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Übungen zur Festkörpertheorie II — SS04

2. Übungsblatt

1. Density Correlation Function - Friedel Oscillation

In a free electron gas the retarded density-density correlation function is defined as

$$\chi(x,x') = -i\theta(t-t')\sum_{\sigma\sigma'} < [\Psi^\dagger_\sigma(x)\Psi_\sigma(x)\ ,\ \Psi^\dagger_{\sigma'}(x')\Psi_{\sigma'}(x')] > \tag{1}$$

where x,x' include space and time coordinates.

a) Derive an expression for Eq. 1 in momentum and energy space by applying the diagram technique for T $\neq 0$. The resulting function $L(q,\omega)$ is called Lindhard function.

b) Evaluate and draw $L(q,\omega)$ in the stationary case $\omega = 0$ and for temperature $T \to 0$.

c) Transform L(q) back into coordinate space. In the limit of large r one can see the *Friedel Oscillation* of the stationary density-density correlation function $\chi(\vec{r},0)$.

Effective Interaction - Random Phase Approximation (RPA)

In the first exercise the density-density correlation $\chi^0_q(t,t')$ for the free electron gas has been calculated diagrammatically. Now an arbitrary interaction v(q) is introduced to derive an expression for the full $\chi_q(t,t')$.

a) Draw the diagram series for $\chi_q(t,t')$ up to first order perturbation theory. Show that one can write this series as a Dyson equation by introducing the irreducible polarization part $\Lambda_q(t,t')$, defined as the sum over all closed bubbles in the expansion of $\chi_q(t,t')$. Proof the relation

$$\chi_q = \frac{\Lambda_q}{1 - v(q)\Lambda_q} \tag{2}$$

b) Show by drawing the corresponding diagram series, that similiar to a) one can define an effective interaction by

$$V_q^{eff} = \frac{v(q)}{1 - v(q)\Lambda_q} \tag{3}$$

The RPA is defined by regarding only the lowest order Λ_q^0 which is exactly the Lindhardt function of exercise 1.).

Remarks:
Depending of the type of interaction v(q) there arise different effects in the electron system. When a local charge is introduced, the effective interaction can be understood due to the interaction of the electron charge density with the local charge.
If one regards two local spins $\vec{S_1}, \vec{S_2}$ placed into an electron gas, V_{eff} describes an effective interaction $J\vec{S_1} \cdot \vec{S_2}$ mediated by the electron spin density. This interaction is known as Rudermann-Kittel-Kasuya-Yosida (RKKY) interaction.