From String Theory to the Supersymmetric Standard Model

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Supersymmetry will forever be connected with the name of

JULIUS WESS
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Where do we find supersymmetry?

It plays an important role in Mathematical Physics and Mathematics.

Could appear at the Planck Scale (String Theory)?

... or at the TeV Scale?

LHC might test this possibility in the near future!
Questions

- What can we learn from strings for particle physics?
- Can we incorporate particle physics models within the framework of string theory?
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- Can we incorporate particle physics models within the framework of string theory?

Recent progress:

- explicit model building towards the MSSM
  - Heterotic brane world
  - local grand unification
- moduli stabilization and Susy breakdown
  - gaugino condensation and uplifting
  - mirage mediation
The road to the Standard Model

What do we want?

- gauge group $SU(3) \times SU(2) \times U(1)$
- 3 families of quarks and leptons
- scalar Higgs doublet
The road to the Standard Model

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- gauge group $SU(3) \times SU(2) \times U(1)$
- 3 families of quarks and leptons
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But there might be more:

- supersymmetry (SM extended to MSSM)
- neutrino masses and mixings

as a hint for a large mass scale around $10^{16}$ GeV
Indirect evidence

Experimental findings suggest the existence of two new scales of physics beyond the standard model

\[ M_{\text{GUT}} \sim 10^{16}\text{GeV} \text{ and } M_{\text{SUSY}} \sim 10^{3}\text{GeV} : \]
Indirect evidence

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\[ M_{\text{GUT}} \sim 10^{16}\text{GeV and } M_{\text{SUSY}} \sim 10^3\text{GeV}: \]

- Neutrino-oscillations and “See-Saw Mechanism”

\[ m_\nu \sim \frac{M_W^2}{M_{\text{GUT}}} \]

\[ m_\nu \sim 10^{-3}\text{eV for } M_W \sim 100\text{GeV}, \]
Indirect evidence

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- **Neutrino-oscillations** and “See-Saw Mechanism”

\[ m_\nu \sim \frac{M_W^2}{M_{\text{GUT}}} \]

\[ m_\nu \sim 10^{-3}\text{eV for } M_W \sim 100\text{GeV}, \]

- **Evolution of couplings constants** of the standard model towards higher energies.
MSSM (supersymmetric)
Standard Model
Grand Unification

This leads to SUSY-GUTs with nice things like

- unified multiplets (e.g. spinors of SO(10))
- gauge coupling unification
- Yukawa unification
- neutrino see-saw mechanism
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- unified multiplets (e.g. spinors of SO(10))
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But there remain a few difficulties:

- breakdown of GUT group (large representations)
- doublet-triplet splitting problem (incomplete multiplets)
- proton stability (need for R-parity)
String Theory

What do we get from string theory?

- supersymmetry
- extra spatial dimensions
- large unified gauge groups
- consistent theory of gravity
String Theory

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- supersymmetry
- extra spatial dimensions
- large unified gauge groups
- consistent theory of gravity

These are the building blocks for a unified theory of all the fundamental interactions. But do they fit together, and if yes how?

We need to understand the mechanism of compactification of the extra spatial dimensions.
Calabi Yau Manifold
Orbifold
Quarks, Leptons and Higgs fields can be localized:

- in the Bulk ($d = 10$ untwisted sector)
- on 3-Branes ($d = 4$ twisted sector fixed points)
- on 5-Branes ($d = 6$ twisted sector fixed tori)
Localization

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- on 3-Branes ($d = 4$ twisted sector fixed points)
- on 5-Branes ($d = 6$ twisted sector fixed tori)

but there is also a “localization” of gauge fields

- $E_8 \times E_8$ in the bulk
- smaller gauge groups on various branes

Observed 4-dimensional gauge group is common subgroup of the various localized gauge groups!
Localized gauge symmetries

\[ SU(4)^2 \]

\[ SU(6) \times SU(2) \]

\[ SO(10) \]

\[ SU(6) \times SU(2) \]
Local Grand Unification

In fact string theory gives us a variant of GUTs

- complete multiplets for fermion families
- split multiplets for gauge- and Higgs-bosons
- partial Yukawa unification
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Key properties of the theory depend on the geography of the fields in extra dimensions.

This geometrical set-up called local GUTs, can be realized in the framework of the “heterotic braneworld”.

