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Condensed Matter Theory I — WS05/06

Exercise 7

(Please return your solutions before 07.02., 13:00 h)

7.1 Meissner Effect (10 points)

The London equations of superconductivity are

$$\frac{\partial}{\partial t}\left(\Lambda\vec{j}\right) = \vec{E} \qquad \vec{\nabla}\times\left(\Lambda\vec{j}\right) = -\vec{H} \qquad \Lambda = \frac{m}{2n_s e^2}$$

Consider a thin superconducting slab of thickness d which is infinitely extended in y and z direction and which is in a uniform magnetic field $\vec{H}$ parallel to the slab surface.

a) Calculate the magnetic field $\vec{H}(\vec{x})$ inside the slab and draw the result. What is the London penetration depth λ_L?

b) Calculate also the current density inside the slab and again draw the result.

c) Calculate the average magnetization of the slab as a function of d.

Consider now a infinitely long cylindrical superconducting wire of radius R ($R \gg \lambda_L$) and a current flowing through the wire.

d) Show that the current density distribution $\vec{j}(r)$ obeys a ordinary differential equation of the form (*modified Bessel differential equation*)

$$x^2\frac{d^2 j}{dx^2} + x\frac{dj}{dx} - x^2 j = 0$$

e) Draw the solution of d) (*modified Bessel function of the first kind*) and show that the current flows only in a small layer underneath the surface of the wire.

7.2 Ginzburg-Landau Theory (6 points)

In the lecture the Ginzburg-Landau theory of second order phase transitions for superconductors was discussed.

a) Use the expansion of the free energy density in the Ginzburg-Landau theory to calculate the critical magnetic field $\vec{H}_c$ of the superconductor.

b) Consider again a cylindrical superconducting wire with radius R and an electrical current flowing through this wire. The current induces a magnetic field outside the wire. When the current exceeds a critical value I_c the magnetic field at the surface will exceed $\vec{H}_c$ and superconductivity will collapse. Calculate I_c.