}]}$$

$$S_{eff}[\Delta, \bar{\Delta}] = \int_0^\beta d\tau \frac{\bar{\Delta}\Delta}{g} + \sum_{\mathbf{k}} Tr \ln(\partial_\tau + h_{\mathbf{k}})$$

Application to BCS-Hamiltonian

To proceed we must invoke some approximation, we expect that **fluctuations will be small**
$\rightarrow$ integral will be dominated by minimal value of S_{eff}
$\rightarrow$ **saddlepoint approximation** $\mathcal{Z} = e^{-S_{eff}[\Delta_o, \bar{\Delta}_o]}$
Expect that Δ_o is independent of τ because of translational invariance.

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$$\Psi_{\mathbf{k}}(\tau) = \frac{1}{\sqrt{\beta}} \sum_n \Psi_{\mathbf{k}} e^{-i\omega_n \tau}$$

with the Matsubara frequencies $\omega_n = (2n+1)\frac{\pi}{\beta}$

$$\det[\partial_\tau + h_{\mathbf{k}}] = \prod_n \det[-i\omega_n + h_{\mathbf{k}}] = \prod_n [\omega_n^2 + \epsilon_{\mathbf{k}}^2 + |\Delta|^2]$$

Application to BCS-Hamiltonian

Inserting this into S_{eff} yields:

$$\frac{S_{eff}}{\beta} = -T \sum_{\mathbf{k}n} \ln[\omega_n^2 + \epsilon_{\mathbf{k}}^2 + |\Delta|^2] + \frac{\Delta^2}{g} = F_{eff}$$

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Minimizing wrt Δ:

$$\frac{\partial F_{eff}}{\partial \Delta} = -T \sum_{\mathbf{k}n} \frac{\Delta}{\omega_n^2 + E_{\mathbf{k}}^2} + \frac{\Delta}{g} = 0$$

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BCS Gap equation

$$\frac{1}{g} = \frac{1}{\beta} \sum_{\mathbf{k}n} \frac{1}{\omega_n^2 + E_{\mathbf{k}}^2}$$

$$E_{\mathbf{k}} = \sqrt{\epsilon_{\mathbf{k}}^2 + |\Delta|^2}$$

Application to BCS-Hamiltonian

From this follows (after some work):

Equations for Δ and T_c

$$\Delta = 2\omega_D e^{-\frac{1}{gN(0)}}$$

$$T_c = \frac{e^{-\Psi(\frac{1}{2})}}{2\pi}\omega_D e^{-\frac{1}{gN(0)}}$$

With N: density of states and $\Psi(z) = \frac{d}{dz}\ln\Gamma(z)$ the digamma function.

Ginzburg-Landau theory

Since transition is continuous close to T_c, expand S_{eff} for **small** Δ.

$$S_{eff}[\Delta,\bar{\Delta}] = \int_0^\beta d\tau \frac{\bar{\Delta}\Delta}{g} + \sum_{\mathbf{k}} Tr\ln(\partial_\tau + h_{\mathbf{k}})$$

with $$\partial_\tau + h_{\mathbf{k}} = \begin{pmatrix} \partial_\tau + \epsilon_{\mathbf{k}} & \Delta(\tau) \\ \bar{\Delta}(\tau) & \partial_\tau - \epsilon_{\mathbf{k}} \end{pmatrix} = \mathcal{G}^{-1}$$

$$\mathcal{G}^{-1} := \begin{pmatrix} [\mathcal{G}_o^p]^{-1} & \Delta \\ \bar{\Delta} & [\mathcal{G}_o^h]^{-1} \end{pmatrix} = \mathcal{G}_o^{-1}\left[1 + \mathcal{G}_o \begin{pmatrix} 0 & \Delta \\ \bar{\Delta} & 0 \end{pmatrix}\right]$$

$$Tr\ln(\mathcal{G}^{-1}) = Tr\ln(\mathcal{G}_0^{-1}) - \frac{1}{2} Tr[\mathcal{G}_o \begin{pmatrix} 0 & \Delta \\ \bar{\Delta} & 0 \end{pmatrix}]^2 + \ldots$$

