One-loop Radiative Corrections to the Higgs Boson Mass Sum Rules in the MSSM

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Ich versichere, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate kenntlich gemacht habe.

Referent:	Prof. Dr. Hans-Peter Nilles
Korreferent:	Prof. Dr. Herbert K. Dreiner

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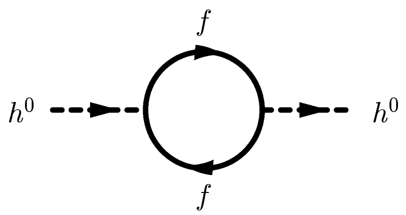
Chapter 1

Introduction

1.1 Motivation for Supersymmetry

Most precision measurements agree remarkably well with the predictions of the Standard Model(SM). Although there is no experimental evidence for supersymmetry to be realized in Nature, a number of unsolved theoretical problems has lead physicists to think that new physics beyond the SM must exist at an energy scale of 1 TeV or below.

The only particle which is predicted by the SM and has escaped detection so far is the Higgs boson to which all particles owe their masses. If found, the Higgs boson will be the only scalar particle in the theory. Scalar particles have the undesirable property that their masses are subject to huge radiative corrections. The calculation of the first-order correction to the squared Higgs boson mass gives a quadratically divergent expression arising from SM fermions in the loops.

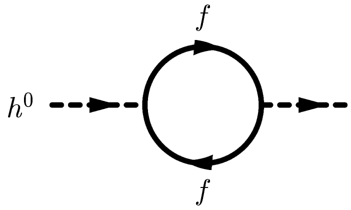


$$\propto \quad \Lambda^2 + c \log \Lambda$$

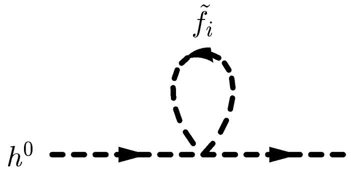
The cutoff Λ is the energy scale at which new physics enters and up to which the momentum integration in the loop integrals is performed. In some sense, the cutoff codes our ignorance of the physics at high energy scales. For the Standard Model itself, this is not really a problem. The divergences are taken care of by the standard techniques of renormalization, and the renormalized masses are input parameters of the theory. As soon as the Standard Model is embedded into a more fundamental framework, the quadratic dependence of the Higgs boson mass on Λ reveals its disturbing sensitivity to new physics beyond the cutoff. Quantum corrections "drag" the Higgs boson mass to values of order Λ which could in principle be the grand unification scale $\sim 10^{16}$ GeV, the Planck scale $\sim 10^{19}$ GeV or any intermediate scale, so it is not natural to have a light Higgs boson. This is the so-called "Naturalness Problem" .

Supersymmetry [1] is one of the most promising ideas that attempt to solve this problem. The Minimal Supersymmetric Standard Model (MSSM) is the most economical among the supersymmetric extensions of the SM. In the MSSM, one adds a supersymmetric partner to each quark, lepton and gauge boson. To give masses to both up-type and down-type quarks, and to avoid gauge anomalies introduced by the fermionic superpartners of the Higgs boson, one has to add two Higgs doublets.

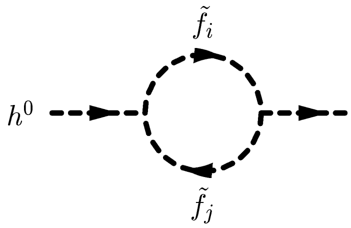
With the new particles present, the radiative corrections to scalar masses can be tamed. For each fermion, we have two complex scalars, the so-called superpartners. These particles also contribute to the correction of the mass. Due to supersymmetry, the couplings of the fermions and the scalars to the Higgs boson will be related, and fermions running in a loop give an additional minus sign, so it turns out that the quadratic divergence is exactly canceled.



$$\propto \quad -2\Lambda^2 + c_1 \log \Lambda$$



$$\propto \quad \Lambda^2 + c_2 \log \Lambda, \quad i = 1, 2$$



$$\propto \quad c_3 \log \Lambda$$

We have so far only considered the effects at one-loop. Clearly, we need this successful cancellation to persist also to higher order, because fermions can also indirectly couple to the Higgs boson, giving rise to contributions also quadratic in Λ. Fortunately, once supersymmetry is assumed, the cancellation of such contributions is not only possible, but actually unavoidable. The nice ultraviolet behavior of its theories is the main theoretical motivation for supersymmetry.

1.2 Why are Mass Sum Rules Interesting?

Supersymmetry decouples from standard model physics if the scale of supersymmetry breaking is more than a few hundred GeV. Future collider experiments will discover or rule out supersymmetry and determine which supersymmetric model is valid.

Once the Higgs boson is discovered, it will be crucial to have precise predictions for the properties of its properties in order to determine whether it is of supersymmetric origin or not. Radiative corrections to Higgs boson masses and decay rates play an essential role.

Supersymmetry tightly constrains the Higgs sector and thus gives rise to relations among the masses. At tree level, the mass sum rule for the neutral Higgs bosons is

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2.$$

This relation is remarkable for a number of reasons. First, the mass sum rule is naturally satisfied. Second, only physical Higgs boson masses enter the sum rule, and the validity of this relation can be checked without measuring parameters such as mixing angles or couplings. The gauge boson masses, m_Z and m_W, are already known to a very high precision, and the masses of the Higgs bosons will hopefully be determined very precisely in the future. Whenever dealing with high-precision measurements, the need of considering radiative corrections arises. The tree level mass sum rule is thus modified:

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2 + \Delta$$

Checking the validity of this relation reliably tests supersymmetry. The yet to be discovered Higgs sector could be due to a general two doublet model. If the mass sum rule is satisfied, we know that the Higgs sector is supersymmetric.

