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Übungen zur Festkörpertheorie II — SS04

3. Übungsblatt

1. Density Correlation Function - 1d

To complete the discussion about the density-density correlation function of the last exercise, calculate the following expression in 1d :

$$\chi_{\rho\rho}^{\sigma\sigma'}(q) = \delta_{\sigma\sigma'} \sum_k \frac{f(\epsilon_k) - f(\epsilon_{k+q})}{\epsilon_k - \epsilon_{k+q}} \tag{1}$$

Show, that in 1d Eq. 1 shows a real divergence at $q = 2k_F$ (in 3d the function was non-analytical but not diverging).

2. Pair-Pair Correlation function

The pair-pair correlation function, in the Matsubara formalism is defined as

$$\Pi_{\sigma\sigma'}(1,2) = - < T(\Psi_{\sigma'}(1)\Psi_\sigma(1)\Psi_\sigma^\dagger(2)\Psi_{\sigma'}^\dagger(2) > \tag{2}$$

a) Perform the Fourier transform of Eq. 2 and derive an expression for $\Pi_{\sigma\sigma'}(q, \omega = 0)$ using the diagram technique.

b) Evaluate this equation in 1d. Show that the results diverges like 1/q.

c) Now an attractive pair-pair interaction is included. This interaction can be local in space and time. Show that summing a RPA-like perturbation series for q=0 leads to a critical temperature T_c. At T_c the pair-pair correlation function diverges.