Ginzburg-Landau theory

$$Tr(\mathcal{G}_o^p \Delta \mathcal{G}_o^h \Delta) = \sum_{\mathbf{kk}'} \mathcal{G}_o^p < k|\Delta|k' > \mathcal{G}_o^h < k'|\bar{\Delta}|k >$$

$$\underbrace{=}_{q=k-k'} \sum_q \Delta_q \bar{\Delta}_{-q} \frac{1}{\beta L^d} \underbrace{\sum_k \mathcal{G}_o^p(k) \mathcal{G}_o^h(k+q)}_{\Pi(q)\text{pairing susceptibility}}$$

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Apart from the term $Tr \ln \mathcal{G}_o^{-1}$ S_{eff} contains:

$$S_{eff} \supset \sum_{\omega_n \mathbf{q}} [\frac{1}{g} + \Pi(\omega_n, \mathbf{q})] |\Delta_{\omega_n,\mathbf{q}}|^2 + \mathcal{O}(\Delta^4)$$

Ginzburg-Landau theory

Approximate Π

$$\Pi(\omega_n, \mathbf{q}) = \Pi(0,0) + \frac{\mathbf{q}^2}{2}\partial_q^2 \Pi(0,0) + \mathcal{O}(\omega_n, \mathbf{q}^4)$$

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Transform to position representation (and include a term of order Δ^4):

$$S_{eff} \supset \beta \int d^d r \left[\frac{t}{2}|\Delta|^2 + \frac{K}{2}|\partial\Delta|^2 + u|\Delta|^4 \right]$$

$$\frac{t}{2} = \frac{1}{g} + \Pi(0,0) \qquad K = \partial_q \Pi(0,0) > 0 \qquad u > 0$$

Ginzburg-Landau theory

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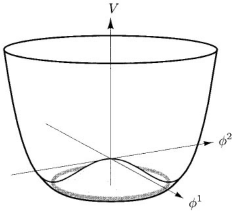
$$\frac{t}{2} = \frac{1}{g} + \Pi(0,0) \qquad K = \partial_q \Pi(0,0) > 0 \qquad u > 0$$

We again assume that $\mathcal{Z}$ will be dominated by the minimal action:

$$\partial\Delta = 0$$

$$\frac{\delta S_{eff}}{\delta|\Delta|} \overbrace{=}^{!} 0 = \frac{\delta}{\delta|\Delta|}\left(\frac{t}{2}|\Delta|^2 + u|\Delta|^4\right)$$

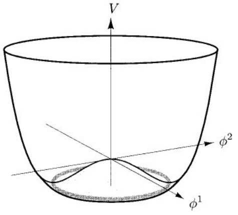
Ginzburg-Landau theory



$$\Rightarrow |\Delta|(t + 4u|\Delta|^2) = 0 \qquad |\Delta| = \begin{cases} 0 & t > 0 \\ \sqrt{\frac{t}{4u}} & t < 0 \end{cases}$$

For $t < 0$ U(1) symmetry is spontaneously broken.
$\Delta = |\Delta| e^{\Phi} \rightarrow \Phi$ field remains massless/gapless.

Ginzburg-Landau theory



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Goldstone theorem

Every time a continous global symmetry gets spontaneous broken there exists a gapless exitation $\rightarrow$ Goldstone mode.

Ginzburg-Landau theory

By calculating $\Pi(0,0)$ we can relate t with T:

$$T_c = \pi\omega_D \exp(-\frac{1}{N(0)g})$$
$$\frac{t}{2} \approx N(0)\frac{T - T_c}{T_c}$$

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The Ginzburg-Landau theory of superconductors was known (1956) before BCS-theory (1957) and predicted the right results. Parameters are unknown if you start with a GL theory but meaningfull predictions are possible.