The mass sum rule probes for the presence of undetected, heavier Higgs bosons. In the case of a general supersymmetric $2N$ doublet model, the mass sum rule is changed to

$$\sum m_{H_i}^2 = \sum m_{A_i}^2 + m_Z^2 + \Delta,$$

where on the left hand side we sum over the CP-even Higgs bosons, and on the right hand side we sum over the CP-odd states. Violation of our original mass sum rule could thus hint to the existence of undetected heavier Higgs bosons. It should be pointed out that in other than the minimal extension of the standard model Higgs singlet states could be present which would change the mass sum rule already at tree level.

By a precise determination of Δ, CP violation in the Higgs sector can be probed. If CP is violated, the CP eigenstates H and A can mix, and this will lead to a shift in the relative position of the mass eigenstates and hence in Δ.

As a result, precision measurements of the Higgs mass rule will give a reliable test of the supersymmetric extension of the standard model and of the question whether it is minimal or not. For comparison with experiment, calculation of the radiative corrections plays a fundamental role.

1.3 Different Approaches to Calculating Radiative Corrections

It is known since quite a long time that radiative corrections substantially modify the tree level predictions for the Higgs boson masses. Much effort has been paid to evaluate these corrections and various techniques have been developed

- The effective potential approach [2, 3, 4]:
  The effective potential approach consists in minimizing the one-loop effective potential V_{eff}. The one-loop corrected squared mass matrix is identified with the second derivatives of V_{eff} at the minimum. The resulting matrix is diagonalized by an effective mixing angle α_{eff} and the eigenvalues are the one-loop corrected masses. The effective potential approach is an approximation. In calculating the contributions from loop diagrams, the external momenta of the particles are being neglected. As a consequence, the physical mass and the one-loop corrected mass differ by the self-energy terms $\Pi(m^2) - \Pi(0)$, and threshold effects are neglected.

- Renormalization group equations [5]:
  When supersymmetry is unbroken, the Higgs self-couplings are related to the gauge couplings. The parameters of any theory are in general dependent on the energy scale where they are evaluated. The relation between the parameters at two energy scales is given by the renormalization group equations. The gauge couplings are now run from their experimentally measuered values at the electroweak scale up to M_{SUSY}, where we have defined M_{SUSY} to be the mass scale of the

supersymmetric particles which are assumed to be roughly of the same order. At M_{SUSY}, we impose the supersymmetry boundary conditions, i.e. the relations between the gauge couplings and the Higgs self-couplings pointed out above for the case of unbroken supersymmetry. We now evolve the Higgs self-couplings down to the electroweak scale by integrating the renormalization group equations and compute the masses of the Higgs bosons which are given in terms of the gauge and Higgs couplings. Using this method, large logarithms of the ratio between M_{SUSY} and the electroweak scale are resummed to all orders, but this method may not detect all significant one-loop corrections. Effects from realistic particle spectra are not covered.

- The full diagrammatic calculation [6, 7, 8]:
  The complete diagrammatic calculation is more complicated than the approaches described above. The physical masses are identified with the poles of real part of the propagators according to the standard definition. Threshold effects from particle creation which are neglected in the effective potential apprroach are taken ito account and show up as characteristic cusps in the graphs of the one-loop corrected masses. Furthermore, effects from realistic particle spectra are covered, and not only leading, but all one-loop contributions are considered.

As the preceding discussion has made clear, every method has its advantages and disadvantages. The most reliable and accordingly most challenging way to evaluate the corrections is a diagrammatic calculation. Because of the great number of contributing diagrams, much attention should be paid to finding the dominant contributions which should then be evaluated first. It will turn out that the most important effects arise from the third generation of quarks and squarks due to their large Yukawa couplings.

It should be noted that going beyond one-loop corrections is desirable, but a highly non-trivial task. Analytic results for the two loop corrections to the MSSM Higgs masses exist only in the limit of vanishing external momenta, in which case the results from the diagrammatic and the effective potential approach are perfectly equivalent.

1.4 Overview of the Thesis

This work is organized as follows. In chapter 2, we describe the derivation of the tree level mass sum rule for the neutral Higgs bosons and its one-loop renormalization. The dominant corrections are diagrammatically evaluated and the final result is numerically analyzed. In chapter 3, we performed the same analysis for the charged Higgs boson mass sum rule.

The details of the calculations are in both cases deferred to the appendices D and E. In appendix F we list, for convenience and further reference, the results of the diagrammatic calculation for the Feynman diagrams used in our analyses.

Reading appendix A before proceeding with the main text is recommended.

We assume that the reader is familiar with supersymmetry and especially with the Higgs sector of the MSSM at tree level. We give a short overview of the MSSM in appendix B, mainly to fix conventions and to introduce our notation.

Chapter 2

The Neutral Higgs Boson Mass Sum Rule and its Renormalization

The mass sum rule involving the neutral Higgs bosons in the Minimal Supersymmetric Standard Model is derived and renormalized at one-loop. The radiative corrections arising from top, stop, bottom, and sbottom loops are calculated, and the results are numerically analyzed.

2.1 The Mass Sum Rule

The Higgs sector of the MSSM is tightly constrained by supersymmetry. The cubic and quartic couplings in the Higgs potential (see eq. B.1) are related to the gauge coupling constants g' and g. The Higgs sector can thus be described by only two additional parameters with respect to the Standard Model (SM), which are conventionally chosen to be the ratio of the vacuum expectation values, $\tan\beta = v_2/v_1$, and the mass of the CP-odd Higgs boson, m_A. For details, the reader is referred to section B.2. Since the four scalar masses m_h, m_H, m_A, $m_{H^\pm}$ are all determined from just two parameters, m_A and $\tan\beta$, we get mass relations [9, 10] among the Higgs bosons. The masses of the light and heavy CP-even Higgs bosons are

$$m_{h^0}^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right]. \qquad (2.1)$$

$$m_H^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 + \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta}\right], \qquad (2.2)$$

