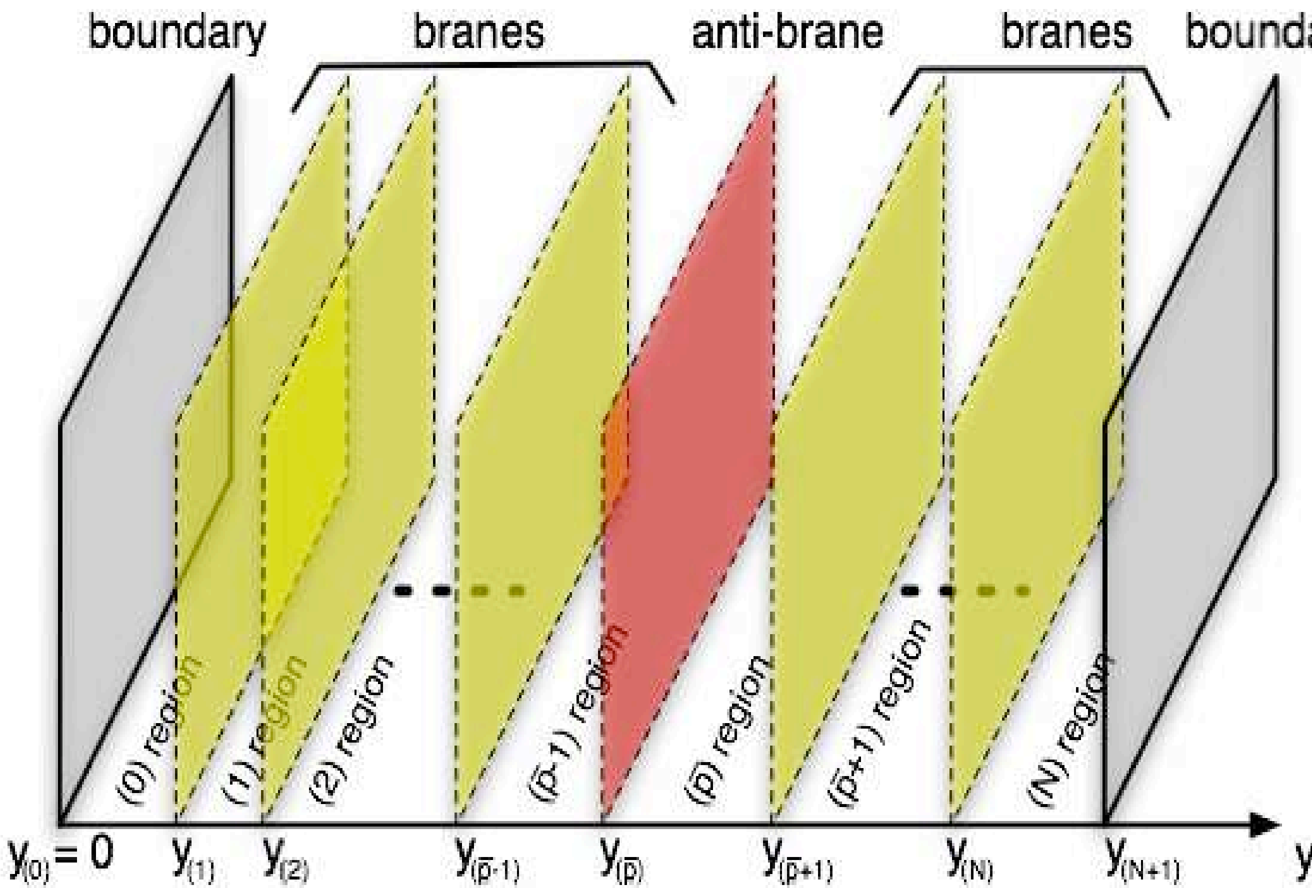
Anti-Branes in Heterotic M-theory

James Gray - University of Oxford.

Based on work with André Lukas and Burt Ovrut.

hep-th/0701025
arXiv:0709.2914

5D Heterotic M-theory including anti-branes.



What I say today will be valid for a single anti-brane, as shown, but generalizes trivially to an arbitrary number of these objects.

