

# Strings, Exceptional Groups and Grand Unification

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# Outline

- Grand Unification and
- global symmetries

play an important role in particle physics research

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- Where do they come from?
- The nature of “global” symmetries
- The need for an ultraviolet completion
- Strings and local grand unification
- Discrete (gauge) symmetries
- The fate of “local model building”

# GUT groups

GUT motivation comes from bottom-up picture

- GUTs need SUSY
- SU(5) as the minimal extension
- SO(10) allows complete family in **16**

# GUT groups

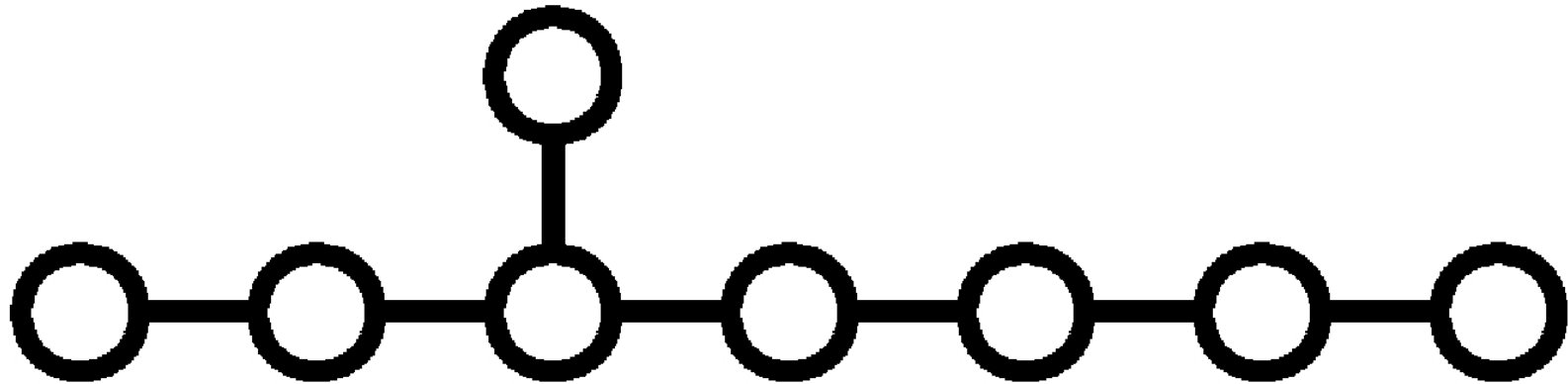
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- GUTs need SUSY
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From the mathematical structure we would prefer exceptional groups

- There is a maximal group:  $E_8$ ,
- but  $E_8$  and  $E_7$  do not allow chiral fermions in  $d = 4$ .
- How does this fit with SU(5) and SO(10)?

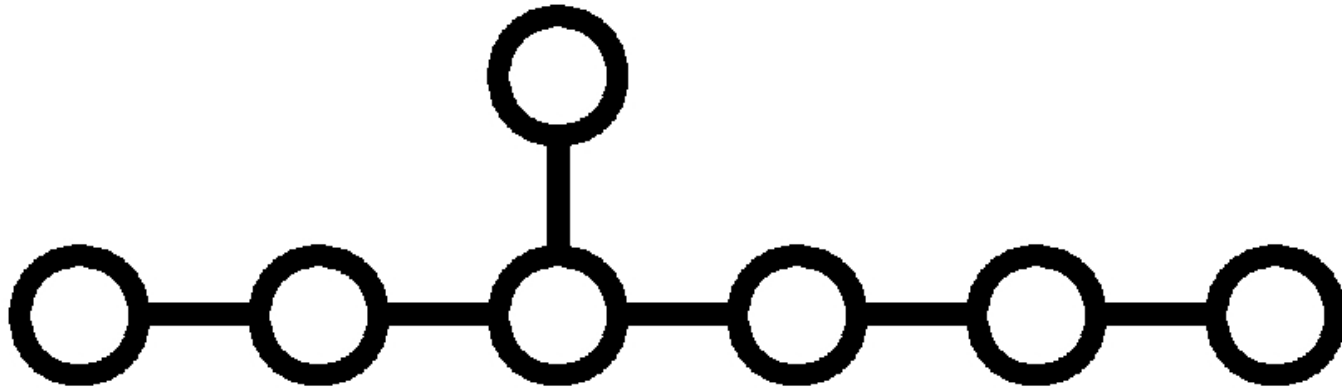
# Maximal Group



$E_8$  is the maximal group.

There are, however, no chiral representations in  $d = 4$ .

