Axion Inflation in String Theory

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0705.3253 [hep-th] JHEP

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Introduction and Motivation

➪ Observational Cosmology:

- Inflation as a promising scenario
  - explains qualitative properties of universe (flatness, homogeneity, isotropy etc.)
  - growing experimental evidence from measurements of cosmological observables (e.g. WMAP data extracted from the CMB)

- Primordial gravitational waves: tensor to scalar ratio r

$$\boxed{r = P_g/P_{\mathcal{R}} \quad < \quad 0.3}$$ (current observational bound)

Future experiments: (e.g. Planck satellite) test r down to $\boxed{r > 0.01}$

➪ Important task: **Embed Inflation into String Theory**

- Challenge:

  Identify inflatons with sufficiently flat potentials in controlled compactification, e.g.

Brane inflation	Kähler moduli inflation	Racetrack inflation
Dvali,Tye and many others	Conlon, Quevedo; ...	Blanco-Pillado et al; ...

- Explicit realizations in controlled compactifications remain hard to construct

- Models reproducing known cosmological observable are not necessarily able to incorporate future observations

  Example:

  Primordial gravity waves – most string models only allow for unobservable small r

  Baumann,McAllister; Bean,Shandera,Tye,Xu; Kallosh,Linde

  Recently: first efforts to explore embeddings with possibly observable r

  Krause;Becker,Leblond,Shandera;Kallosh,Sivanandam,Soroush

Goal of the Talk

Study possibility of realizing inflation driven by a large number of axion fields within Type IIB string theory. Such models are interesting from conceptional point of view, since intrinsically stringy corrections need to be incorporated. They also can allow for a detectable amount of gravity waves $r < 0.14$.

Outline of the Talk

- Review: Axion inflation in supergravity
- Axion decay constants and stringy corrections
- Axion potentials: D1 instantons and D5 gaugino condensates
- Axion inflation in $\mathcal{N} = 1$ orientifold compactifications

Review: Axion Inflation in supergravity

➪ Assisted inflation of N axions: N-flation

Dimopoulos,Kachru,McGreevy,Wacker

see also: Kim,Nilles,Peloso

- effective 4D Lagrangian for N axion fields c^a:

$$\mathcal{L} \;=\; \tfrac{1}{2}\sum_a f_a^2\, \partial_\mu c^a \partial^\mu c^a - V_{\text{eff}}(c^a)$$

f_a are the axion decay constants

- canonically normalized axions: $\theta^a = f_a\ c^a$

$$-\pi \;<\; c^a \;\leq\; \pi \qquad\qquad -f_a\pi \;<\; \theta^a \;\leq\; f_a\pi$$

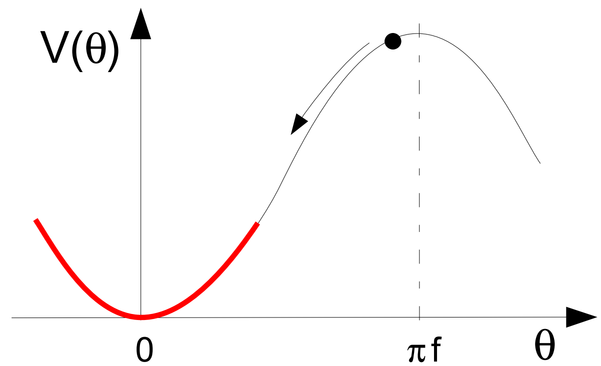
- effective potential:

$$V_{\text{eff}}(\theta^a) = C + \sum_{a=1}^{N} \Lambda_a^4 \Big(1 - \cos\Big[\frac{\mu^a\theta^a}{f_a}\Big]\Big)$$

$\Rightarrow$ μ^a slight extension of the original proposal of Dimopoulos et al.