A Toy Model of the Warping in Heterotic.

$$S \sim \int d^5x \left[\partial_\alpha \Phi \partial^\alpha \Phi - \delta(y)\, S_{(0)} \Phi - \delta(y - \pi\rho)\, S_{(N+1)} \Phi - \sum_{p=1}^{N} \delta(y - y_{(p)}) S_{(p)} \Phi \right]$$

Bulk equation of motion: $\Box_5 \Phi = 0$

At the boundaries: $D_y \Phi|_{y=0} = -S_{(0)}$, $D_y \Phi|_{y=\pi\rho} = +S_{(N+1)}$

At the branes: $-D_y \Phi|_{y=y_{(p)}+} + D_y \Phi|_{y=y_{(p)}-} = S_{(p)}$

Perform the split: $\Phi = \phi_0(x^\mu) + \phi(x^\mu + y)$

$$\int_0^{\pi\rho} \phi \, dy = 0$$

Bulk equation becomes: $\Box_4 \phi_0 + \Box_4 \phi + D_y^2 \phi = 0$

Integrate the bulk equations across the orbifold and use boundary conditions:-

$$\Box_4 \phi_0 + \sum_p S_{(p)} = 0$$

Look at the case where warping is weak and 4d derivatives are small and substitute this back into the bulk equation:-

$$D_y^2 \phi = \sum_p S_{(p)}.$$

So in the end we have a system of equations for the warping:-

Bulk :

$$D_y^2\phi = \sum_p S_{(p)}$$

Boundaries:-

$$D_y\phi|_{y=0} = -S_{(0)} \ , \qquad D_y\phi|_{y=\pi\rho} = +S_{(N+1)}$$

Branes:-

$$-D_y\phi|_{y=y_{(p)}+} + D_y\phi|_{y=y_{(p)}-} = S_{(p)}$$

E.G. 1: The supersymmetric vacuum

- Sources S are the tensions of the branes.
- Sum of the charges on the compact space is zero.
- Objects are BPS so tension = charge.
- Therefore $\sum S = 0$ and the bulk equation becomes $D_y^2 \phi = 0$.

The warping in the supersymmetric vacuum is linear in y.

E.G. 2: Warping due to matter fluctuations

- The sources S are now the kinetic and potential terms for brane and boundary localized fields.
- Therefore $\sum S \neq 0$ (matter on different objects is independent).
- So in this case bulk equation is $D_y^2 \phi = \sum S$.

The warping due to matter field fluctuations is quadratic. This is typical of any change away from the pure tension vacuum in heterotic M-theory.

E.G. 3: Anti-branes in heterotic M-theory.

- Sources S are the tensions of the branes and anti-branes.
- Sum of the charges on the compact space is zero.
- For an anti-brane charge = − tension.
- So tensions do not sum to zero and we have in the bulk $D_y^2\phi = \sum S$.

Thus the warping due to the anti-brane is quadratic in y.

Heterotic M-theory in five dimensions:

The bulk theory:

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{4}G_{kl}(b)\partial b^k\partial b^l + \frac{1}{2}G_{kl}(b)\mathcal{F}^k_{\alpha\beta}\mathcal{F}^{l\alpha\beta} + \frac{1}{4}V^{-2}(\partial V)^2 + \lambda(d_{ijk}b^ib^jb^k - 6)\right.$$
$$+\frac{1}{4}\mathcal{K}_{a\bar{b}}(\mathfrak{z})\partial\mathfrak{z}^a\partial\bar{\mathfrak{z}}^{\bar{b}} - V^{-1}(\tilde{\mathcal{X}}_{A\alpha} - \bar{M}_{AB}(\mathfrak{z})\mathcal{X}^B_\alpha)([\Im(M(\mathfrak{z}))]^{-1})^{AC}(\tilde{\mathcal{X}}^\alpha_C - M_{CD}(\mathfrak{z})\mathcal{X}^{D\alpha})$$
$$\left.+\frac{1}{4!}V^2G_{\alpha\beta\gamma\delta}G^{\alpha\beta\gamma\delta} + m^2V^{-2}G^{kl}(b)\hat{\beta}_k\hat{\beta}_l\right]$$
$$-\frac{1}{2\kappa_5^2}\int\left(\frac{2}{3}d_{klm}\mathcal{A}^k\wedge\mathcal{F}^l\wedge\mathcal{F}^m + 2G\wedge((\xi^A\tilde{\mathcal{X}}_A - \tilde{\xi}_A\mathcal{X}^A) - 2m\hat{\beta}_k\mathcal{A}^k)\right)$$

- V :Volume of the Calabi-Yau.
- b^k : Shape of the Calabi-Yau.