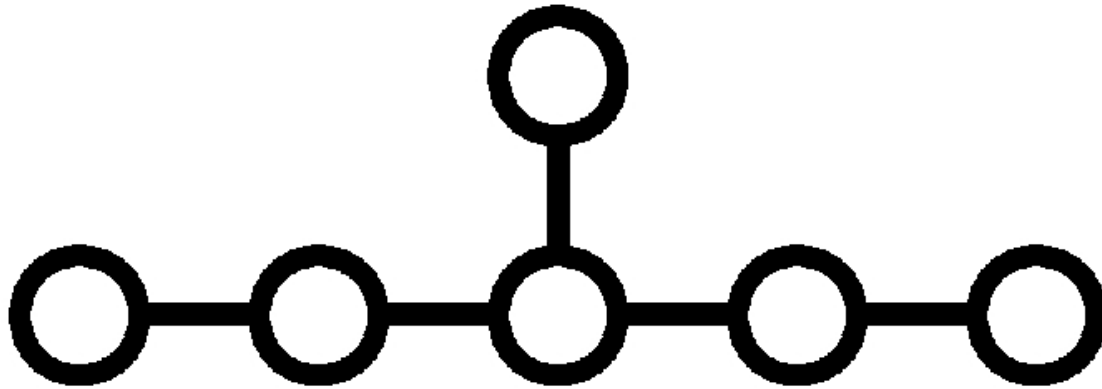
$E_7$



Next smaller is  $E_6$ .

No chiral representations in  $d = 4$  either

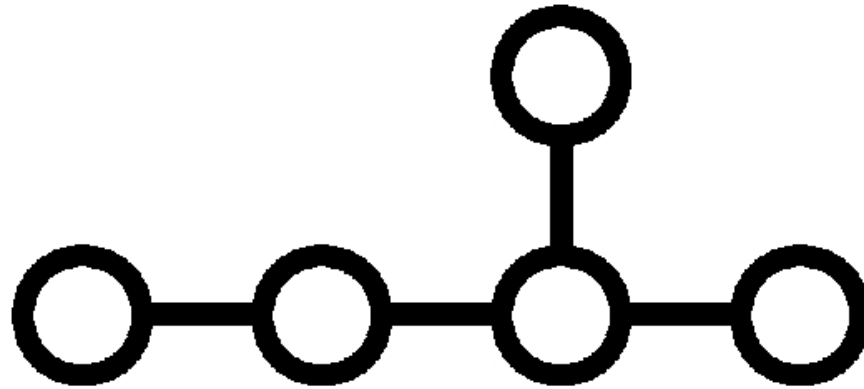
$E_6$



$E_6$  allows for chiral representations even in  $d = 4$ .



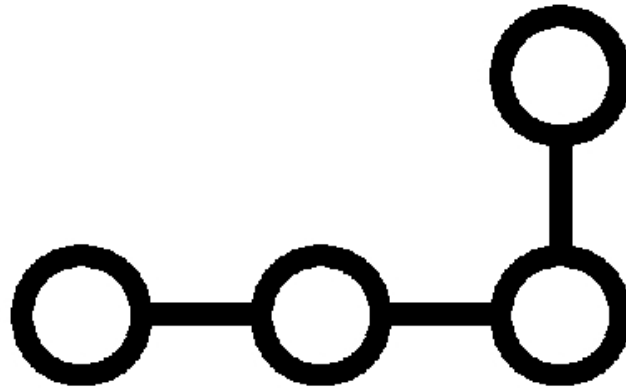
$$E_5 = D_5$$



$E_5$  is usually not called exceptional.

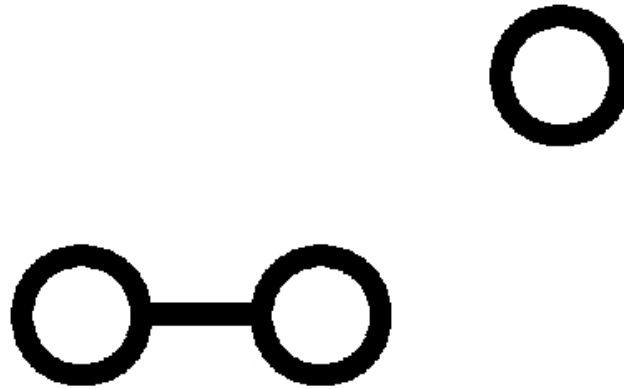
It coincides with  $D_5 = SO(10)$ .

$$E_4 = A_4$$



$E_4$  coincides with  $A_4 = SU(5)$

$E_3$



$E_3$  coincides with  $A_2 \times A_1$  which is  $SU(3) \times SU(2)$ .

# Exceptional groups in string theory

String theory favours  $E_8$

- $E_8 \times E_8$  heterotic string
- $E_8$  enhancement at a local point in F-theory

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Strings live in higher dimensions:

- chiral spectrum possible even with  $E_8$
- $E_8$  broken in process of compactification
- provides source for more (discrete) symmetries
- from  $E_8/SO(10)$  and  $SO(6)$  of the higher dimensional Lorentz group

# The use of additional symmetries

Symmetries are very useful for

- absence of FCNC (solve **flavour problem**)
- **Yukawa textures** à la Frogatt-Nielsen
- solutions to the  **$\mu$  problem**
- creation of hierarchies
- proton stability

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But they might be destroyed by gravitational effects:

- **we need a UV-completion of the theory**
- **with a consistent incorporation of gravity**
- **(discrete) gauge symmetries are safe**

# String theory as UV-completion

What do we get from string theory?