From eqs. 2.2, 2.1 we immediately get the mass sum rule

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2, \qquad (2.3)$$

which is valid at tree level. The mass sum rule is subject to large radiative corrections. We will describe the renormalization scheme in section 2.2. The renormalized version of eq. 2.3 is

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2 + \Delta. \qquad (2.4)$$

The one-loop correction Δ to the mass sum rule can be expressed entirely in terms of self energies:

$$\Delta = -\mathrm{Re}\ \Pi_{HH}(m_H^2) - \mathrm{Re}\ \Pi_{hh}(m_h^2) + \mathrm{Re}\ \Pi_{AA}(m_A^2) + \mathrm{Re}\ \Pi_{GG}(0) - \mathrm{Re}\ \Pi_{ZZ}(m_Z^2) \qquad (2.5)$$

After discussing the dominant contributions, the diagrammatic calculation of the correction Δ is performed in section 2.3 and our results are numerically analyzed in section 2.5.

2.2 The Renormalization Scheme

The renormalization scheme we describe follows ref. [7]. We have made no attempt to give a self-contained account of renormalization. For the basics, the reader is referred to ref. [11] and to standard textbooks on quantum field theory [12, 13].

We now proceed to describe the renormalization scheme. In this section, we denote bare quantities with a subscript 'b', renormalized ones with a subscript 'r'. All calculations will be performed in Landau gauge ($\xi = 0$). The bare Higgs potential is given by eq. B.1:

$$\begin{aligned} V_{\text{Higgs},b} = & \, m_{1b}^2 H_1^{i*} H_1^i + m_{2b}^2 H_2^{i*} H_2^i - m_{12b}^2 (\epsilon_{ij} H_1^i H_1^j + \epsilon_{ij} H_1^{i*} H_1^{j*}) \\ & + \frac{1}{8}(g_b^2 + g_b'^2)[H_1^{i*} H_1^i - H_2^{j*} H_2^j]^2 + \frac{1}{2} g_b^2 |H_1^{i*} H_2^i|^2 \end{aligned}$$

We have absorbed the $|\mu|^2$ into the definition of m_1, m_2. Substituting

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\zeta_1 \\ \varphi_1^- \end{pmatrix} + \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_2^+ \\ \eta_2 + i\zeta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

and rearranging the terms in powers of the fields gives:

$$\begin{aligned} V_{\text{Higgs},b} = & \, \text{const} + \tau_{1b}\eta_1 + \tau_{2b}\eta_2 + \frac{1}{2}\eta_i (M_{\eta_i\eta_j})_b \eta_j + \frac{1}{2}\zeta_i (M_{\zeta_i\zeta_j})_b \zeta_j \\ & + \text{terms of higher order in } \eta_i, \zeta_i + \text{terms in charged fields } \varphi_i^{\pm} \end{aligned}$$

The coefficients τ_{1b}, τ_{2b} of the terms linear in the fields are given in eqs. B.6, B.7. Note that the usual tree level minimization condition gives $\tau_{1b} = \tau_{2b} = 0$, so at tree level, terms linear in the fields are absent. At one-loop level, tadpole diagrams iT_1, iT_2 must be taken into account, as they shift the minimum. We will impose

$$T_1 = \tau_{1b}, \quad T_2 = \tau_{2b} \tag{2.6}$$

as a renormalization condition, which ensures that the renormalized vacua still minimize the one-loop potential.

We define $m^2 \equiv m_1^2 + m_2^2$ and $v^2 \equiv v_1^2 + v_2^2$ to simplify the notation. The mass matrices $(M_{\zeta_i\zeta_j})_b$, $(M_{\eta_i\eta_j})_b$ are given by eqs. B.11, B.15:

$$(M_{\zeta_i\zeta_j})_b = \begin{pmatrix} m_b^2 \sin^2\beta_b + \frac{1}{\sqrt{2}v_b}(\cos\beta_b T_1 - \sin\beta_b T_2) & m_b^2 \sin\beta_b \cos\beta_b - \frac{1}{\sqrt{2}v_b}(\sin\beta_b T_1 + \cos\beta_b T_2) \\ m_b^2 \sin\beta_b \cos\beta_b - \frac{1}{\sqrt{2}v_b}(\sin\beta_b T_1 + \cos\beta_b T_2) & m_b^2 \cos^2\beta_b - \frac{1}{\sqrt{2}v_b}(\cos\beta_b T_1 - \sin\beta_b T_2) \end{pmatrix}, \tag{2.7}$$

$$(M_{\eta_i\eta_j})_b = \begin{pmatrix} (M_{\zeta_1\zeta_1})_b + m_{Zb}^2 \cos^2\beta_b & -(M_{\zeta_1\zeta_2})_b - m_{Zb}^2 \sin\beta_b \cos\beta_b \\ -(M_{\zeta_2\zeta_1})_b - m_{Zb}^2 \sin\beta_b \cos\beta_b & (M_{\zeta_2\zeta_2})_b + m_{Zb}^2 \sin^2\beta_b \end{pmatrix}, \tag{2.8}$$

We have used eq. 2.6 to substitute T_1, T_2 for τ_{1b}, τ_{2b}, respectively.

We write each bare parameter as the sum of the renormalized parameter and the counterterm:

$$v_{ib} = v_{ir} + \delta v_i, \quad m_{ib} = m_{ir} + \delta m_i, \quad m_{12b} = m_{12r} + \delta m_{12}, \quad g_b' = g_r' + \delta g', \quad g_b = g_r + \delta g \tag{2.9}$$

All other renormalized parameters and counterterms are derived from those given in eq. 2.9. Note that the field renormalization enters in the renormalization of the vacua. The renormalized parameters and counterterms in eq. 2.9 are fixed by the renormalization conditions which will be given below.