- For one axion with $\mu^a = \mu = 1$:

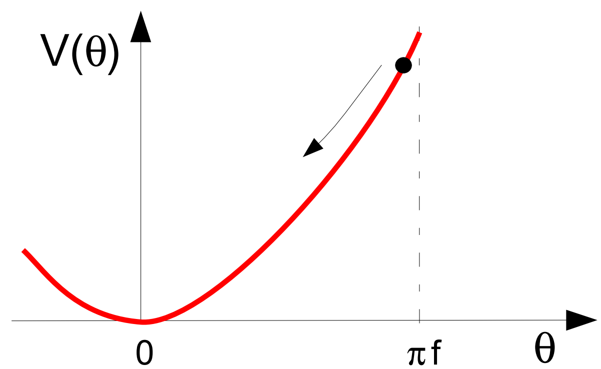
quadratic regime
(chaotic inflation)



- For one axion with $\mu^a = \mu < 1/3$:

quadratic in whole
field range of axion

(chaotic inflation)



➪ Assistance effect: Liddle,Mazumdar,Schunck; Kanti,Olive

Eqn. of motion for axions

$$\ddot{\theta}^a + 3H\ \dot{\theta}^a + \partial_{\theta^a} V = 0 \qquad H^2 = \frac{1}{3M_P^2}\left(\tfrac{1}{2}(\dot{\theta}^a)^2 + V\right)$$

Hubble friction H: contains the whole potential V of all fields θ^a

Downward force $\partial_{\theta^a} V$: contains only ath potential term

➪ Slow roll conditions: $\epsilon < 1\ , \quad |\eta| < 1$

$$\epsilon = \frac{M_P^2}{2}\sum_a \left(\frac{\partial_{\theta^a} V}{V}\right)^2 \qquad \eta = M_P^2\ \min_a \left(\frac{\partial^2_{\theta^a} V}{V}\right)$$

➪ Quadratic regime (for simplicity): chaotic inflation $\theta^a \approx \alpha\, M_P$

$$\epsilon = \frac{2}{N\alpha^2} \qquad \eta = \frac{2}{N\alpha^2}$$

$\Rightarrow$ slow roll inflation needs large α, N

➪ Large N ?

String compactifications can admit $N = 10^4$ or more axions!

➪ Large α ?

Recall: $\theta^a = \alpha M_P$ $\quad |\theta^a| < f_a \pi$ $\quad$ (f_a axion decay constants - kinetic terms)

Need to find:

String compactification: large number of axions N & large axion decay constants f_a.
During inflation an effective theory for only axions has to be valid.

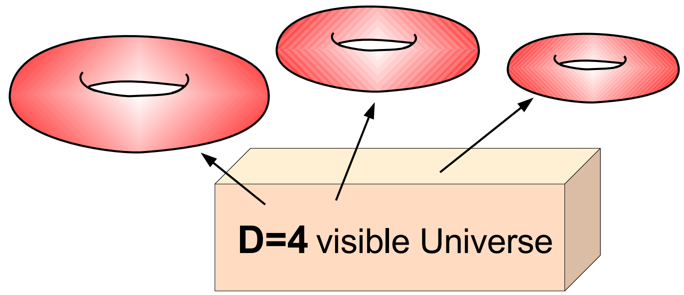
➪ Implications (chaotic regime) $\quad \alpha_{\rm in}$ starting point of inflation, $\quad m$ axion mass

$$N_e = \frac{N\alpha_{\rm in}^2}{4} \qquad \frac{\delta\rho}{\rho} = 4\frac{m}{M_P} N_e \qquad r = \frac{8}{N_e}$$

$\Rightarrow$ for $N_e \approx 55$ the tensor scalar ratio is $\boxed{r = 0.14}$ (maximal accessible value)

Axion decay constants and stringy corrections

➪ Compactify Type IIB string theory on Calabi-Yau manifold



➪ Axions from ten-dimensional p-form fields

$$B_2 = b^A \omega_A \qquad C_2 = c^A \omega_A \qquad C_4 = \rho_A \tilde{\omega}^A$$

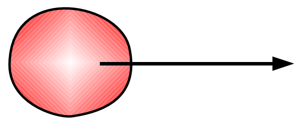
NS-NS B-field — R-R two and four-form form

$b^A,\ c^A,\ \rho_A$ scalar axions - coefficients in Kaluza-Klein expansion

$\omega_A, \tilde{\omega}^A$ two and four forms on Calabi-Yau manifold
(zero modes - lowest Kaluza-Klein modes)