- supersymmetry
- extra spatial dimensions
- (large unified) gauge groups
- consistent theory of gravity
- a plenitude of discrete symmetries
- no global continuous symmetries



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String theory serves as a UV-completion with a consistent incorporation of gravity, and thus able to provide reliable symmetries.

# Grand Unification in String Theory

In fact string theory gives us a variant of GUTs

- complete (or split) multiplets for fermion families
- split multiplets for gauge- and Higgs-bosons
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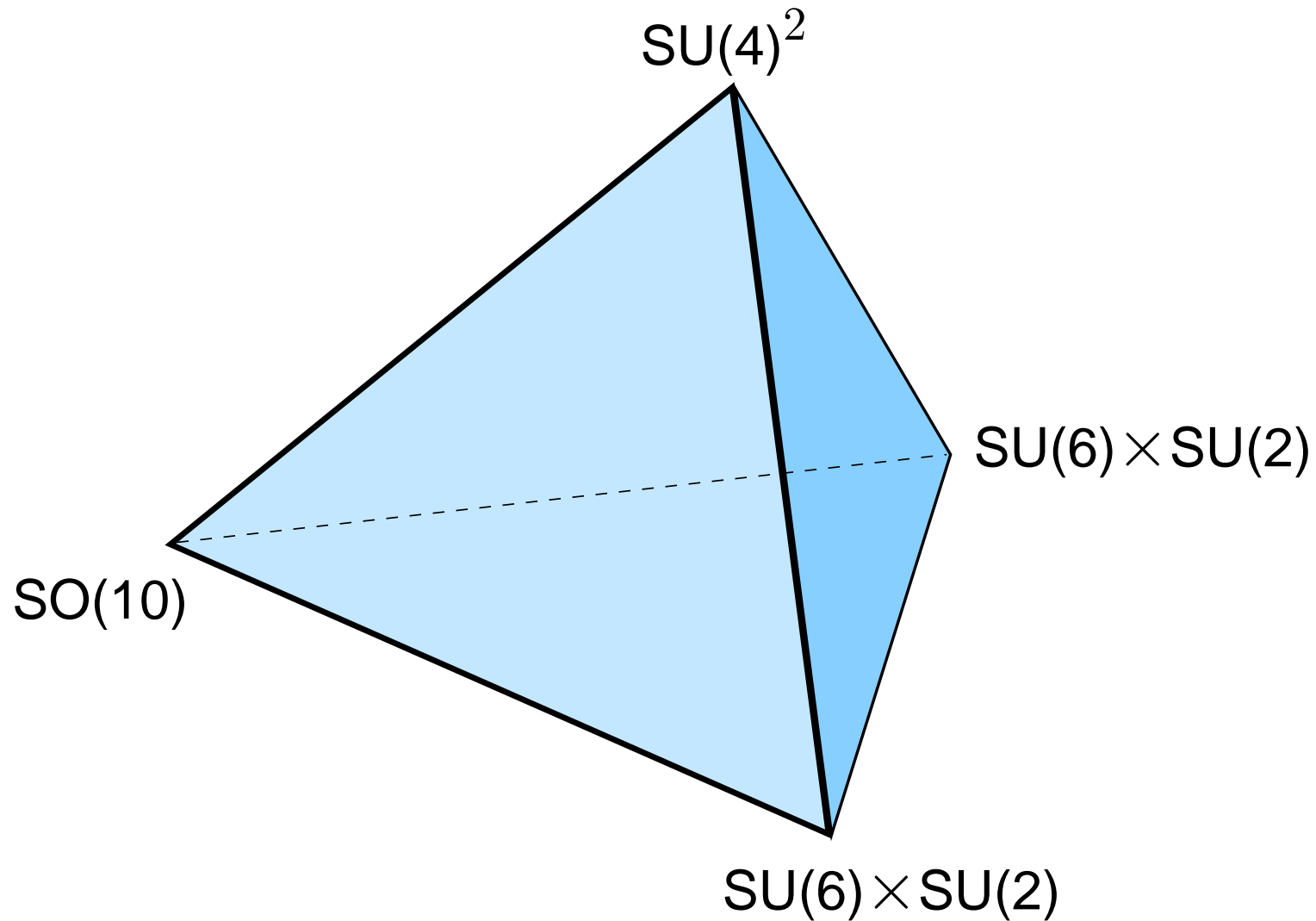
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Key properties of the theory depend on the **geography** of the fields in extra dimensions.