Boundary theories:

$$-\int d^5x\,\delta(y)\sqrt{-h_{(0)}}\,\Big[\frac{m}{\kappa_5^2}V^{-1}b^k\tau_k^{(0)}+\frac{1}{16\pi\alpha_{\rm GUT}}V\,{\rm tr}({\rm F}^2_{(0)})+{\rm G}_{(0){\rm IJ}}{\rm D}_\mu{\rm C}^{\rm Ix}_{(0)}{\rm D}^\mu\bar{\rm C}^{\rm J}_{(0){\rm x}}$$
$$+V^{-1}G^{IJ}_{(0)}\frac{\partial W_{(0)}}{\partial C^{Ix}_{(0)}}\frac{\partial\bar W_{(0)}}{\partial\bar C^{J}_{(0)x}}+{\rm tr}(D^2_{(0)})\Big]$$

and similarly on the other boundary

Brane theories:

$$-\frac{1}{2\kappa_5^2}\int d^5x\,\Bigg\{\sum_{p=1}^{N}(\delta(y-y_{(p)})+\delta(y+y_{(p)}))\sqrt{-h_{(p)}}\,\Bigg[mV^{-1}\tau_k^{(p)}b^k+\frac{2m(n^k_{(p)}\tau_k^{(p)})^2}{V(\tau_l^{(p)}b^l)}j_{(p)\mu}j^\mu_{(p)}$$
$$+[\Im\Pi]_{(p)uw}E^u_{(p)\mu\nu}E^{w\mu\nu}_{(p)}\Bigg]-4m\hat C_{(p)}\wedge\tau_k^{(p)}d(n^k_{(p)}s_{(p)})-2[\Re\Pi]_{(p)uw}E^u_{(p)}\wedge E^w_{(p)}\Bigg\}$$

where $\quad j_{(p)\mu}=\dfrac{\beta_k^{(p)}}{n^l_{(p)}\beta_l^{(p)}}(d(n^k_{(p)}s_{(p)})-\hat{\mathcal A}^k_{(p)})_\mu$.

These actions are supplemented by some Bianchi identities:

$$\begin{aligned}
(dG)_{y\mu\nu\gamma\rho} &= -4\kappa_5^2(J^{(0)}_{4\mu\nu\gamma\rho}\delta(y) + J^{(N+1)}_{4\mu\nu\gamma\rho}\delta(y-\pi\rho)) \\
(d\mathcal{F}^k)_{y\mu\nu} &= -4\kappa_5^2(J^{(0)k}_{2\mu\nu}\delta(y) + J^{(N+1)k}_{2\mu\nu}\delta(y-\pi\rho)) \\
(d\mathcal{X}^A\mathcal{G}_A - d\tilde{\mathcal{X}}_B\mathcal{Z}^B)_{y\mu} &= -4\kappa_5^2(J^{(0)}_{1\mu}\delta(y) + J^{(N+1)}_{1\mu}\delta(y-\pi\rho))
\end{aligned}$$

Where the magnetic sources here are determined by the matter and gauge field fluctuations.

$$\begin{aligned}
J^{(p)}_{4\mu\nu\gamma\rho} &= \frac{1}{16\pi\alpha_{\text{ GUT}}}\text{tr}(F_{(p)}\wedge F_{(p)})_{\mu\nu\gamma\rho} \\
J^{(p)k}_{2\mu\nu} &= -i\sum_{I,J}\Gamma^k_{(p)IJ}(D_\mu C^{Ix}_{(p)}D_\nu\bar{C}^J_{(p)x} - D_\mu\bar{C}^I_{(p)x}D_\nu C^{Jx}_{(p)}) \\
J^{(p)}_{1\mu} &= \frac{e^{-\mathcal{K}}}{2V}\sum_{I,J,K}\lambda_{IJK}f^{(IJK)}_{xyz}C^{Ix}_{(p)}C^{Jy}_{(p)}D_\mu C^{Kz}_{(p)}
\end{aligned}$$

So we just follow a very similar procedure to that shown in the toy model:

- We need a metric ansatz:

$$\begin{aligned} ds_5^2 &= a^2(y,x^\mu) g_{4\mu\nu} dx^\mu dx^\nu + b^2(y,x^\mu) dy^2 \\ V &= V(y,x^\mu) \\ b^k &= b^k(y,x^\mu)\ . \end{aligned}$$

- We need embeddings for the branes (appears in the induced metric etc.).

$$X^\mu = \sigma^\mu \qquad\qquad Y = y_{(p)}(\sigma^\mu)$$

Solve for the warping as before:

$$\begin{aligned} \frac{a_{(p)}}{a_0} &= 1 - \epsilon_0 \frac{b_0}{3V_0} b_0^k \left[h_{(p)k} - \delta_k \left(z^2 - \frac{1}{3} \right) \right] \\ \frac{V_{(p)}}{V_0} &= 1 - 2\epsilon_0 \frac{b_0}{V_0} b_0^k \left[h_{(p)k} - \delta_k \left(z^2 - \frac{1}{3} \right) \right] \\ b_{(p)}^k &= b_0^k + 2\epsilon_0 \frac{b_0}{V_0} \left[\left(h_{(p)}^k - \frac{1}{3} h_{(p)l} b_0^k b_0^l \right) - \left(\delta^k - \frac{1}{3} \delta_l b_0^k b_0^l \right) \left(z^2 - \frac{1}{3} \right) \right] \end{aligned}$$

Here y has been rescaled to give z and h is a linear function in z.

$$h_{(p)k}(z) = \sum_{q=0}^{p} \tau_k^{(q)} (z - z_{(q)}) - \frac{1}{2} \sum_{q=0}^{N+1} \tau_k^{(q)} z_{(q)} (z_{(q)} - 2) - \delta_k$$

Points to notice:

- The warping is quadratic as promised. As we turn the anti-brane into a brane ($\delta \longrightarrow 0$) then it goes back to being linear.