The divergent loop integrals must be regularized. To avoid complications in the calculation, the regularization should respect the symmetries of the Lagrangian. Dimensional regularization [14] preserves the gauge symmetry, but breaks supersymmetry. A regularization method, called dimensional reduction [15, 16], explicitly preserves supersymmetry. For doing calculations, dimensional reduction amounts to usual dimensional regularization where the Dirac algebra is performed in four (and not n) dimensions.

Renormalization schemes differ in the way they remove the divergence. In the minimal subtraction scheme (MS), based on dimensional regularization, only the pole $2/\epsilon$ is removed, whereas in the modified minimal subtraction scheme ($\overline{\text{MS}}$), also the artifacts of dimensional regularization is removed, i.e. $2/\epsilon - \gamma + \log 4\pi$ is subtracted. The corresponding schemes using dimensional reduction are called DR and $\overline{\text{DR}}$, respectively. In the following, we will denote self-energies and tadpoles which are renormalized using $\overline{\text{DR}}$ with a hat, and the $\overline{\text{DR}}$ renormalized masses and the angle β will be given the subscript 'r'. We will later in the text establish the connection between the $\overline{\text{DR}}$ renormalized parameters and the physical ones.

We substitute the decomposition of the bare parameters eq. 2.9 in eqs. 2.7, 2.8 and fix the counterterms to cancel the poles according to the $\overline{\text{DR}}$ scheme:

$$(M_{\zeta_i\zeta_j})_r = \begin{pmatrix} m_r^2 \sin^2\beta_r + \frac{1}{\sqrt{2}v_r}(\cos\beta_r \hat{T}_1 - \sin\beta_r \hat{T}_2) & m_r^2 \sin\beta_r\cos\beta_r - \frac{1}{\sqrt{2}v_r}(\sin\beta_r \hat{T}_1 + \cos\beta_r \hat{T}_2) \\ m_r^2 \sin\beta_r\cos\beta_r - \frac{1}{\sqrt{2}v_r}(\sin\beta_r \hat{T}_1 + \cos\beta_r \hat{T}_2) & m_r^2 \cos^2\beta_r - \frac{1}{\sqrt{2}v_r}(\cos\beta_r \hat{T}_1 - \sin\beta_r \hat{T}_2) \end{pmatrix}, \tag{2.10}$$

$$(M_{\eta_i\eta_j})_r = \begin{pmatrix} (M_{\zeta_1\zeta_1})_r + m_{Zr}^2\cos^2\beta_r & -(M_{\zeta_1\zeta_2})_r - m_{Zr}^2 \sin\beta_r\cos\beta_r \\ -(M_{\zeta_2\zeta_1})_r - m_{Zr}^2\sin\beta_r\cos\beta_r & (M_{\zeta_2\zeta_2})_r + m_{Zr}^2\sin^2\beta_r \end{pmatrix}. \tag{2.11}$$

The one-loop $\overline{\text{DR}}$-renormalized inverse propagator matrices [11] are given by:

$$\Gamma_{\zeta_i\zeta_j}(p^2) = p^2\delta_{ij} - (M_{\zeta_i\zeta_j})_r + \hat{\Pi}_{\zeta_i\zeta_j}(p^2), \tag{2.12}$$
$$\Gamma_{\eta_i\eta_j}(p^2) = p^2\delta_{ij} - (M_{\eta_i\eta_j})_r + \hat{\Pi}_{\eta_i\eta_j}(p^2). \tag{2.13}$$

The $\overline{\text{DR}}$-renormalized masses will be related to physical ones below. The vacua implicitly depend on the renormalization scale, thus $\tan\beta = v_2/v_1$ is a running quantity in our scheme. We will henceforth drop the index 'r' of v_1, v_2, β.

Next, eliminate the tadpoles by expressing them in terms of self-energies. We note that $\Gamma_{\zeta_i\zeta_j}(0)$ has a zero eigenvalue [4, 17] which corresponds to the Goldstone field $G = -c_\beta\zeta_1 + s_\beta\zeta_2$, i.e. $\Gamma_{GG}(0) = \Gamma_{GA}(0) = 0$. The orthogonal state is the CP-odd Higgs boson $A = s_\beta\zeta_1 + c_\beta\zeta_2$. The change of basis is then given by

$$\begin{pmatrix} G \\ A \end{pmatrix} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix}. \tag{2.14}$$

Let us denote the inverse propagator matrix of the CP-odd Higgs boson in the basis ζ_1, ζ_2 by

$$\Gamma_{ij}(p^2) = p^2\delta_{ij} - M_{ij} + \hat{\Pi}_{ij}(p^2), \quad i,j = \zeta_1,\zeta_2$$

and in the G, A basis by

$$\Gamma_{\alpha\beta}(p^2) = p^2\delta_{\alpha\beta} - M_{\alpha\beta} + \hat{\Pi}_{\alpha\beta}(p^2), \quad \alpha,\beta = G, A.$$

The expressions for the inverse propagator matrix of the CP-odd Higgs boson in the two bases are related by

$$U_{\alpha i}\,\Gamma_{ij}(p^2)\,U^{-1}_{j\beta} = \Gamma_{\alpha\beta}(p^2), \tag{2.15}$$

where U is the matrix in eq. 2.14. Now we use $\Gamma_{GG}(0) = 0$. Specializing to $p^2 = 0$ and $\alpha = \beta = G$, eq. 2.15 reads

$$U_{Gi}\,\Gamma_{ij}(0)\,U^{-1}_{jG} = \Gamma_{GG}(0) \quad \leftrightarrow \quad -U_{Gi}M_{ij}U^{-1}_{jG} + U_{Gi}\hat{\Pi}_{ij}(0)U^{-1}_{jG} = \Gamma_{GG}(0). \tag{2.16}$$