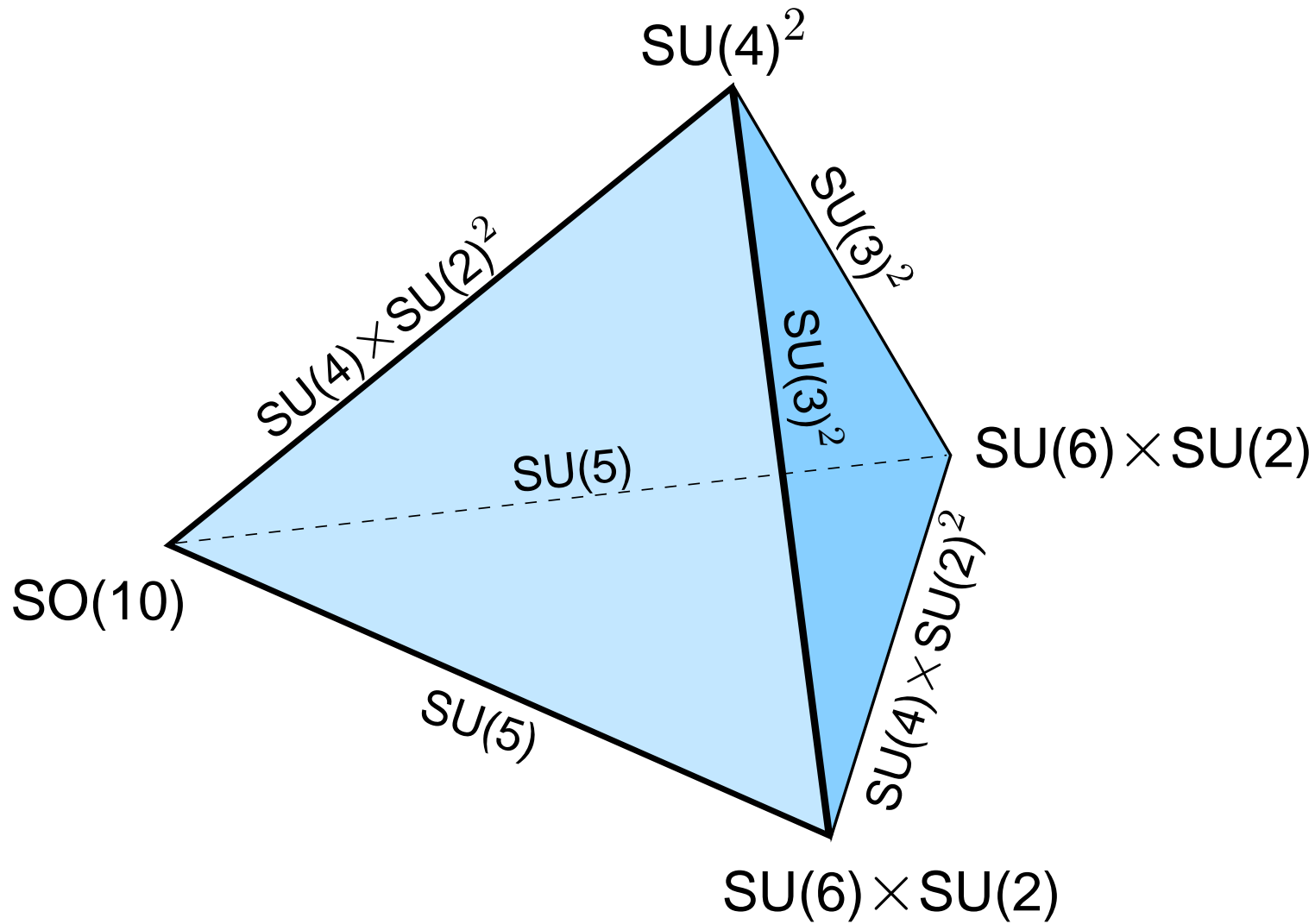
This geometrical set-up called **local grand unification**, and can be realized in the framework of the **“heterotic braneworld”**.

(Förste, HPN, Vaudrevange, Wingerter, 2004; Buchmüller, Hamaguchi, Lebedev, Ratz, 2004)

# Localized gauge symmetries



# Standard Model Gauge Group



# A word of clarification

Do not confuse

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# Symmetries

String theory gives us

- **gauge** symmetries
- **discrete** symmetries from geometry and stringy selection rules  
(Kobayashi, HPN, Plöger, Raby, Ratz, 2006)
- **accidental global**  $U(1)$  symmetries in the low energy effective action  
(Choi, Kim, Kim, 2006; Choi, HPN, Ramos-Sanchez, Vaudrevange, 2008)