➪ geometrical moduli: volumes of two-cycles v^A

Two cycle – (e.g. two sphere)



volume v^A

Associate:

R-R two-form axion:	c^A
R-R four-form axion:	ρ_A
NS-NS axion:	b^A

hyper multiplet to each two-cycle: (v^A, b^A, c^A, ρ_A)

➪ Axion decay constants: focus on axions c^A from R-R two-form C_2

dimensional reduction

$$\mathcal{L} = \frac{1}{2} f_{AB}^2 \partial_\mu c^A \partial^\mu c^B$$

$$\boxed{\frac{f_{AB}^2}{M_P^2} = g_s^2 \, G_{A\bar{B}}}$$

moduli space metric $G_{A\bar{B}}$

Moduli space geometry:

➪ $G_{A\bar{B}}$ metric on special Kähler manifold inside the moduli space of hyper multiplets:

- metric depends on v^A, b^A through complex coordinates $t^A = -b^A + iv^A$
- metric is Kähler

$$G_{A\bar{B}} = \partial_{t^A}\partial_{\bar{t}^B} K \qquad\qquad K = -\log \mathcal{V}$$

$\mathcal{V}(t,\bar{t})$ - quantum volume of Calabi-Yau manifold

- metric can be obtained from holomorphic pre-potential $\mathcal{F}(t)$ containing classical and α' quantum effects

$$\mathcal{V} = 2i(\mathcal{F} - \bar{\mathcal{F}}) - i(\partial_{t^A}\mathcal{F} + \partial_{\bar{t}^A}\bar{\mathcal{F}})(t^A - \bar{t}^A)$$

➪ <u>in general:</u> hyper multiplet moduli space has quaternionic geometry

Evaluating the axion decay constants:

➪ leading axion decay constants f^2_{AB}

$$\frac{f^2_{AB}}{M_P^2} = g_s^2 \left[-\frac{\partial_{t^A}\partial_{\bar{t}^B}\mathcal{V}}{\mathcal{V}} + \frac{\partial_{t^A}\mathcal{V}\,\partial_{\bar{t}^B}\mathcal{V}}{\mathcal{V}^2} \right]$$

f_{AB} is small for:
(1) small string coupling g_s
(2) large quantum volume $\mathcal{V}$

$\Rightarrow$ large number N of two-cyles bigger than string length ℓ_s^2

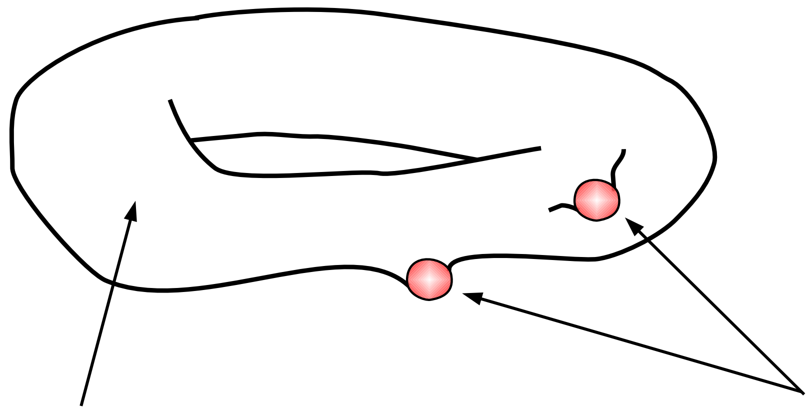
$\Rightarrow$ leads to a large volume $\mathcal{V}$ scaling with N

$\Rightarrow$ No assisted axion inflation!

