Lecture Theoretical Physics IV - SS 2005 - Prof. H. Kroha

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Exercises 1

Exercices on April 17th - 21st.

Exercice 1.1 Maxwell relations

The function $f(x, y)$ has exact differential

$$df = u(x, y)dx + v(x, y)dy.$$

Show that

$$(\frac{\partial u}{\partial y})_x = (\frac{\partial v}{\partial x})_y.$$

Remark: f has an exact differential, if it holds: $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.

Aufgabe 1.2 Probability

a) What is the probability for two of ten students of having their birthday on the same day? By how many students is the probability that at least two of them have their birthday on the same day $\frac{1}{2}$?

b) One takes three cards out of a 32-card deck, one at time. What is the probability that the third card is a queen?
One takes now eight cards out of a 52-card deck. Find the probabilities of taking
   - four aces and two kings,
   - at least one ace.

Exercice 1.3 Physical system with auxiliary conditions

The method of Lagrange multiplicators

a) Classical mechanics: catenary curve. A rope is hung from two points at the same height suspended in a gravitational field. Which curve describes the equilibrium position of the rope? We remain that in equilibrium the potential energy of the rope is minimal.

b) Quantum mechanics: let H be a Hamilton operator. Determine the minimum of $\langle\Psi|H|\Psi\rangle$ in the Hilbert space of all states $|\Psi\rangle$ under the auxiliary condition, that the $|\Psi\rangle$ are normed. Show that

$$E_0 = \min_{\{|\Psi\rangle\},\lambda} \langle\Psi|H - \lambda|\Psi\rangle + \lambda$$

gives the lowest energy eigenvalue, where the minimum of the expectation value will be taken over all states $|\Psi\rangle$ and real parameters λ.

Good luck!