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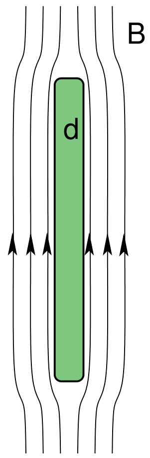
Übungen zur Festkörpertheorie I — WS03/04

6. Übungsblatt

Stationary electrodynamics of type I superconductors

1. Meißner effect in a thin superconducting slab in parallel magnetic field

Consider a thin superconducting slab of thickness d, which is infinitely extended in the y and z direction, in a uniform magnetic field $\vec{B}$ parallel to the slab surface. The superconductor is described by the London equations



$$\begin{aligned} \frac{\partial}{\partial t}\left(\Lambda\vec{j}\right) &= \vec{E}\ , \quad \Lambda = \frac{m}{n_s q^2} \\ \nabla \times \vec{H} &= \frac{4\pi}{c}\vec{j}\ . \end{aligned}$$

a) Calculate the magnetic field distribution $H(x)$ in the slab. What is the London penetration depth λ_L?

b) Calculate the average magnetization of the slab in dependence of its thickness.

2. Critical field and critical current of a type I superconducting wire

a) Calculate the critical field H_c of of a macroscopic sample of a type one superconductor, using the Ginzburg-Landau theory.

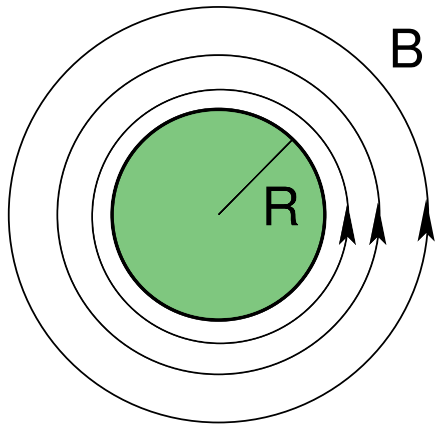
The current flowing through a superconducting wire generates a magnetic field B outside of the wire. When at the surface of the wire this field reaches the critical value H_c, the superconductivity will collapse. Therefore, the critical field and, hence, the critical current depend on the geometry of the wire. We consider a cylindrical wire of radius R.

b) Show that the magnetic field lines form concentrical circles around the wire, and calculate the field strength B at the wire surface. Determine the critical current in the wire at which the superconductivity is destroyed?

— Please turn —

c) Calculate the current density distribution $j(r)$ in the wire, and show that the current flows only in layer of thickness λ_L underneath the surface of the wire.

d) Typical values for the London penetration depth and for the critical field are $\lambda_L = 500$ Å and $H_c = 500$ Œ, respectively. What is the current density averaged over a surface layer of thickness λ_L?



BCS theory

3. Temperature dependence of the superconducting order parameter Δ

Using the BCS self-consistency equation

$$\frac{1}{|\lambda|N(0)} = \frac{1}{2}\int_0^{\hbar\omega_D} d\xi \frac{\tanh(\xi/2k_BT)}{\sqrt{\xi^2 - \Delta^2}}$$

determine the temperature dependence of the order parameter Δ

a) for $T \to 0$ and

b) for $t \to T_c$.