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Condensed Matter Theory I — WS05/06

Exercise 4

(Please return your solutions before 06.12., 13:00 h)

4.1 Spectral Function (3 points)

In the lecture the spectral function $A(\omega)$ was defined. Show that this function is normalized, i.e.

$$\int \frac{d\omega}{2\pi} A(\omega) = 1$$

4.2 Spectral Weight (4 points)

As you learnt in the lecture the bare one-particle Green's function in Matsubara representation is given by

$$G^0(k, i\omega) = \frac{1}{i\omega - (\varepsilon_k - \mu)}$$

whereas the full one-particle Green's function differs from the bare one by the *self energy* contribution $\Sigma(k, i\omega)$

$$G(k, i\omega) = \frac{1}{i\omega - (\varepsilon_k - \mu) - \Sigma(k, i\omega)}$$

At low temperature only the processes around the Fermi edge $(\omega = 0, k = k_F)$ are physically relevant. Assume that the non-interacting states are free particles

$$\varepsilon_k = \frac{k^2}{2m}$$

and assume furthermore that one can expand the denominator in the vicinity of $(0, k_F)$. Show that in first order the full Green's function can be written as

$$G(k, i\omega) = \frac{z}{i\omega - (\varepsilon_k^* - \mu^*)} \qquad \text{with}: \quad \varepsilon_k^* = \frac{k^2}{2m^*}$$

Express z, μ^* and m^* as functions of (the derivatives of) Σ. Which values can z take?

4.3 Large-ω Behaviour of the Green's Function (3 points)

Show that the full one-particle Green's function obeys the following rule:

$$G(\omega) \;\rightarrow\; \frac{1}{\omega} \qquad \text{for } \omega \rightarrow \infty$$

4.4 Diagram Technique I (4 points)

In the lecture the rules for calculating time-domain Feynman diagrams were presented. Show by Fourier transformation that for a translational invariant potential $V(x, y) = V(x - y)$ both energy and momentum are conserved at each vertex.

4.5 Diagram Technique II (4 points)

Consider the local potential from exercise 3.3 and calculate the self energy in first order pertubation theory (Fig. 1).
Replace now the bare propagator by the full one (Fig. 2) and compare the result with the one from the mean field approximation 3.3.d.

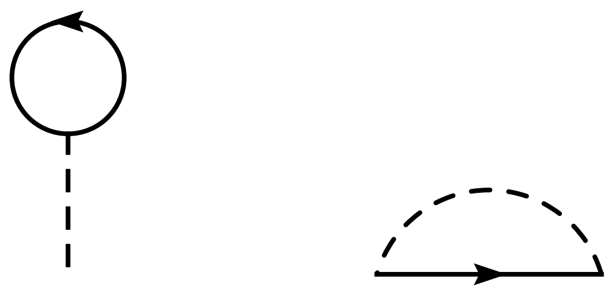

Figure 1: First Order Self Energy Diagrams

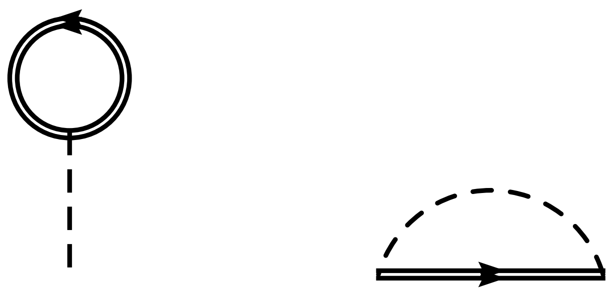

Figure 2: Renormalised First Order Self Energy Diagrams