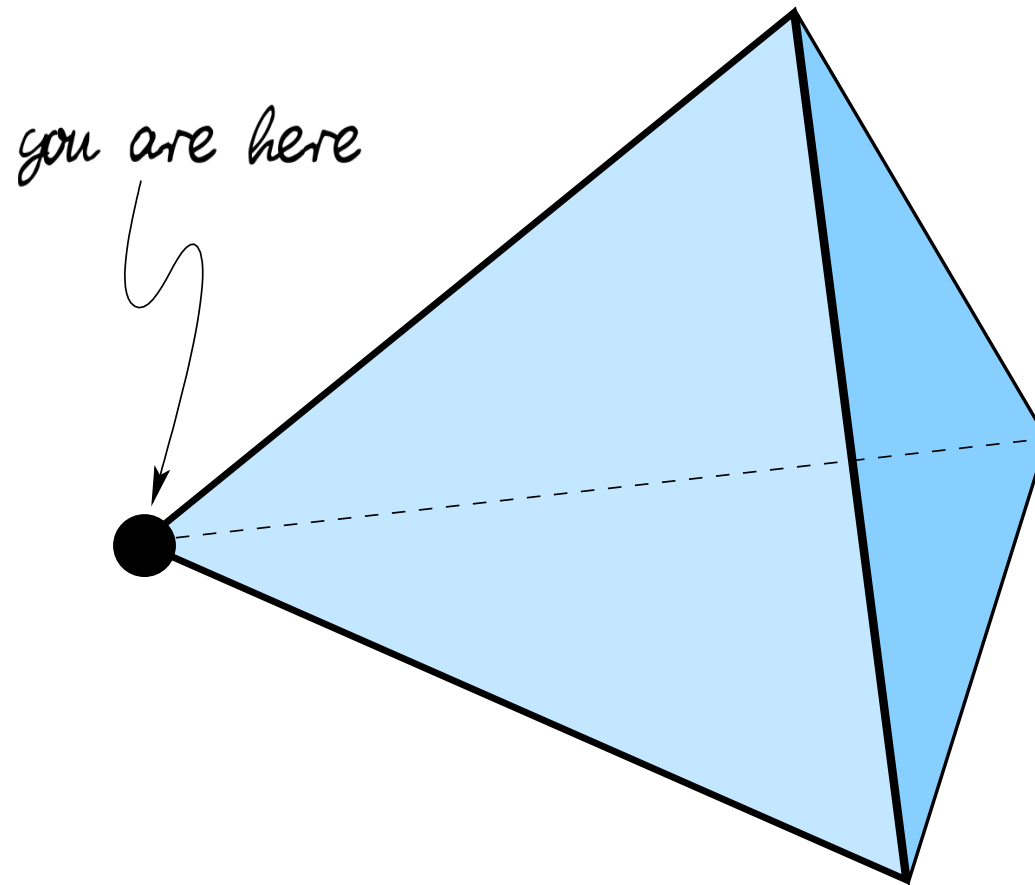
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# Location matters



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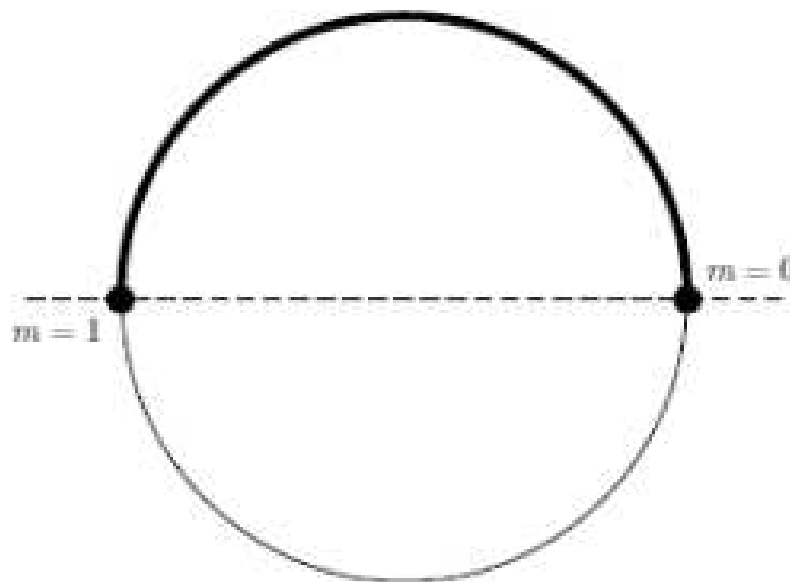
These symmetries can be trusted as we are working within a consistent theory of gravity (global model)!.

# Applications of global symmetries

## Applications of discrete and accidental global symmetries:

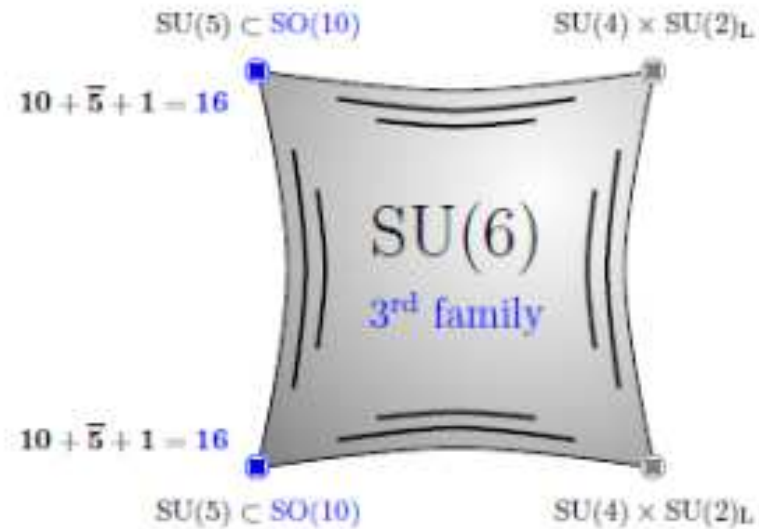
- (nonabelian) family symmetries (and FCNC)  
(Ko, Kobayashi, Park, Raby, 2007)
- Yukawa textures (via Frogatt-Nielsen mechanism)
- a solution to the  $\mu$ -problem  
(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)
- creation of hierarchies  
(Kappl, HPN, Ramos-Sanchez, Ratz, Schmidt-Hoberg, Vaudrevange, 2008)
- proton stability via “Proton Hexality” or  $Z_4^R$   
(Förste et al. 2010; Lee et al. 2011)
- approximate global  $U(1)$  for a QCD action  
(Choi, Kim, Kim, 2006; Choi, HPN, Ramos-Sanchez, Vaudrevange, 2008)