Noting that $U_{Gi}\hat{\Pi}_{ij}(0)U^{-1}_{jG}$ can be rewritten as $\hat{\Pi}_{GG}(0)$ and

$$UM_{ij}U^{-1} = \begin{pmatrix} c^2_\beta M_{11} - 2s_\beta c_\beta M_{12} + s^2_\beta M_{22} & -s_\beta c_\beta M_{11} - (c^2_\beta - s^2_\beta) + s_\beta c_\beta M_{22} \\ -s_\beta c_\beta M_{11} - (c^2_\beta - s^2_\beta) + s_\beta c_\beta M_{22} & s^2_\beta M_{11} + 2s_\beta c_\beta M_{12} + c^2_\beta M_{22} \end{pmatrix},$$

eq. 2.16 takes on the form

$$c^2_\beta M_{11} - 2s_\beta c_\beta M_{12} + s^2_\beta M_{22} + \hat{\Pi}_{GG}(0) = \Gamma_{GG}(0). \tag{2.17}$$

Substituting the expressions for M_{ij} as given in eq. 2.10 into eq. 2.17 immediately gives

$$\hat{\Pi}_{GG}(0) = \frac{1}{\sqrt{2}v}(c_\beta \hat{T}_1 + s_\beta \hat{T}_2). \tag{2.18}$$

Starting with $\Gamma_{GA}(0) = 0$ and specializing to $p^2 = 0$, $\alpha = G$, $\beta = A$ gives after completely analogous calculations

$$\hat{\Pi}_{GA}(0) = \frac{1}{\sqrt{2}v}(-s_\beta \hat{T}_1 + c_\beta \hat{T}_2). \tag{2.19}$$

Solving eqs. 2.18, 2.19 for $\hat{T}_1$, $\hat{T}_2$ gives:

$$\hat{T}_1 = \sqrt{2}v\left[c_\beta \hat{\Pi}_{GG}(0) - s_\beta \hat{\Pi}_{GA}(0)\right], \quad \hat{T}_2 = \sqrt{2}v\left[s_\beta \hat{\Pi}_{GG}(0) + c_\beta \hat{\Pi}_{GA}(0)\right]. \tag{2.20}$$

The physical mass of the A boson, m_A, is the pole of the real part of the propagator, or the zero of the inverse propagator, i.e. Re $\Gamma_{AA}(m^2_A) = 0$. In the following, we will not take the real part for the sake of brevity, it being understood that for the masses, only the real part of the self-energy makes a contribution. Using again eq. 2.16 and specializing to $p^2 = m^2_A$, $\alpha = \beta = A$ gives

$$m^2_A + \hat{\Pi}_{AA}(m^2_A) = s^2_\beta M_{11} + 2s_\beta c_\beta M_{12} + c^2_\beta M_{22}.$$

Inserting the expressions for M_{ij} as given in eq. 2.10 yields

$$m^2_A = m^2 - \frac{1}{\sqrt{2}v}(c_\beta \hat{T}_1 + s_\beta \hat{T}_2) - \hat{\Pi}_{AA}(m^2_A). \tag{2.21}$$

m_A is the physical mass, and we drop the index r.

The Z mass introduced above is defined by the renormalized vacua and gauge couplings,

$$m_{Zr} = \frac{1}{2}(g'^2_r + g^2_r)(v^2_{1r} + v^2_{2r}).$$

The connection between m_{Zr} and the physical mass is given by

$$m_Z = m_{Zb} + \Pi_{ZZ}(m^2_Z) = m_{Zr} + \delta m_Z + \frac{2}{\bar{\epsilon}} + \hat{\Pi}_{ZZ}(m^2_Z) = m_{Zr} + \hat{\Pi}_{ZZ}(m^2_Z). \tag{2.22}$$

The masses of the CP-even Higgs bosons, m_h and m_H, are by definition the real parts of the poles of the propagator matrix $\Gamma^{-1}_{\eta_i\eta_j}(p^2)$, so they are the zeros of the equation

$$\det\, \Gamma_{\eta_i\eta_j}(p^2) = 0. \tag{2.23}$$

As we want to express m_h, m_H in terms of the input parameters of the theory, we rewrite $M_{\zeta_i\zeta_j}$ as given in 2.10 using eqs. 2.20, 2.21, 2.22:

$$M_{\zeta_i\zeta_j} = \begin{pmatrix} \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] s_\beta^2 + \hat{\Pi}_{\zeta_1\zeta_1}(0) & \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] s_\beta c_\beta + \hat{\Pi}_{\zeta_1\zeta_2}(0) \\ \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] s_\beta c_\beta + \hat{\Pi}_{\zeta_1\zeta_2}(0) & \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] c_\beta^2 + \hat{\Pi}_{\zeta_2\zeta_2}(0) \end{pmatrix}, \tag{2.24}$$

where we have used the abbreviation

$$\Delta\hat{\Pi}(p^2) \equiv \hat{\Pi}(p^2) - \hat{\Pi}(0). \tag{2.25}$$

The calculation is straightforward, and the only point to be observed is that we used $\hat{\Pi}_{ij} = U^{-1}\hat{\Pi}_{\alpha\beta}U$, $i,j = \zeta_1, \zeta_2$, $\alpha, \beta = \eta_1, \eta_2$, i.e.

$$\hat{\Pi}_{ij} = \begin{pmatrix} c_\beta^2\hat{\Pi}_{GG} - 2s_\beta c_\beta\hat{\Pi}_{GA} + s_\beta^2\hat{\Pi}_{AA} & -s_\beta c_\beta\hat{\Pi}_{GG} - (c_\beta^2 - s_\beta^2)\hat{\Pi}_{GA} + s_\beta c_\beta\hat{\Pi}_{AA} \\ -s_\beta c_\beta\hat{\Pi}_{GG} - (c_\beta^2 - s_\beta^2)\hat{\Pi}_{GA} + s_\beta c_\beta\hat{\Pi}_{AA} & s_\beta^2\hat{\Pi}_{GG} + 2s_\beta c_\beta\hat{\Pi}_{GA} + c_\beta^2\hat{\Pi}_{AA} \end{pmatrix}.$$

to write the result in a more compact form.