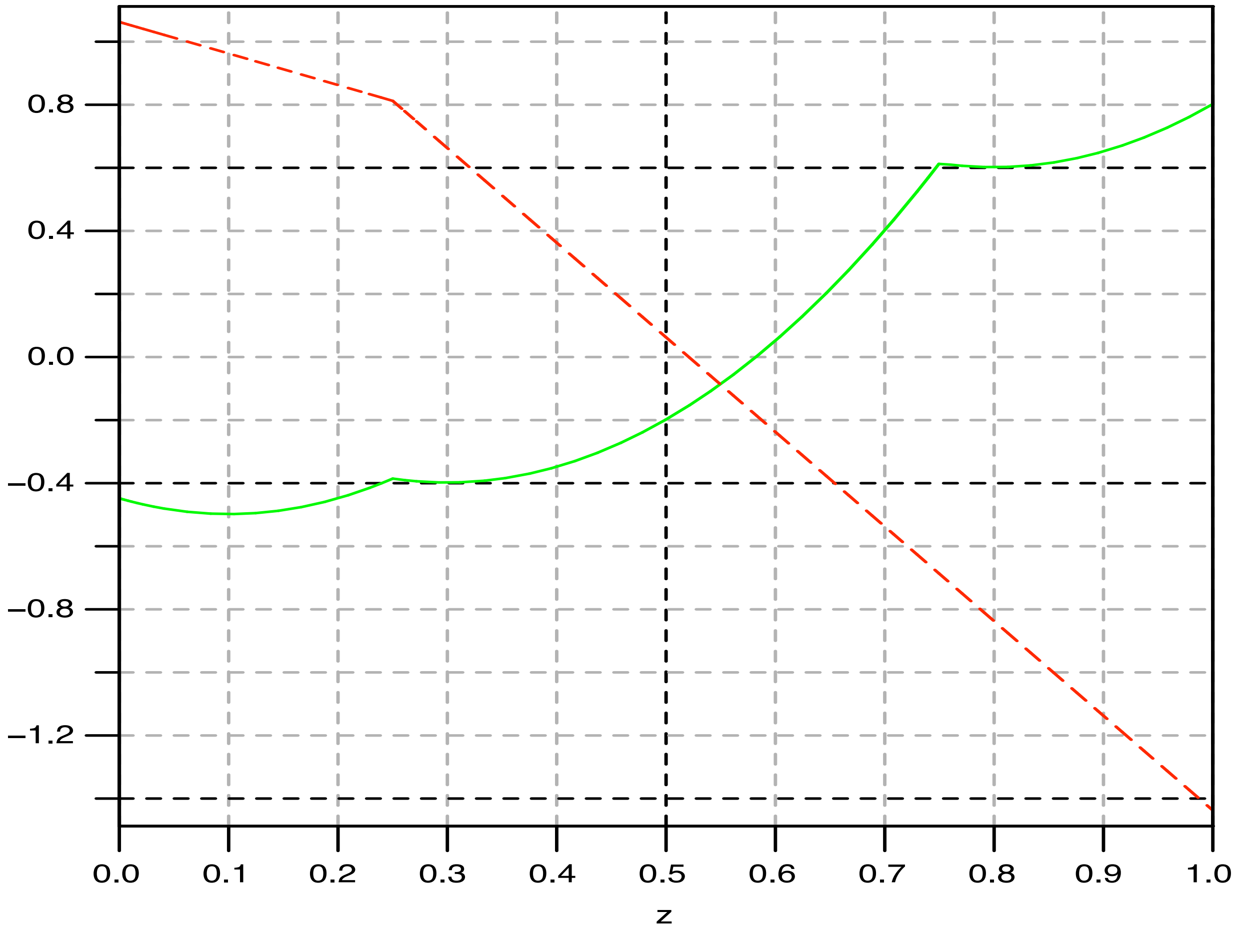
- The orbifold average of the z dependent parts are zero.

- The warpings are all controlled by the parameter:

$$\epsilon_S = \epsilon_0 \frac{b_0}{V_0}$$

Four dimensional heterotic M-theory is constructed as an expansion in this quantity.