# Origin of discrete symmetries



The semidirect product of  $Z_2 \times Z_2$  and  $S_2$   
leads to the nonabelian group  $D_4$

# Local GUT picture



Family symmetries in local GUT models

# The $\mu$ problem

In general we have to worry about

- doublet-triplet splitting
- mass term for additional doublets
- the appearance of “naturally” light doublets

In the heterotic braneworld we find models

- with only 2 doublets
- which are neutral under all selection rules
- if  $M(s_i)$  allowed in superpotential
- then  $M(s_i)H_uH_d$  is allowed as well



# The $\mu$ problem II

We have verified that (up to order 8 in the superpotential)

- $F_i = 0$  implies automatically
- $M(s_i) = 0$  for all allowed terms  $M(s_i)$  in the superpotential  $W$

Therefore

- $W = 0$  in the supersymmetric (Minkowski) vacuum
- as well as  $\mu = \partial^2 W / \partial H_u \partial H_d = 0$ , while all the vectorlike exotics decouple
- with broken supersymmetry  $\mu \sim m_{3/2} \sim \langle W \rangle$

This solves the  $\mu$ -problem

(Casas, Munoz, 1993)

# The creation of the hierarchy

Is there an explanation for a vanishing  $\mu$ :

- string miracle or an underlying symmetry?

The  $\mu$ -term is in fact forbidden by an R-symmetry.

For a continuous R-symmetry we would have

- a supersymmetric ground state with  $W = 0$  and  $U(1)_R$  spontaneously broken
- a problematic R-Goldstone-Boson

However, the above R-symmetry appears as an accidental continuous symmetry resulting from an exact discrete symmetry of (high) order  $N$

# Hierarchy

Such accidental symmetries lead to

- creation of a **small constant in the superpotential**
- explanation of a **small  $\mu$  term**

(Kappl, HPN, Ramos-Sanchez, Ratz, Schmidt-Hoberg, Vaudrevange, 2008)

Even with a moderate hierarchy like  $\phi/M_P \sim 10^{-2}$  one can generate small values for  $\mu$  and  $\langle W \rangle$

$$m_{3/2} \sim W_{\text{eff}} = c + A e^{-a S}$$

The second term in  $W_{\text{eff}}$  could be protected by an **anomalous R-symmetry** like e.g.  $Z_4^R$

(Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange; 2010)

# Proton stability

In the standard model Baryon number  $U(1)_B$  is not a good symmetry

- Baryon and lepton number are anomalous
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Baryon number violation is needed for baryogenesis.

- Grand unification addresses these questions
- proton decay via dimension-6 operators
- GUT scale has to be sufficiently high

# GUTs need SUSY

Grand unification most natural in the framework of SUSY

- evolution of gauge couplings
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But there is a problem

- dimension-4 and -5 operators
- more symmetries needed
- matter parity (or R-parity)
- baryon triality, proton hexality

(Ibanez, Ross, 1991; Dreiner, Luhn, Thormeier, 2005)

# MSSM

The **minimal particle content** of the susy extension of the standard model includes chiral superfields

- $Q, \bar{U}, \bar{D}$  for quarks and partners
- $L, \bar{E}$  for leptons and partners
- $H_d, H_u$  Higgs supermultiplets



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with superpotential

$$W = QH_d\bar{D} + QH_u\bar{U} + LH_d\bar{E} + \mu H_u H_d.$$

Also allowed (but problematic) are dimension-4 operators

$$\bar{U}\bar{D}\bar{D} + QL\bar{D} + LL\bar{E}.$$

# The question of proton stability

These dimension-4 operators could be forbidden by some symmetry

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Where does this symmetry come from?

- it could be a subgroup of  $SO(10)$
- in consistent heterotic constructions it comes from  $(E_8 \times E_8)/SO(10)$
- in local F-theory construction from  $E_8/SO(10)$

# Proton Hexality

But there are in addition dimension-5 operators that might mediate too fast proton decay  $QQQL + \bar{U}\bar{U}\bar{D}\bar{E}$

	$Q$	$\bar{U}$	$\bar{D}$	$L$	$\bar{E}$	$H_u$	$H_d$	$\bar{\nu}$
$6 Y$	1	-4	2	-3	6	3	-3	0
$\mathbb{Z}_2^{\text{matter}}$	1	1	1	1	1	0	0	1
$B_3$	0	-1	1	-1	2	1	-1	0
$P_6$	0	1	-1	-2	1	-1	1	3

Proton hexality is exactly what we need:

- dangerous dimension 4 and 5 operators forbidden
- neutrino Majorana masses allowed ( $LLH_uH_u$ )

(Dreiner, Luhn, Thormeier, 2005)

# GUTs and Hexality

Combination of GUTs and proton hexality is perfect

But GUTs and Hexality are incompatible (Luhn, Thormeier, 2007)