➪ relax condition that all cycles are bigger than string scale

Scenarios with vanishing/small two-cycles:

➪ consider scenarios with two scales:

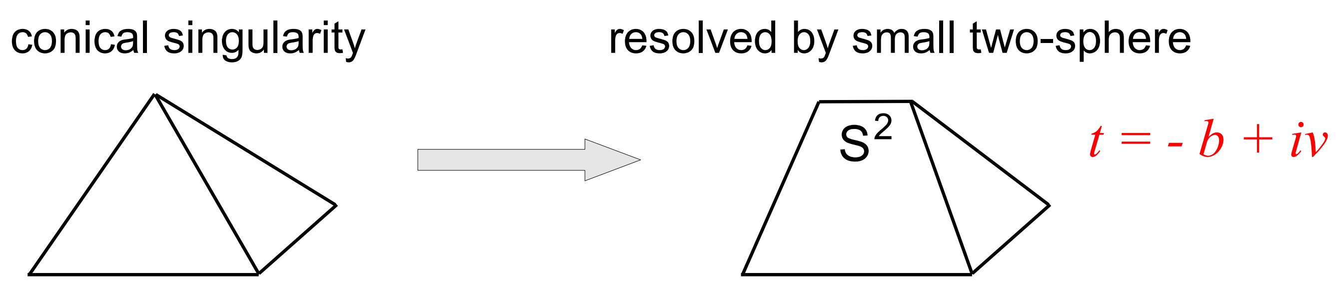


➪ large cycles with radius: $R/\ell_s > 1$ (keep total volume above string scale)

➪ large number N of small cycles of size: $|t| = |-b + iv| < 1$

$\Rightarrow$ N R-R two-form axions (two-volumes below string scale)

➪ Standard example of blown-up singularity: Resolved conifold



$\Rightarrow$ Vol$S^2 < \ell_s^2$ - strings start to wrap on small S^2

$\Rightarrow$ calculable α' corrections to the quantum volume:

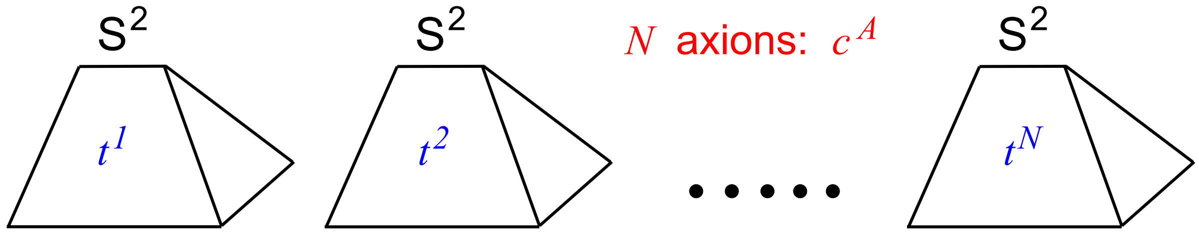
$$\mathcal{F}_{\rm cone} = -\frac{i}{2} t^2 \log t + \ldots \qquad \mathcal{V} = \mathcal{V}_{\rm class}(R) + |t|^2 \log |t| + \ldots$$

$\Rightarrow$ presence of small cycles does not increase $\mathcal{V}$

Compute: Large axion decay constants in resolved geometries

However: $f_a < M_P$ Banks, Dine, Fox, Gorbatov

➪ Generalize to: Set-ups with N vanishing cycles



➪ What about other stringy corrections ?

D1 branes wrapped around the S^2's can become light and correct the theory

Strominger; Becker,Becker,Strominger

Instanton contribution: $$\exp\Big(-\frac{|t^A|}{g_s}+ic^A\Big)$$

$\Rightarrow$ have to make sure that $|t|/g_s > 1$ small string coupling

$\Rightarrow$ D1 instantons are subleading in f_a $\Rightarrow$ f_a is independent of the axions c^A

➪ <u>However:</u> D1 instanton corrections are the leading corrections to the scalar potential

Axion potentials: D1 instantons and D5 gaugino condensates

➪ <u>Recall:</u> Axion N-flation requires a potential of the form

$$V_{\rm eff}(\theta^a) = C + \sum_{a=1}^{N} \Lambda_a^4 \Big(1 - \cos\Big[\frac{\mu^a \theta^a}{f_a}\Big]\Big) \qquad\qquad \theta^a = f_a\, c^a$$

➪ To discuss potentials:

$\Rightarrow$ Break supersymmetry to $\mathcal{N}=1$ in four space-time dimensions

$\Rightarrow$ Inclusion of D-branes and orientifold planes

➪ Scalar potential in $\mathcal{N}=1$ supergravity theory:

$$V = e^{K/M_P^2}\big(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2/M_P^2\big) + \text{D-terms}$$