An example:



- Red line is a supersymmetric case with one brane.
- The green line is what happens if you add an anti-brane.

Results: The four dimensional effective theory.

- We can use these warpings to systematically derive the four dimensional effective theory by dimensional reduction.
- Today I will present parts of the bosonic action. I will start with zeroth and first order in ϵ_S and then move on to some terms at second order.

Split up the first order result into pieces which contain the sum of the : tensions and pieces which do not.

$$S = S_{\delta^0} + S_{\delta^1}$$

$$S_{\delta^0} = S_4^{\rm moduli} + S_4^{\rm gauge} + S_4^{\rm matter}$$

$$\begin{aligned}
S_4^{\rm moduli} &= -\frac{1}{2\kappa_4^2}\int d^4x\sqrt{-g_4}\,\Bigg[\frac{1}{2}R_4 + \frac{3}{4}(\partial\beta)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{4}e^{-2\phi}(\partial\sigma)^2 + \frac{1}{4}G_{kl}\partial b^k\partial b^l + e^{-2\beta}G_{kl}\partial\chi^k\partial\chi^l \\
&\quad + \frac{1}{4}\mathcal{K}_{a\bar{b}}(\mathfrak{z})\partial\mathfrak{z}^a\partial\bar{\mathfrak{z}}^{\bar{b}} + 2\epsilon_0\sum_{p=1}^{N}\tau_k^{(p)}z_{(p)}e^{-2\phi}\partial\sigma\partial(n_{(p)}^k\nu_{(p)}) + \frac{\epsilon_0}{2}\sum_{p=1}^{N}b^k\tau_k^{(p)}e^{\beta-\phi}(\partial z_{(p)})^2 \qquad (41)\\
&\quad + 2\epsilon_0\sum_{p=1}^{N}\frac{\tau_l^{(p)}\tau_k^{(p)}}{\tau_m^{(p)}b^m}e^{-\phi-\beta}\left(\chi^l\chi^k(\partial z_{(p)})^2 - 2\chi^k\partial(n_{(p)}^l\nu_{(p)})\partial z_{(p)} + \partial(n_{(p)}^k\nu_{(p)})\partial(n_{(p)}^l\nu_{(p)})\right) \\
&\quad + \lambda(d_{ijk}b^ib^jb^k - 6)\Bigg] \\
S_4^{\rm gauge} &= -\frac{1}{16\pi\alpha_{\rm GUT}}\int d^4x\sqrt{-g_4}\,\Bigg[e^{\phi}\left({\rm tr}F_{(0)}^2 + {\rm tr}F_{(N+1)}^2\right) - \frac{1}{2}\sigma\epsilon_{\mu\nu\rho\gamma}\left(F_{(0)}^{\mu\nu}F_{(0)}^{\rho\gamma} + F_{(N+1)}^{\mu\nu}F_{(N+1)}^{\rho\gamma}\right) \qquad (42)\\
&\quad + \sum_{p=1}^{N}\left([\Im\Pi]_{(p)uw}E_{(p)}^uE_{(p)}^w - \frac{1}{2}[\Re\Pi]_{(p)uw}\epsilon_{\mu\nu\rho\gamma}E_{(p)}^{u\mu\nu}E_{(p)}^{w\rho\gamma}\right)\Bigg] \\
S_4^{\rm matter} &= -\int d^4x\sqrt{-g_4}\sum_{p=0,N+1}\Bigg[\frac{1}{2}\Big(e^{-\beta}G_{(p)MN}DC_{(p)}^{Mx}D\bar{C}_{(p)x}^{N} - 2e^{-2\beta}G_{kl}\omega_{1\mu}^{(p)k}\partial^\mu\chi^l \qquad (43)\\
&\quad + e^{-\phi-2\beta}G_{(p)}^{MN}\frac{\partial W_{(p)}}{\partial C_{(p)}^{Mx}}\frac{\partial\bar{W}_{(p)}}{\partial\bar{C}_{(p)x}^{M}} + e^{-2\beta}{\rm tr}(D_{(p)}^2)\Big)\Bigg]
\end{aligned}$$

Supersymmetric in form despite containing anti-brane moduli.