(Förste, HPN, Vaudrevange, Wingerter, 2004; Buchmüller, Hamaguchi, Lebedev, Ratz, 2004)
The Remnants of $SO(10)$

- $SO(10)$ is realized in the higher dimensional theory
- broken in $d = 4$
- coexistence of complete and incomplete multiplets
The Remnants of SO(10)

- $SO(10)$ is realized in the higher dimensional theory
- broken in $d = 4$
- coexistence of complete and incomplete multiplets

Still there could be remnants of $SO(10)$ symmetry

- 16 of SO(10) at some branes
- correct hypercharge normalization
- R-parity
- distinction between different families

that are very useful for realistic model building ...
The “fertile patch”: $Z_6$ II orbifold

- provides fixed points and fixed tori
- allows $SO(10)$ gauge group
- allows for localized 16-plets for 2 families
- $SO(10)$ broken via Wilson lines
- nontrivial hidden sector gauge group

(Kobayashi, Raby, Zhang, 2004; Buchmüller, Hamaguchi, Lebedev, Ratz, 2004)
### Selection Strategy

<table>
<thead>
<tr>
<th>criterion</th>
<th>$V^{SO(10),1}$</th>
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<td>② models with 2 Wilson lines</td>
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<td>③ SM gauge group $\subset SO(10)$</td>
<td>3563</td>
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<td>④ 3 net families</td>
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<td>492</td>
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<td>⑤ gauge coupling unification</td>
<td>528</td>
<td>234</td>
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<tr>
<td>⑥ no chiral exotics</td>
<td>128</td>
<td>90</td>
</tr>
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</table>

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingert, 2006)
The road to the MSSM

This scenario leads to

- 200 models with the exact spectrum of the MSSM (absence of chiral exotics)
- local grand unification (by construction)
- gauge- and (partial) Yukawa unification

(Raby, Wingerter, 2007)

- examples of neutrino see-saw mechanism
  (Buchmüller, Hamguchi, Lebedev, Ramos-Sanchez, Ratz, 2007)

- models with R-parity + solution to the $\mu$-problem
  (Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)

- hidden sector gaugino condensation
A Benchmark Model

At the orbifold point the gauge group is

\[ SU(3) \times SU(2) \times U(1)^9 \times SU(4) \times SU(2) \]

- one \( U(1) \) is anomalous
- there are singlets and vectorlike exotics
- decoupling of exotics and breakdown of gauge group has been verified
- remaining gauge group

\[ SU(3) \times SU(2) \times U(1)_Y \times SU(4)_{\text{hidden}} \]

- for discussion of neutrinos and R-parity we keep also the \( U(1)_{B-L} \) charges
<table>
<thead>
<tr>
<th>#</th>
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<td>$(3, 2; 1, 1)_{(1/6,1/3)}$</td>
<td>$q_i$</td>
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<td>$\bar{u}_i$</td>
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<td>$y_i$</td>
</tr>
<tr>
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<td>$f_i$</td>
<td>6</td>
<td>$(1, 1; 4, 1)_{(0,*)}$</td>
<td>$\bar{f}_i$</td>
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<td>$\bar{f}_i^-$</td>
<td>2</td>
<td>$(1, 1; \overline{4}, 1)_{(1/2,1)}$</td>
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<td>$\chi_i$</td>
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<td>$v_i$</td>
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</tbody>
</table>
**Unification**

- Higgs doublets are in untwisted (U3) sector
- Trilinear coupling to the top-quark allowed
- Threshold corrections ("on third torus") allow unification at correct scale around $10^{16}$ GeV
Hidden Sector Susy Breakdown

Gravitino mass $m_{3/2} = \Lambda^3 / M_{\text{Planck}}^2$ is in the TeV range for the hidden sector gauge group $SU(4)$

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006)
See-saw neutrino masses

The see-saw mechanism requires

- right handed neutrinos \((Y = 0 \text{ and } B - L = \pm 1)\),
- heavy Majorana neutrino masses \(M_{\text{Majorana}}\),
- Dirac neutrino masses \(M_{\text{Dirac}}\).

The benchmark model has 49 right handed neutrinos:

- the left handed neutrino mass is \(m_\nu \sim M_{\text{Dirac}}^2/M_{\text{eff}}\)
- with \(M_{\text{eff}} < M_{\text{Majorana}}\) and depends on the number of right handed neutrinos.

(Buchmüller, Hamguchi, Lebedev, Ramos-Sanchez, Ratz, 2007; Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)
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<td>vi</td>
<td>2</td>
<td>(3, 1; 1, 1)_{(1/6, -2/3)}</td>
<td>vi</td>
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</table>
R-parity

- R-parity allows the distinction between Higgs bosons and sleptons
- \( SO(10) \) contains R-parity as a discrete subgroup of
  \( U(1)_{B-L} \).
- In conventional “field theory GUTs” one needs large representations to break
  \( U(1)_{B-L} \) (\( \geq 126 \) dimensional)
- In heterotic string models one has more candidates for R-parity (and generalizations thereof)
- One just needs singlets with an even \( B - L \) charge that break
  \( U(1)_{B-L} \) down to R-parity

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)
Discrete Symmetries

There are numerous discrete symmetries

- from geometry
- and from stringy selection rules,
- both of abelian and nonabelian nature.