$M_{\eta_i\eta_j}$ is given by the same expression as in eq. 2.11, but $M_{\zeta_i\zeta_j}$ replaced by the values in eq. 2.24 and m_{Zr} replaced by eq. 2.22:

$$M_{\eta_i\eta_j} = \begin{pmatrix} M_{\zeta_1\zeta_1} + \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] c_\beta^2 & -M_{\zeta_1\zeta_2} - \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] s_\beta c_\beta \\ -M_{\zeta_2\zeta_1} - \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] s_\beta c_\beta & M_{\zeta_2\zeta_2} + \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] s_\beta^2 \end{pmatrix} \tag{2.26}$$

The inverse propagator matrix for the CP-even Higgs is given by

$$\Gamma_{\eta_i\eta_j}(p^2) = p^2\delta_{ij} - M_{\eta_i\eta_j} + \hat{\Pi}_{\eta_i\eta_j}(p^2). \tag{2.27}$$

Substituting eqs. 2.26, 2.24 into eq. 2.27 immediately gives

$$\Gamma_{\eta_1\eta_1}(p^2) = p^2 - \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] c_\beta^2 - \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] s_\beta^2 - \hat{\Delta}_{11} + \Delta\hat{\Pi}_{\eta_1\eta_1}(p^2), \tag{2.28}$$
$$\Gamma_{\eta_1\eta_2}(p^2) = \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] s_\beta c_\beta + \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] s_\beta c_\beta - \hat{\Delta}_{12} + \Delta\hat{\Pi}_{\eta_1\eta_2}(p^2), \tag{2.29}$$
$$\Gamma_{\eta_2\eta_2}(p^2) = p^2 - \left[m_Z^2 - \hat{\Pi}_{ZZ}(m_Z^2)\right] s_\beta^2 - \left[m_A^2 + \Delta\hat{\Pi}_{AA}(m_A^2)\right] c_\beta^2 - \hat{\Delta}_{22} + \Delta\hat{\Pi}_{\eta_2\eta_2}(p^2), \tag{2.30}$$

where we have used the abbreviation

$$\hat{\Delta}_{11} \equiv \hat{\Pi}_{\zeta_1\zeta_1}(0) - \hat{\Pi}_{\eta_1\eta_1}(0), \quad \hat{\Delta}_{22} \equiv \hat{\Pi}_{\zeta_2\zeta_2}(0) - \hat{\Pi}_{\eta_2\eta_2}(0), \quad \hat{\Delta}_{12} \equiv -\hat{\Pi}_{\zeta_1\zeta_2}(0) - \hat{\Pi}_{\eta_1\eta_2}(0) \tag{2.31}$$

to simplify the notation.

Now we write eq. 2.23 in the form

$$\det\,(p^2 - M[p^2]) = 0. \tag{2.32}$$

We will denote the dependence of M on p^2 by squared brackets for notational convenience. Exact solutions of eq. 2.32 can only be found numerically. There are explicit expressions for approximate solutions [18]. We will follow another approach [7] and obtain the solutions using matrix perturbation theory. Split $M[p^2]$ in a part independent of p^2 and one dependent on p^2:

$$M[p^2] = M^0 + \Delta M[p^2]$$

Explicitly, the decomposition is given by:

$$M^0_{11} = m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 + \hat{\Delta}_{11}, \quad M^0_{22} = m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 + \hat{\Delta}_{22}, \tag{2.33}$$

$$M^0_{12} = -(m_Z^2 + m_A^2) s_\beta c_\beta + \hat{\Delta}_{12}, \tag{2.34}$$

$$\Delta M_{11}[p^2] = -\hat{\Pi}_{ZZ}(m_Z^2) c_\beta^2 + \Delta\hat{\Pi}_{AA}(m_A^2) s_\beta^2 - \Delta\hat{\Pi}_{\eta_1\eta_1}(p^2), \tag{2.35}$$

$$\Delta M_{22}[p^2] = -\hat{\Pi}_{ZZ}(m_Z^2) s_\beta^2 + \Delta\hat{\Pi}_{AA}(m_A^2) c_\beta^2 - \Delta\hat{\Pi}_{\eta_2\eta_2}(p^2), \tag{2.36}$$

$$\Delta M_{12}[p^2] = \hat{\Pi}_{ZZ}(m_Z^2) s_\beta c_\beta - \Delta\hat{\Pi}_{AA}(m_A^2) s_\beta c_\beta - \Delta\hat{\Pi}_{\eta_1\eta_2}(p^2). \tag{2.37}$$

This decomposition is immediately verified by substituting eqs. 2.33-2.37 in

$$\Gamma_{\eta_i\eta_j}(p^2) = p^2 - M[p^2]$$

and comparing the result to eqs. 2.28-2.30. The decomposition as given in eqs. 2.33-2.37 is not unique. It is inspired by the effective potential approach.