Axion potential only generated by non-perturbative effects:

Superpotentials from D1 instantons or gaugino condensates on D5 branes

➪ D1 instantons on vanishing cycles:

Couple to (instanton action): $G^a = c^a - i|t^a|/g_s$

- Type I: orientifolds with O9 planes — Witten

$$W_{\rm D1} = \sum_a B_a \, e^{-iG^a}$$

$\Rightarrow$ generalize to other orientifold scenarios (Type IIB with O5 planes)

- Type IIB / F-theory: orientifolds with O3/O7 planes — Witten

$\Rightarrow$ superpotential due to D3 instantons

$$W_{\rm D3} = \sum_\alpha A_\alpha \, \Theta_\alpha(\tau, G^a) \; e^{T_\alpha}$$

$\Rightarrow$ D1 instanton dependence - Exp $(-iG^a)$ - through determinants

Witten; Ganor; TG

➪ Gaugino condensates on space-time filling D5 branes wrapped on vanishing cycle

Gauge coupling: $G = c - i|t|/g_s$

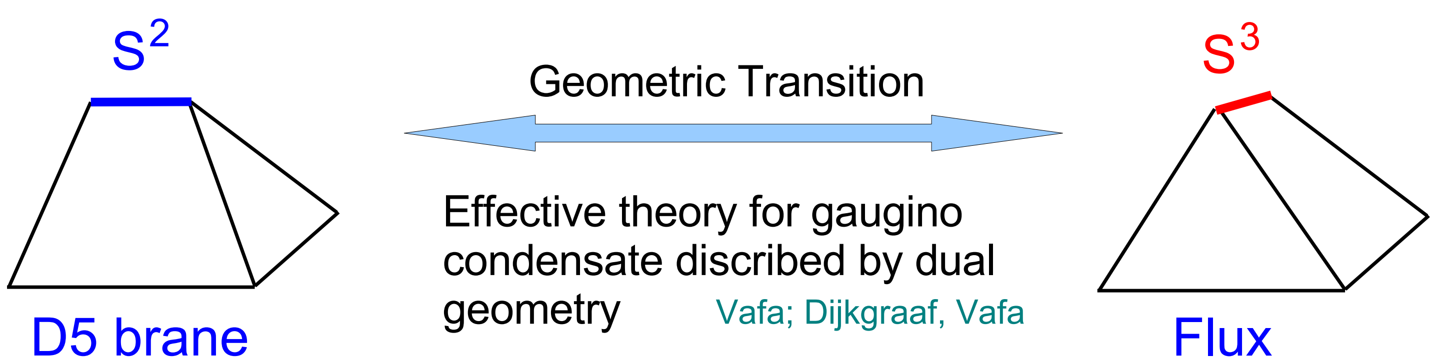
- gaugino condensate S of $U(K)$ gauge group: Veneziano-Yankielowicz sup.

$$W_{\rm VY} = G\, S + \frac{1}{2\pi i} K\ S\ \left(\log(S/\Lambda_0^3) - 1\right) \quad \xrightarrow{\text{eliminate } S} \quad W_{\rm D5} = \Lambda_0^3\, e^{-iG/K}$$

$\Rightarrow \quad \mu = 1/K$, but potential remains 2π periodic — Witten

- recently: Computation of axion potentials using geometric transition

Vafa,Heckman,Seo; Aganagic,Beem,Kachru



Axion inflation in $\mathcal{N} = 1$ compactifications

➪ Embed scenario into $\mathcal{N}=1$ orientifold comapctification with O3/O7 planes

Effective $\mathcal{N}=1$ theory can be compute including all $\mathcal{N}=2$ world-sheet corrections

TG,Louis

projection: $\mathcal{O}=(-)^{F_L}\Omega_p\sigma^*$ $\sigma^* J = J$ $\sigma^*\Omega=-\Omega$

