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Übungen zur Festkörpertheorie I — WS03/04

5. Übungsblatt

1. Phonons in a metal: Kohn anomaly

   In metals one expects that the harmonic interactions between the ions of a lattice are not only between nearest neigbors, but are longranged. The physical origin is that a displacement of an ionic charge induces an electric charge polarization in the electron sea which is longranged and oscillatory in space. This oscillatory polarization acts on the other ions in the lattice. The oscillations are induced by the fact that the electronic states are filled up to the Fermi wave number k_F, and the wave number of the resulting oscillatory ion-ion potential is $2k_F$ ("Friedel oscillations").

   We now consider a one-dimensional linear chain of ions in an electron sea, the lattice constant is a. The force constant between an ion on lattice site i and an ion on lattice site j is

   $$\kappa_{i-j} = \kappa_0 \, \frac{\sin[2k_F a(i-j)]}{k_F a(i-j)}$$

   a) Write down the Langrange function for the system of ions and derive the equations of motion.

   b) Make an ansatz of plane wave solutions for the ion displacements, $q_j(t) = q_0 \exp[i(kx_j - \omega t)]$, $x_j = ja$, and derive an expression for $\omega(k)^2$ and $\partial\omega^2/\partial k$. Show that $\omega(k)^2$ has a cusp (divergent slope) at $k = 2k_F$. This effect is called the Kohn anomaly after Walter Kohn, who predicted it in 1959.

   c) Draw the curve $\omega(k)$.

2. Second quantization

   During the exercise the concept of second quantization will be discussed.