In lowest order of matrix perturbation theory, eq. 2.32 is simply an eigenvalue problem:

$$\det\,(p^2 - M^0) = 0$$

The eigenvalues and eigenstates are well-known:

$$(m_h^2)^0 = \frac{1}{2}\left[M^0_{11} + M^0_{22} - \sqrt{(M^0_{11} - M^0_{22})^2 + 4(M^0)^2_{12}}\right] \tag{2.38}$$

$$(m_H^2)^0 = \frac{1}{2}\left[M^0_{11} + M^0_{22} + \sqrt{(M^0_{11} - M^0_{22})^2 + 4(M^0)^2_{12}}\right] \tag{2.39}$$

$$h = -s_\alpha\eta_1 + c_\alpha\eta_2, \quad H = c_\alpha\eta_1 + s_\alpha\eta_2, \quad \tan\alpha = \frac{2M^0_{12}}{M^0_{11} - M^0_{22}} \tag{2.40}$$

At first order in perturbation theory, we obtain corrections to the eigenvalues:

$$(m_h^2)^1 = (m_h^2)^0 + \Delta^1 m_h^2, \quad (m_H^2)^1 = (m_H^2)^1 + \Delta^1 m_H^2, \tag{2.41}$$

$$\Delta^1 m_h^2 = \langle h|\Delta M[(m_h^2)^0]|h\rangle, \quad \Delta^1 m_H^2 = \langle H|\Delta M[(m_H^2)^0]|H\rangle. \tag{2.42}$$

We will evaluate the correction to $(m_h^2)^0$ explicitly. Substituting the expression for h as given in eq. 2.40 yields

$$\Delta^1 m_h^2 = \begin{pmatrix} -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \Delta M_{11}[(m_h^2)^0] & \Delta M_{12}[(m_h^2)^0] \\ \Delta M_{12}[(m_h^2)^0] & \Delta M_{22}[(m_h^2)^0] \end{pmatrix} \begin{pmatrix} -s_\alpha \\ c_\alpha \end{pmatrix},$$

and the result of the calculation is

$$\Delta^1 m_h^2 = s_\alpha^2 \Delta M_{11}[(m_h^2)^0] - 2s_\alpha c_\alpha \Delta M_{12}[(m_h^2)^0] + c_\alpha^2 \Delta M_{22}[(m_h^2)^0].$$

Inserting the expression for $\Delta M_{ij}(p^2)$ in eqs. 2.35-2.37 gives after a simple calculation

$$\begin{aligned}\Delta^1 m_h^2 = {} & \Delta\hat{\Pi}_{AA}(m_A^2)\cos^2(\beta-\alpha) - \hat{\Pi}_{ZZ}(m_Z^2)\sin^2(\beta+\alpha) \\ & - s_\alpha^2 \Delta\hat{\Pi}_{\eta_1\eta_1}(m_h^2) + 2s_\alpha c_\alpha \Delta\hat{\Pi}_{\eta_1\eta_2}(m_h^2) - c_\alpha^2 \Delta\hat{\Pi}_{\eta_2\eta_2}(m_h^2).\end{aligned}$$

The last three terms can be rewritten in terms of the self-energy of the mass eigenstate h. To do so, note that the change of basis from η_1, η_2 to h, H is given by

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

as one can immediately see from eq. 2.40. Changing the basis in the self-energy matrix gives

$$\begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \hat{\Pi}_{\eta_1\eta_1} & \hat{\Pi}_{\eta_1\eta_2} \\ \hat{\Pi}_{\eta_1\eta_2} & \hat{\Pi}_{\eta_2\eta_2} \end{pmatrix} \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} = \begin{pmatrix} \hat{\Pi}_{HH} & \hat{\Pi}_{Hh} \\ \hat{\Pi}_{Hh} & \hat{\Pi}_{hh} \end{pmatrix},$$

which yields the expression

$$\Delta\hat{\Pi}_{hh}(m_h^2) = s_\alpha^2 \Delta\hat{\Pi}_{\eta_1\eta_1}(m_h^2) - 2s_\alpha c_\alpha \Delta\hat{\Pi}_{\eta_1\eta_2}(m_h^2) + c_\alpha^2 \Delta\hat{\Pi}_{\eta_2\eta_2}(m_h^2).$$

The final result for the correction to m_h^2 can now be cast into the form

$$\Delta^1 m_h^2 = -\sin^2(\beta+\alpha)\hat{\Pi}_{ZZ}(m_Z^2) + \cos^2(\beta-\alpha)\Delta\hat{\Pi}_{AA}(m_A^2) - \Delta\hat{\Pi}_{hh}(m_h^2). \tag{2.43}$$

Completely analogous calculations give

$$\Delta^1 m_H^2 = -\cos^2(\beta+\alpha)\hat{\Pi}_{ZZ}(m_Z^2) + \sin^2(\beta-\alpha)\Delta\hat{\Pi}_{AA}(m_A^2) - \Delta\hat{\Pi}_{HH}(m_H^2). \tag{2.44}$$

Substituting eqs. 2.38, 2.39, 2.43, 2.44 in eq. 2.41, taking the sum and renaming $(m_h^2)^1$, $(m_H^2)^1$ to m_h^2, m_H^2, we arrive at

$$m_h^2 + m_H^2 = M_{11}^0 + M_{22}^0 - \hat{\Pi}_{ZZ}(m_Z^2) + \Delta\hat{\Pi}_{AA}(m_A^2) - \Delta\hat{\Pi}_{HH}(m_H^2) - \Delta\hat{\Pi}_{hh}(m_h^2).$$

Substituting the expressions for M_{11}^0, M_{22}^0 given in eq. 2.33 and using the definition eq. 2.25, we have

$$\begin{aligned} m_h^2 + m_H^2 = m_Z^2 + m_A^2 + \hat{\Delta}_{11} + \hat{\Delta}_{22} - \hat{\Pi}_{ZZ}(m_Z^2) + \hat{\Pi}_{AA}(m_A^2) - \hat{\Pi}_{AA}(0) \\ - \hat{\Pi}_{HH}(m_H^2) + \hat{\Pi}_{HH}(0) - \hat{\Pi}_{hh}(m_h^2) + \hat{\Pi}_{hh}(0). \end{aligned} \tag{2.45}$$

Using eq. 2.31, we have

$$\hat{\Delta}_{11} + \hat{\Delta}_{22} = \hat{\Pi}_{\zeta_1\zeta_1}(0) + \hat{\Pi}_{\zeta_2\zeta_2}(0) - \hat{\Pi}_{\eta_1\eta_1}(0) - \hat{\Pi}_{\eta_2\eta_2}(0). \tag{2.46}$$