- split of cohomology: $H^{(1,1)} = H^{(1,1)}_- \oplus H^{(1,1)}_+$
- split of basis: $\omega^-_a \quad a = 1...h^{(1,1)}_-$ $\omega^+_\alpha \quad \alpha = 1...h^{(1,1)}_+$
- new special coordinates associated to split basis:

$$-B_2 + iJ \ = \ t^a_- \ \omega^-_a + t^\alpha_+ \ \omega^+_\alpha$$

- $\mathcal{N}=2$ pre-potential: $\mathcal{F}(\, t^a_-,\, t^\alpha_+ \,)$

➪ $\mathcal{N}=1$ complex coordinates: complex dilaton $\tau = C_0 + ie^{-\phi}$

$$G^a = c^a + ie^{-\phi}\text{Re}\, t_-^a \qquad T_\alpha = \rho_\alpha + ie^{-\phi}\text{Re}\,\partial_{t_+^\alpha}\mathcal{F}$$

c^a R-R two-form axions $\qquad$ ρ_α R-R four-form axions

➪ $\mathcal{N}=1$ Kähler potential:

$$K_{\text{q}}(\tau, G, T) = -2\ln\left[ie^{-2\phi}\big(2(\mathcal{F}-\bar{\mathcal{F}}) - (\mathcal{F}_A + \bar{\mathcal{F}}_A)(t^A - \bar{t}^A)\big)\right]$$

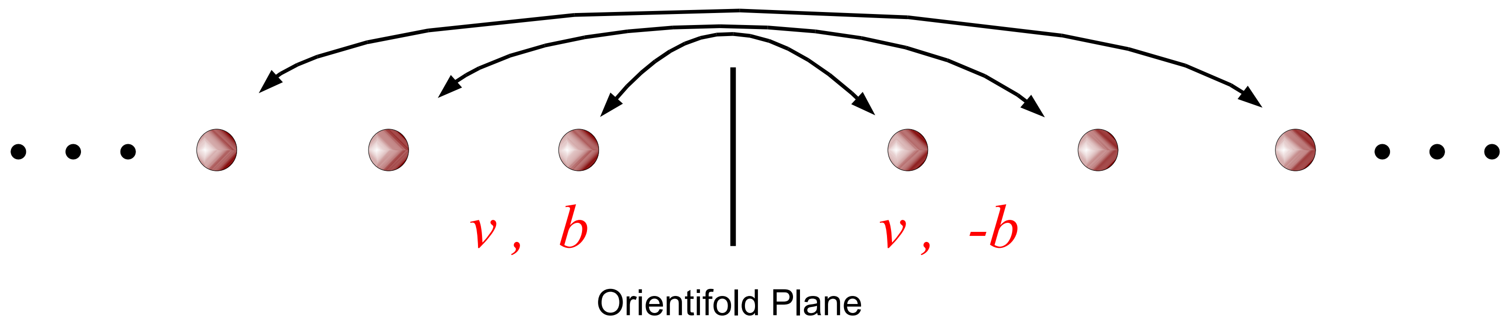
- Kähler potential is complicated implicit function of $\mathcal{N}=1$ coordinates
- derivatives of K_{q} determined by Legendre transform in $\mathcal{N}=2$ or work of Hitchin

➪ $\mathcal{N}=1$ superpotential: flux background + D-instanton corrections

$$W = \int G_3 \wedge \Omega + \sum_\alpha \Theta_\alpha(\tau, G^a)\, e^{iT_\alpha}$$

Gukov,Vafa,Witten; Witten; Ganor; TG

➪ Example: Simplistic orientifold of the N conifold toy model



➪ general $\mathcal{N}=1$ formulas can be applied to N conifold example

special coordinates: $t^j_+ = iv$ $t^j_- = -b$ $t_{\mathrm{R}} = i(R/\ell_s)^2$

simplistic pre-potential: (further perturbative and non-perturbative corrections expected)

$$\mathcal{F} = -\tfrac{1}{3!} t_{\mathrm{R}}^3 + t_{\mathrm{R}} \sum_{i=1}^{N} \left[(t^i_+)^2 + (t^i_-)^2 \right]$$
$$- \tfrac{i}{2} \sum_{i=1}^{N} (t^i_+ + t^i_-)^2 \log(t^i_+ + t^i_-) - \tfrac{i}{2} \sum_{i=1}^{N} (t^i_+ - t^i_-)^2 \log(t^i_+ - t^i_-) \ .$$