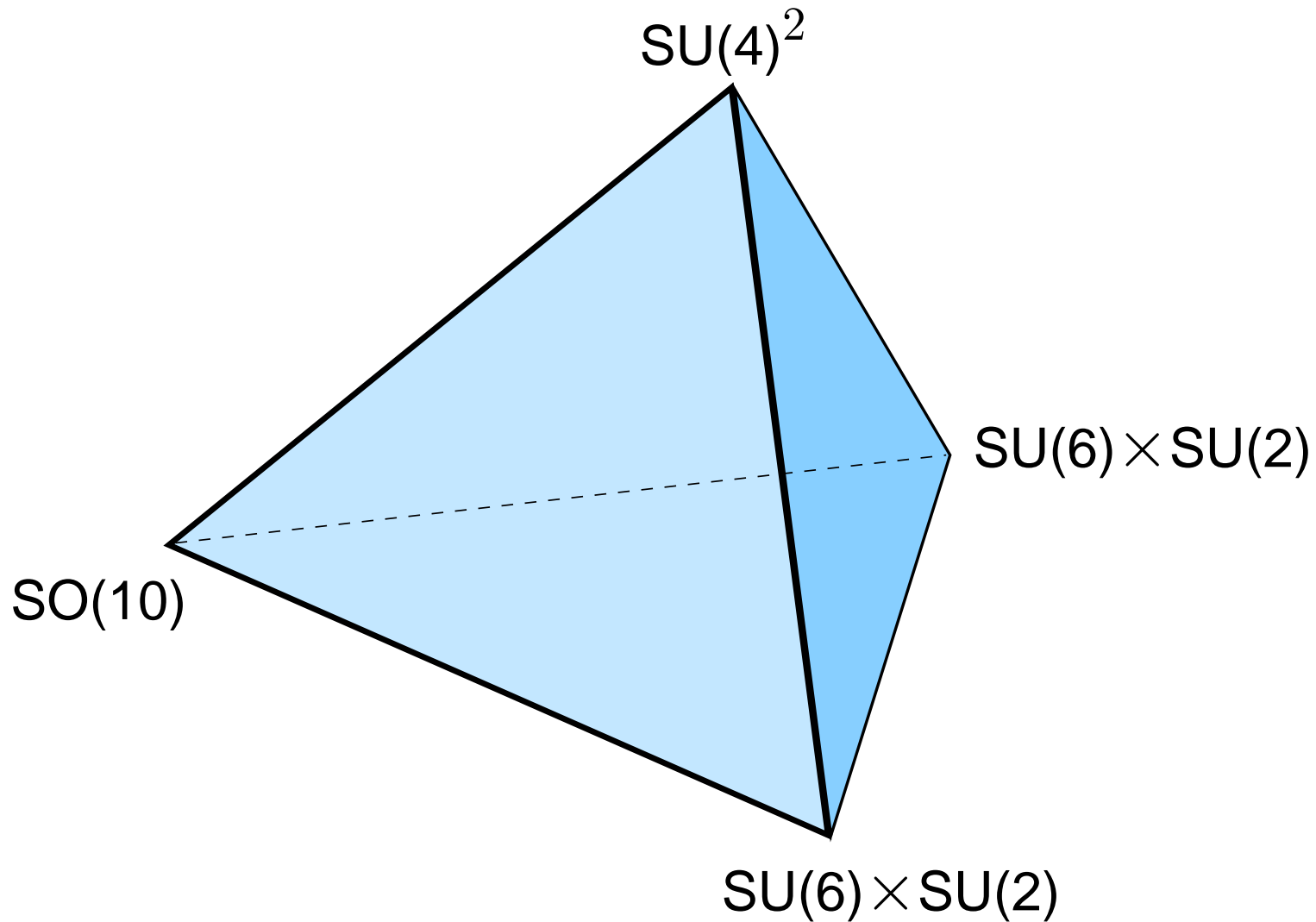
Example:

the 10-dimensional representation of SU(5) includes  $\bar{U}$ ,  $Q$  and  $\bar{E}$  and they cannot all have the same charge under hexality.