(Kobayashi, HPN, Plöger, Raby, Ratz, 2006)

Possible applications:

- (nonabelian) family symmetries
- Yukawa textures
- approximate global $U(1)$ for a QCD axion

(Choi, Kim, Kim, 2006; Choi, HPN, Ramos-Sanchez, Vaudrevange, 2008)
The $\mu$ problem

In general we have to worry about

- doublet-triplet splitting
- mass term for additional doublets
- the appearance of "naturally" light doublets

In the benchmark model we have

- only 2 doublets
- which are neutral under all selection rules
- if $M(s_i)$ allowed in superpotential
- then $M(s_i)H_uH_d$ is allowed as well
The $\mu$ problem II

We have verified that (up to order 6 in the singlets)

- $F_i = 0$ implies automatically
- $M(s_i) = 0$ for all allowed terms $M(s_i)$ in the superpotential $W$

Therefore

- $W = 0$ in the supersymmetric (Minkowski) vacuum
- as well as $\mu = \partial^2 W / \partial H_u \partial H_d = 0$, while all the vectorlike exotics decouple
- with broken supersymmetry $\mu \sim m_{3/2} \sim < W >$

This solves the $\mu$-problem

(Casas, Munoz, 1993)
Gaugino Condensation

Gravitino mass $m_{3/2} = \Lambda^3 / M_{\text{Planck}}^2$ and $\Lambda \sim \exp(-S)$

We need to fix the dilaton!

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006)
Run-away potential

\begin{center}
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=\textwidth,
\addplot[domain=2.1:2.4,samples=100]{2*10^(-32)*x^4};
\end{axis}
\end{tikzpicture}
\end{center}

\begin{align*}
N &= 4 & A &= 4.9 & d &= 0 & p &= 0 & b &= 0
\end{align*}
Corrections to Kähler potential

\[ N = 4 \quad A = 4.9 \quad d = 5.7391 \quad p = 1.1 \quad b = 1 \]

\[ V \times 10^{-28} \]

\[ Re S \]

(Casas, 1996; Barreiro, de Carlos, Copeland, 1998)
Dilaton Domination?

This is known as the dilaton domination scenario, but there are problems to remove the vacuum energy.

One needs a “downlifting” mechanism:

- the analogue to the F-term “uplifting” in the type IIB case (Gomez-Reino, Scrucca, 2006; Lebedev, HPN, Ratz, 2006)
- “downlifting” mechanism fixes $S$ as well (no need for nonperturbative corrections to the Kähler potential) (Löwen, HPN, 2008)
- mirage mediation for gaugino masses
Sequestered sector “downlifting”

\[ N = 4 \quad A = 4.9 \quad C_0 = 0.73 \]

(Lebedev, HPN, Ratz, 2006; Löwen, HPN, 2008)
Metastable “Minkowski” vacuum

(Löwen, HPN, 2008)
Evolution of couplings

\[ \log_{10}(\mu/\text{GeV}) \]

\[ \alpha_i \]

The Heterotic MSSM, WessMemorialConference, München, November 2008 – p.36/40
The Mirage Scale

\[ \log_{10} \left( \frac{\mu}{\text{GeV}} \right) \]

\[ M_3 \]

\[ M_2 \]

\[ M_1 \]

\[ M_i / \text{GeV} \]
Constraints on the mixing parameter

\[ \tan \beta = 5 \quad \eta = 4 \quad \eta' = 6 \]

\( m_{3/2} \) [TeV]

\( \tilde{g} \) LSP

No EWSB

\( \Omega > \Omega_{\text{WMAP}} \)

\( \Omega < \Omega_{\text{WMAP}} \)

Below LEP

(Löwen, HPN, 2008)
Constraints on the mixing parameter

![Graph showing constraints on the mixing parameter](image)

\[
\tan \beta = 30 \quad \eta = 4 \quad \eta' = 6
\]

- No EWSB
- $\tilde{g}$ LSP
- $\tilde{\chi}^+ + \text{LSP}$
- $\Omega > \Omega_{\text{WMAP}}$
- $\Omega < \Omega_{\text{WMAP}}$
- Below LEP

(Löwen, HPN, 2008)
Conclusion

String theory provides us with new ideas for particle physics model building, leading to concepts such as

- Local Grand Unification
- realistic MSSM candidates

Geography of extra dimensions plays a crucial role:

- localization of fields on branes,
- sequestered sectors and mirage mediation

LHC might help us to verify some of these ideas!