➪ $\mathcal{N} = 1$ complex coordinates:

D3-coupling: $T_{\rm R} \leftrightarrow t_{\rm R}$ $\quad T_i \leftrightarrow t^i_+$ D1-coupling: $G^i \leftrightarrow t^i_-$

➪ Moduli stabilization: background flux and D-instantons

(1) stabilize dilaton and complex structure moduli Giddings,Kachru,Polchinski; KKLT

(2) effective superpotential depending on $T_{\rm R}$, T_j and G^j

$$W = W_0 + \sum_{j=1}^{N} e^{iT_j} + e^{iT_{\rm R}} + e^{-1/g_s} \sum_{j=1}^{N} e^{-iG^j} e^{iT_{\rm R}}$$

⇒ axion potential: R-R two-form axions Re G^j

R-R four-form axions Re $T_{\rm R}$, Re T_j

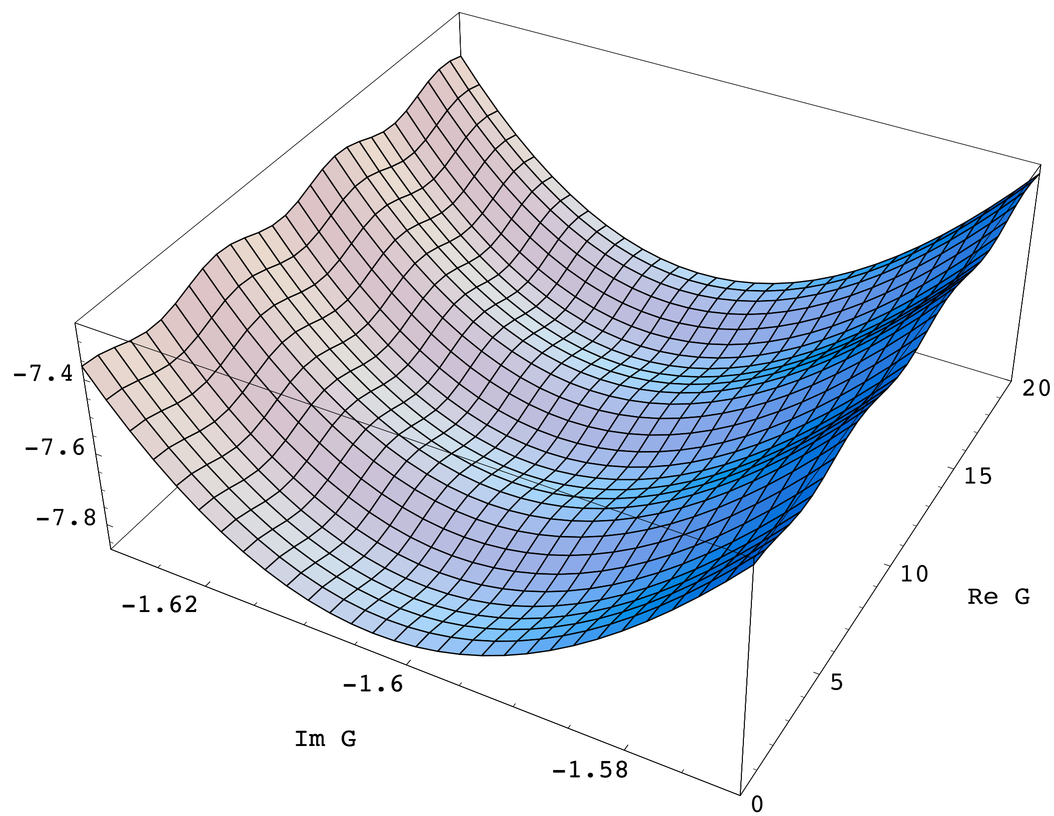
➪ numerical minimization: minimum for $\langle T_j \rangle$, $\langle T_{\rm R} \rangle$ and $\langle G^j \rangle$