Given these definitions of complex fields:

$$
\begin{aligned}
S &= e^{\phi} + \epsilon_0 e^{\beta} \sum_{p=1}^{N} (\tau_k^{(p)} b^k) z_{(p)}^2 + i \left(\sigma + 2\epsilon_0 \sum_{p=1}^{N} \tau_k^{(p)} \chi^k z_{(p)}^2 \right) \\
&= e^{\phi} + i\sigma + \epsilon_0 \sum_{p=1}^{N} \tau_k^{(p)} z_{(p)}^2 T^k \\
T^k &= e^{\beta} b^k + 2i\chi^k \\
Z_{(p)} &= \tau_k^{(p)} b^k e^{\beta} z_{(p)} + 2i\tau_k^{(p)} (-n_{(p)}^k \nu_{(p)} + \chi^k z_{(p)}) \\
&= z_{(p)} \tau_k^{(p)} T^k - 2i\tau_k^{(p)} n_{(p)}^k \nu_{(p)},
\end{aligned}
$$

The kinetic terms on the previous slide are reproduced by the following Kahler and super potentials.

$$\kappa_4^2\, K_{\text{scalar}} = K_D + K_T + \mathcal{K} + K_{\text{matter}} \ ,$$

$$\begin{aligned}
K_D &= -\ln\left[S + \bar{S} - \epsilon_0 \sum_{p=1}^{N} \frac{(Z_{(p)} + \bar{Z}_{(p)})^2}{\tau_k^{(p)}(T^k + \bar{T}^k)} \right] \ , \\
K_T &= -\ln\left[\frac{1}{48} d_{klm}(T^k + \bar{T}^k)(T^l + \bar{T}^l)(T^m + \bar{T}^m) \right] \\
\mathcal{K}(\mathfrak{z}) &= -\ln\left[2i(\mathcal{G} - \bar{\mathcal{G}}) - i(\mathfrak{z}^p - \bar{\mathfrak{z}}^p)\left(\frac{\partial \mathcal{G}}{\partial \mathfrak{z}^p} + \frac{\partial \bar{\mathcal{G}}}{\partial \bar{\mathfrak{z}}^p} \right) \right] \\
K_{\text{matter}} &= e^{K_T/3} \sum_{p=0,N+1} G_{(p)MN} C_{(p)}^{Mx} \bar{C}_{(p)x}^{N} \ ,
\end{aligned}$$

$$W_{(p)} = \sqrt{4\pi\alpha_{\text{GUT}}} \sum_{I,J,K} \lambda_{IJK} f_{xyz}^{(IJK)} C_{(p)}^{Ix} C_{(p)}^{Jy} C_{(p)}^{Kz}$$

$$S_{\delta^1} = -\int d^4x\sqrt{-g_4}\,\mathcal{V}_1$$

$$\mathcal{V}_1 = \kappa_4^{-2}\frac{\epsilon_0}{(\pi\rho)^2}e^{-\phi-2\beta}b^k\delta_k$$

- This, up to a factor of 2, is just the tension of the anti-brane.
- So, from the point of view of moduli stabilization, the `KKLT' procedure of just adding the anti-brane energy to the supersymmetric theory is exact to first order....
- ....up to a correction to the gauge kinetic functions which I will present now, and corrections to the matter Lagrangian.

4d gauge kinetic terms to first order:

$$
\begin{aligned}
S_4^{\mathrm{GKF}} &= \frac{-1}{32\pi\alpha_{\mathrm{GUT}}}\int d^4x\sqrt{-g_4}\Bigg[\left(\left(e^\phi+\epsilon_0 e^\beta b^k\left(\sum_{p=0}^{N+1}\tau_k^{(p)}\left(z_{(p)}^2-2z_{(p)}\right)+\frac{4}{3}\delta_k\right)\right)\right)\mathrm{tr}(F_{(0)}^2) \\
&\qquad + \left(e^\phi+\epsilon_0 e^\beta b^k\left(\sum_{p=0}^{N+1}\tau_k^{(p)}z_{(p)}^2-\frac{2}{3}\delta_k\right)\right)\mathrm{tr}(F_{(N+1)}^2) \\
&\quad -\frac{1}{2}\left(\sigma+2\epsilon_0\left(\sum_{p=1}^{N}\beta_k^{(p)}\chi^k\left(z_{(p)}^2-2z_{(p)}\right)-\beta_k^{(N+1)}\chi^k+2\sum_{p=1}^{N}\tau_k^{(p)}(n_{(p)}^k\nu_{(p)})\right)\right)\epsilon_{\mu\nu\rho\sigma}F_{(0)}^{\mu\nu}F_{(0)}^{\rho\sigma} \\
&\quad -\frac{1}{2}\left(\sigma+2\epsilon_0\left(\sum_{p=1}^{N}\beta_k^{(p)}\chi^k z_{(p)}^2+\beta_k^{(N+2)}\chi^k\right)\right)\epsilon_{\mu\nu\rho\sigma}F_{(N+1)}^{\mu\nu}F_{(N+1)}^{\rho\sigma}\Bigg]. \qquad (63)
\end{aligned}
$$

- The changes from the supersymmetric case have important physical consequences: they can change which boundary undergoes gaugino condensation for example.