The problem is solved in

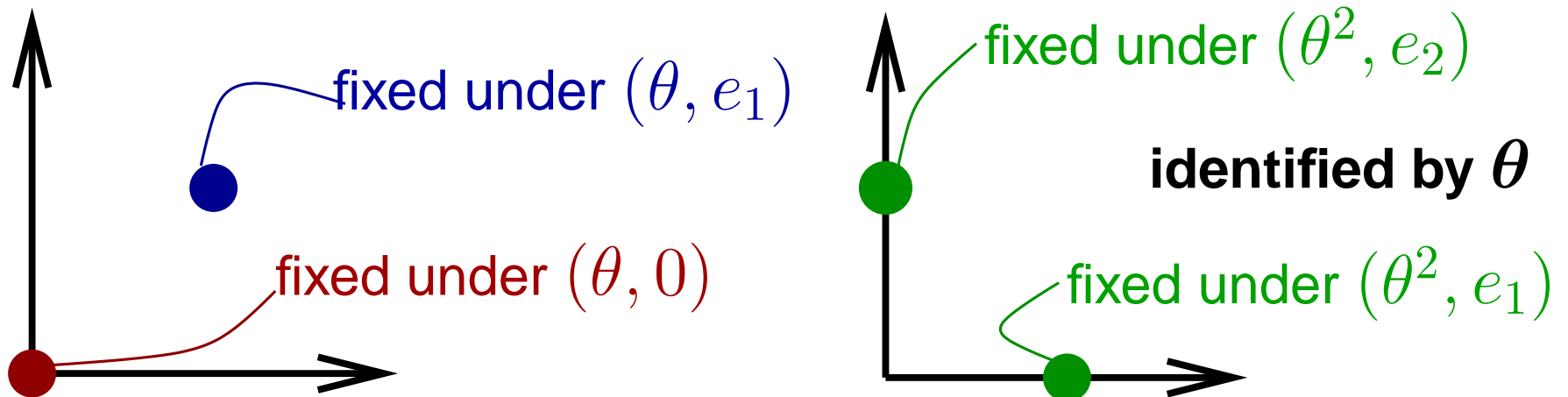
- Local Grand Unification
- need split multiplets for matter fields
- nonlocal structure of matter fields in compactified dimensions

# Localized gauge symmetries



# A $T_2/Z_4$ toy example

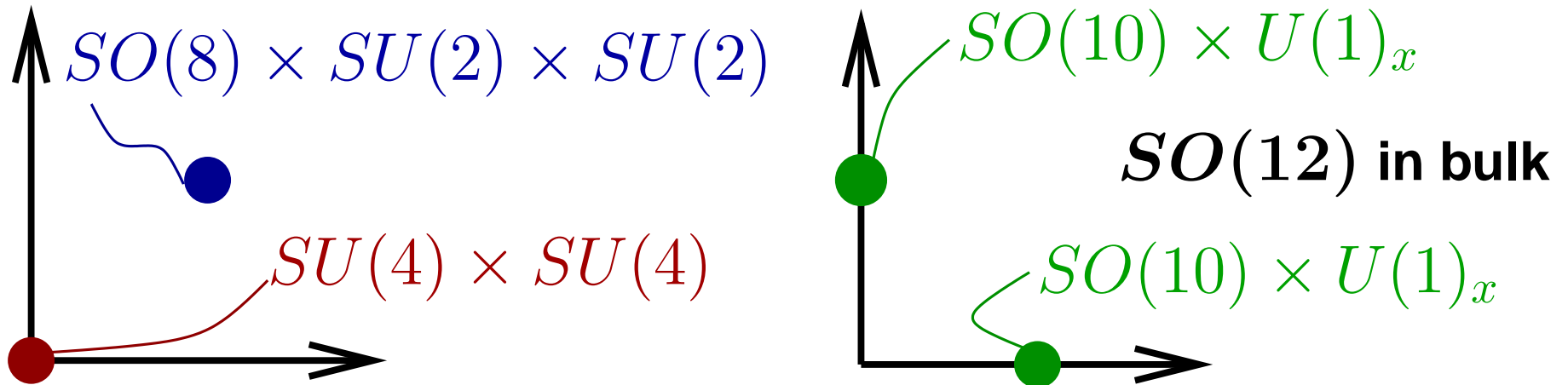
Consider the  $T_2/Z_4$  orbifold, where we have two different types of fixed points



under rotation of  $\theta = \pi/2$  and shift of the lattice vectors.

# A $T_2/Z_4$ toy example

For a suitable embedding of twist and shift in the gauge group  $SO(12)$  we have the following **local gauge group structure**



This allows **split representations compatible with  $P_6$**  and does not require huge representations for the breakdown of  $SO(12)$ .



# Lessons from the heterotic braneworld

The concept of local GUTs leads to a nontrivial structure of matter distribution in extra dimensions

- R-symmetries as subgroup of  $SO(6)$  to solve the  $\mu$  problem

- split multiplets for proton hexality

- $Z_4^R$  consistent with 4d-GUTs

(Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange; 2010)

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Note that we have consistent string models in a global construction. There is a plenitude of (discrete) gauge symmetries, both abelian and nonabelian.

(Kobayashi et al., 2006; Araki et al., 2008)

# The fate of local model building

F-theory constructions are currently relying on  
“local models”

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Symmetries there all originate from

- a single gauge group like  $E_8$
- matter parity as a subgroup of  $E_8$
- but there is no global completion, the symmetry is inconsistent in the presence of gravity.

( Lüdeling, HPN, Stephan, 2011)

Predictions of “Local Models” are not reliable.

# Conclusion

String theory might provide us with a **consistent** UV-completion of the MSSM including

- **Local Grand Unification as a result of a consistent global construction,**
- **a plenitude of discrete symmetries,**
- **originating from some non-localities of matter distribution in extra dimensions.**

**Geography of extra dimensions plays a crucial role.**

Local Grand Unification is the right way to proceed.

**Discrete symmetries as subgroups of  $E_8 \times E_8 \times SO(6)$  as a crucial prediction of string theory!**

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