➪ focus on R-R two-form axions in G^i: have strong exponential suppression in W

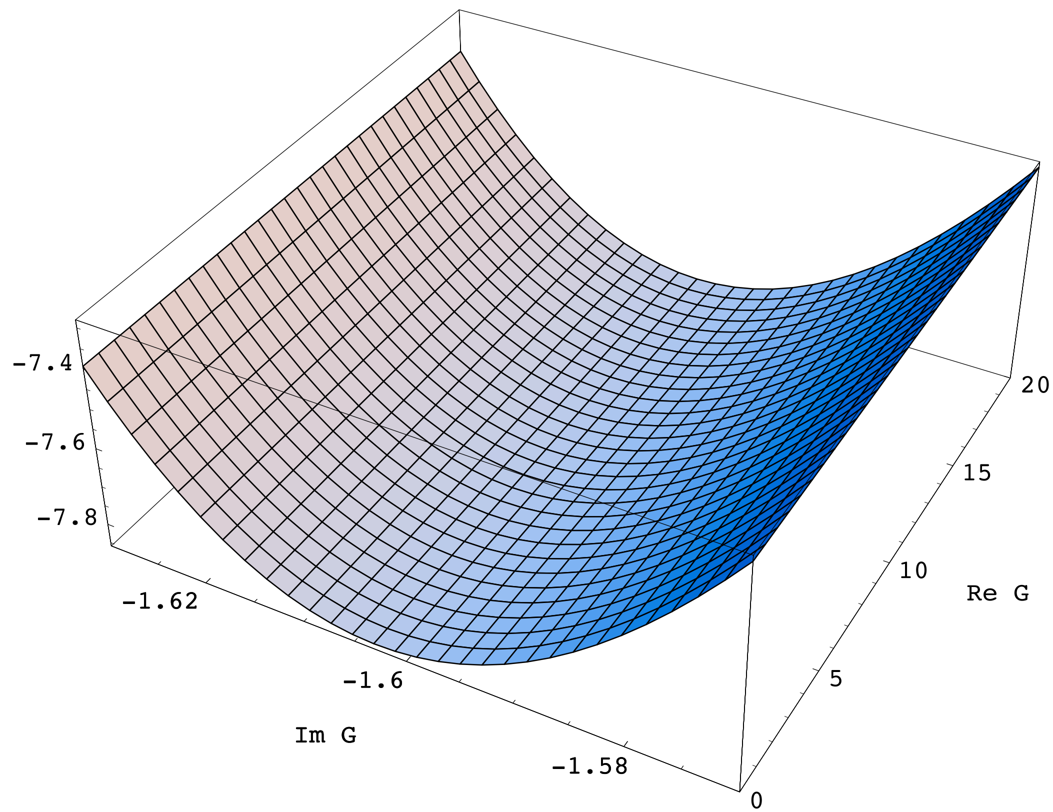
➪ numerical analysis allows to illustrate mass hierarchy:

axion $\mathrm{Re}\, G^i = c$ vs. non-axionic partner $\mathrm{Im}\, G^i = -b/g_s$

➪ Potential $V_{\mathrm{eff}}(G, \bar{G})$ at fixed $\langle T_{\mathrm{R}} \rangle$, $\langle T_j \rangle$: with G^i dependent W



➪ Potential $V_{\text{eff}}(G,\bar{G})$ at fixed $\langle T_{\text{R}}\rangle$, $\langle T_j\rangle$: without G^i dependent W



➪ non-axionic field $\text{Im}\, G^i$ is stabilized by corrections to K (not corrections to W)

➪ axions are lighter than other bulk fields near this vacuum

$\Rightarrow$ axions can potentially drive inflation

Conclusions and outlook

- Discussed specific realization of assisted axion inflation in type IIB string theory:
  - Axion decay constants: (a) close to Planck scale for axions from vanishing cycles
    (b) corrected by string world-sheets
    $\Rightarrow$ $\mathcal{N}=1$ Kähler potential, Kähler stabilization of the saxions
  - Axion potentials: from D1 instantons or gaugino condensates on D5 brane
    $\Rightarrow$ $\mathcal{N}=1$ superpotential, periodic potential for the axions
  - Embedding into $\mathcal{N}=1$ orientifolds: stabilization of all moduli keeping light axions

- Future dircections:
  - construction of explicit semi-realistinc models beyond N conifold scenarios
  - computation of axion potentials using gauge/gravity duality - geometric transitions