These kinetic terms can be written in terms of a `gauge kinetic function' which, in this non-supersymmetric system, is no longer holomorphic in the complex fields.

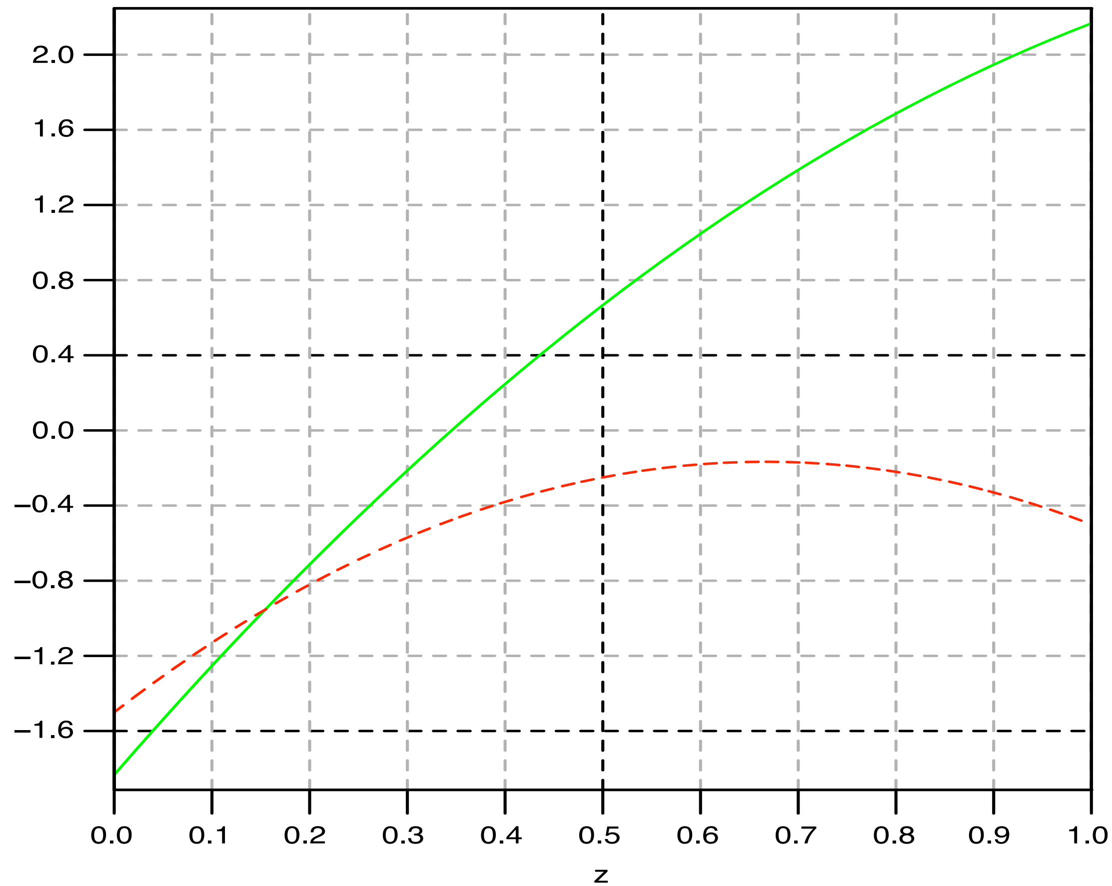
$$
\begin{aligned}
f_{(0)} &= S - \epsilon_0 \left[\tau_k^{(N+1)} T^k + 2\sum_{p=1}^{N} Z_{(p)} - \frac{2}{3}\delta_k (T^k + \bar{T}^k) \right. \\
&\qquad \left. + \delta_k (T^k - \bar{T}^k) \left(\left(\frac{Z_{(\bar{p})} + \bar{Z}_{(\bar{p})}}{\bar{\tau}_k (T^k + \bar{T}^k)} \right)^2 - 2\frac{Z_{(\bar{p})} + \bar{Z}_{(\bar{p})}}{\bar{\tau}_k (T^k + \bar{T}^k)} \right) \right] \\
f_{(N+1)} &= S + \epsilon_0 \left[\tau_k^{(N+1)} T^k - \frac{1}{3}\delta_k (T^k + \bar{T}^k) - \delta_k (T^k - \bar{T}^k) \left(\frac{Z_{(\bar{p})} + \bar{Z}_{(\bar{p})}}{\bar{\tau}_k (T^k + \bar{T}^k)} \right)^2 \right]
\end{aligned}
$$