Anderson-Higgs effect

Inclusion of em fields via minimal coupling:

$$\partial \mapsto \partial + ie\mathbf{A}$$

$$\mathcal{L}_{em} = \frac{-1}{4} F_{\mu\nu} F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu \mathbf{A}_n u - \partial_\nu \mathbf{A}_\mu$$

$$\rightarrow \mathcal{Z} = \int \mathcal{D}\mathbf{A} \int \mathcal{D}[\Delta, \bar{\Delta}] e^{-S_{eff}}$$

Where S_{eff} gets modified:

$$S_{eff} = \beta \int d\mathbf{r} \left[\frac{t}{2} |\Delta|^2 + \frac{K}{2} |(\partial + i2e\mathbf{A})\Delta|^2 + u|\Delta|^4 + \frac{1}{2} (\partial x \mathbf{A})^2 \right]$$

where we used $\qquad A_o = \Phi = 0$

Anderson-Higgs effect

All terms in this Lagrangian are gauge invariant under local gauge transformations:

$$\mathbf{A} \mapsto \mathbf{A} - \partial\Phi(\mathbf{r})$$
$$\Delta \mapsto e^{-2ie\Phi(\mathbf{r})}\Delta$$

Anderson-Higgs effect

All terms in this Lagrangian are gauge invariant under local gauge transformations:

$$\mathbf{A} \mapsto \mathbf{A} - \partial\Phi(\mathbf{r})$$
$$\Delta \mapsto e^{-2ie\Phi(\mathbf{r})}\Delta$$

Write:

$$\Delta(r) = |\Delta(\mathbf{r})|e^{-2ie\Phi(\mathbf{r})}$$

and choose a gauge (unitary gauge):

$$\mathbf{A} \mapsto \mathbf{A} - \partial\Phi(\mathbf{r}) \qquad \Delta \mapsto |\Delta|$$

Anderson-Higgs effect

this results in the action:

$$S_{eff} = \beta \int d\mathbf{r} \left[\frac{t}{2}|\Delta|^2 + \frac{K}{2}(\partial|\Delta|)^2 - \frac{4e^2K|\Delta|^2}{2}\mathbf{A}^2 + u|\Delta|^4 + \frac{1}{2}(\partial x \mathbf{A})^2 \right]$$

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below T_c $\Delta \neq 0 \Rightarrow m_A^2 = 4e^2K|\Delta|^2 \neq= 0$.

Anderson-Higgs effect

The goldstone mode gets eaten by the gauge field which aquires a mass.

Anderson-Higgs effect

The m_A^2-term is responsible for London equation, because minimisation wrt **A**:

$$\partial x \underbrace{(\partial x \mathbf{A})}_{\mathbf{B}} + m_A^2 \mathbf{A} = 0$$

Anderson-Higgs effect

The m_A^2-term is responsible for London equation, because minimisation wrt $\mathbf{A}$:

$$\partial x \underbrace{(\partial x \mathbf{A})}_{\mathbf{B}} + m_A^2 \mathbf{A} = 0$$

With the help of Maxwell equation $\partial x \mathbf{B} = \frac{4\pi}{c}\mathbf{j}$ we obtain:

$$\mathbf{j} = -\frac{c m_A^2}{4\pi}\mathbf{A}$$

This was the basis for the **London equation**.

Conclusions

- We saw how a interacting fermionic system can be rewritten as a system noninteracting fermions in a bosonic field
- We could compute several important quantities like T_c or the gap Δ
- A Ginzburg-Landau ansatz with in orderparameter $< \Psi_\downarrow \Psi_\uparrow$ showed how symmetry breaking occurs
- We saw an example of Goldstones theorem
- By gauging the symmetry we obtained an example of Anderson-Higgs mechanism which is also important in high energy physics

literature

- P. Coleman 04, chapter 12
- Czycholl, Theoretische Festkörperphysik