The second order potential can also be obtained

$$\mathcal{V}_2 = \frac{1}{(\pi\rho)^2\kappa_4^2}\epsilon_0^2 e^{-\beta-2\phi}\, G^{kl}\delta_l \left[\sum_{p=0}^{\bar{p}-1}\tau_k^{(p)}\bar{z} - \sum_{p=\bar{p}+1}^{N+1}\tau_k^{(p)}\bar{z} - \sum_{p=0}^{\bar{p}-1}\tau_k^{(p)} z_{(p)} \right.$$

$$\left. + \sum_{p=\bar{p}+1}^{N+1}\tau_k^{(p)} z_{(p)} + \sum_{p=0}^{N+1}\tau_k^{(p)}(1 - z_{(p)})z_{(p)} - \frac{2}{3}\delta_k \right]$$

- The first four terms are the expected coulomb forces between the branes.
- The last two terms are new. They are in no sense smaller than the coulomb terms.
- We would expect similar terms to arise in other contexts such as type II string theories.



- Note these potentials would be straight lines in the case of the naive coulomb force.

- Note these inter-brane forces are second order in ϵ_S and so relatively weak in any controlled regime of moduli space.

A simple example.

$$S = -\frac{1}{2\kappa_4^2}\int \sqrt{-g}d^4x\left[\frac{R}{2} + \frac{3}{4}(\partial\beta)^2 + \frac{1}{4}(\partial\phi)^2 + \frac{1}{2}|q|e^{\beta-\phi}(\partial\bar{z})^2 + 2|q|e^{-2\beta-\phi} + e^{-\beta-2\phi}\left(6|q|q_1\bar{z} - 6|q|q_2\bar{z} + 6q^2(1-\bar{z})\bar{z} + 6|q|q_2 - \frac{4}{3}q^2\right)\right]$$

- e^{β} gives size of orbifold
- e^{ϕ} gives size of Calabi-Yau
- $\bar{z}$ is position of anti-brane
- q_1, q_2, q are charges of fixed planes and anti-brane respectively ($q_1 + q_2 + q = 0$)

Extensions:

- Flux can be added - resulting potential terms essentially unchanged from SUSY case:

$$W_{\rm flux} = \frac{\sqrt{2}}{\kappa_4^2}\left(\mathcal{X}^{\underline{A}}\mathcal{G}_{\underline{A}} - \tilde{\mathcal{X}}_{\underline{B}}\mathcal{Z}^{\underline{B}}\right)$$

$$W_{\rm flux} = \frac{\sqrt{2}}{\kappa_4^2}\epsilon_0 \frac{v^{\frac{1}{6}}}{(\pi\rho)^2}\left(\frac{1}{6}\tilde{d}_{\underline{abc}}\mathfrak{z}^{\underline{a}}\mathfrak{z}^{\underline{b}}\mathfrak{z}^{\underline{c}}n^0 - \frac{1}{2}\tilde{d}_{\underline{abc}}\mathfrak{z}^{\underline{a}}\mathfrak{z}^{\underline{b}}n^{\underline{c}} - \mathfrak{z}^{\underline{a}}n_{\underline{a}} - n_0\right)$$

$$V_{\rm flux} = e^{\kappa_4^2 K_{\rm mod}}\left(K_{\rm mod}^{i\bar{j}} D_i W_{\rm flux} D_{\bar{j}}\bar{W}_{\rm flux} - 3\kappa_4^2 |W_{\rm flux}|^2\right)$$

- Gaugino condensation can be added. Only change from SUSY case is due to change in G.K.F.

$$V_{\rm c+c/f+f} = \frac{1}{2}e^{\kappa_4^2 K_{\rm mod}}\left(\kappa_4^2|A\epsilon(S+\bar{S})e^{-\epsilon f}|^2 + \bar{A}\kappa_4^2\epsilon(S+\bar{S})e^{-\epsilon\bar{f}}W_{\rm flux} + \kappa_4^2\bar{W}_{\rm flux}A\epsilon(S+\bar{S})e^{-\epsilon f}\right)$$
$$+e^{\kappa_4^2 K_{\rm mod}}\left(K_{\rm mod}^{i\bar{j}} D_i W_{\rm flux} D_{\bar{j}}\bar{W}_{\rm flux} - 3\kappa_4^2|W_{\rm flux}|^2\right)$$

Conclusions

- We have included anti-branes in the vacuum of heterotic M-theory.
- There are unexpected forces between the branes and anti-branes. These are vital in any discussion of the cosmology or stabilization of anti-branes.
- There are corrections to the gauge kinetic functions which change the potential obtained from gaugino condensation.
- One can calculate the supersymmetry breaking seen in the matter sector explicitly (next thing to be done).
- Similar conclusions would be expected in other contexts involving anti-branes.