The trace of a matrix is left invariant under a change of basis, so

$$\hat{\Pi}_{\zeta_1\zeta_1}(0) + \hat{\Pi}_{\zeta_2\zeta_2}(0) = \hat{\Pi}_{AA}(0) + \hat{\Pi}_{GG}(0) \quad \text{and} \quad \hat{\Pi}_{\eta_1\eta_1}(0) + \hat{\Pi}_{\eta_2\eta_2}(0) = \hat{\Pi}_{hh}(0) + \hat{\Pi}_{HH}(0). \tag{2.47}$$

Using eqs. 2.46, 2.47, eq. 2.45 can be cast into the form

$$m_h^2 + m_H^2 = m_Z^2 + m_A^2 - \hat{\Pi}_{ZZ}(m_Z^2) + \hat{\Pi}_{AA}(m_A^2) + \hat{\Pi}_{GG}(0) - \hat{\Pi}_{HH}(m_H^2) - \hat{\Pi}_{hh}(m_h^2), \tag{2.48}$$

and the expression for the correction

$$\Delta = -\mathrm{Re}\,\hat{\Pi}_{HH}(m_H^2) - \mathrm{Re}\,\hat{\Pi}_{hh}(m_h^2) + \mathrm{Re}\,\hat{\Pi}_{AA}(m_A^2) + \mathrm{Re}\,\hat{\Pi}_{GG}(0) - \mathrm{Re}\,\hat{\Pi}_{ZZ}(m_Z^2)$$

as given in eq. 2.5 immediately follows. As mentioned above, we have to take the real parts of the self-energies.

We make some concluding remarks. We have compared our result for Δ to the expression given in ref. [7] and found agreement, apart from an obvious misprint in eq. (33), where the self-energy of the Goldstone boson should be taken at zero momentum transfer,

$$\hat{\Pi}_{GG}(m_A^2) \quad \to \quad \hat{\Pi}_{GG}(0).$$

Notice also that in the footnote on p. 285 in ref. [7] the self-energy diagrams for gauge bosons are denoted

$$+ig_{\mu\nu}\Pi(p^2) + p_\mu p_\nu\text{-terms},$$

this should be changed in

$$-ig_{\mu\nu}\Pi(p^2) + p_\mu p_\nu\text{-terms}$$

to be consistent with eq. (20) and the sign of $\Pi_{ZZ}(m_Z^2)$ in the final result for Δ. This was first pointed out in ref. [19]

We have also checked our expression for Δ against ref. [6] and found complete agreement, altough we do not agree with the result of the diagrammatic calculation, as will be discussed below.

In our renormalization scheme, the self-energies are renormalized via $\overline{\mathrm{DR}}$ and are thus finite. In other schemes, the self-energies are not renormalized and the cancellation of divergences has to be proved explicitly [6]. We have proved the cancellation of divergences in appendix D.

2.3 Diagrammatic Calculation of the Correction

Now we turn our attention to the explicit diagrammatic calculation of the correction

$$\Delta = -\mathrm{Re}\ \Pi_{HH}(m_H^2) - \mathrm{Re}\ \Pi_{hh}(m_h^2) + \mathrm{Re}\ \Pi_{AA}(m_A^2) + \mathrm{Re}\ \Pi_{GG}(0) - \mathrm{Re}\ \Pi_{ZZ}(m_Z^2) \tag{2.49}$$

We have dropped the hat for notational convenience. It is understood that all self-energies are renormalized via $\overline{\mathrm{DR}}$.

We will give an overview of the work which has been done so far by other authors before we proceed to give the details of the present calculation. Ref. [2] is one of the first works which point out that the mass of the lightest Higgs boson, which is constrained to be lighter than the gauge boson Z in supersymmetric two-Higgs-doublet models, can receive large radiative corrections. Using the effective potential approach and assuming $m_t \leq 60$ GeV, they come to the conclusion that the effects of the top and stop are negligible and the mass of the lightest Higgs will not exceed 95 GeV. Ref. [20] performs a diagrammatic calculation of the correction to the Higgs boson mass sum rules including contributions from gauge bosons, gauginos, ghosts, Higgs bosons, Higgsinos in the loops, and show that they are screened against large one-loop corrections. Even a large splitting between the gauge-Higgs and gaugino-Higgsino sector and large supersymmetry breaking mass parameters lead to very small corrections. Refs. [3, 4] also apply an effective potential approach, but stress that ref. [2] assumes that m_t is rather small and radiative corrections to m_h originating from top and stop loops can be large if the top quark is sufficiently heavy. Ref. [21] computes the one-loop radiative correction to m_h diagrammatically under very simplifying assumptions and comes to the conclusion that m_h receives large corrections which grow like m_t^4. Consequently, the light Higgs boson mass can be larger than M_Z. Ref. [6] renormalizes the neutral Higgs boson mass sum rule and follows a diagrammatic approach to compute the correction arising from quarks and squarks running in the loops. The analytic formula for the correction to the mass sum rule given in the appendix does not agree with our results[1], as will be discussed later on in the text. Ref. [7] describes a renormalization procedure for the neutral Higgs boson mass sum rule and calculates the correction arising from quarks and squarks diagrammatically. Refs. [22, 23] describe a complete on-shell renormalization scheme for the Higgs sector of the MSSM at one-loop and list radiative corrections to the neutral Higgs boson masses. The correction to the mass sum rules follow immediately from the corrections to the masses. In evaluating the self-energies of eq. 2.49, we limit ourselves to the contributions from the third generation of quarks and squarks. Due to the large Yukawa couplings, these are the only contributions which modify the mass sum rule significantly as has already become clear in the discussion above.

[1]In email correspondence, M. Berger kindly informed us that the corrections we found to his result are indeed correct, but that these have already been noted some time ago by A. Brignole [7]. We would like to thank M. Berger for this information.

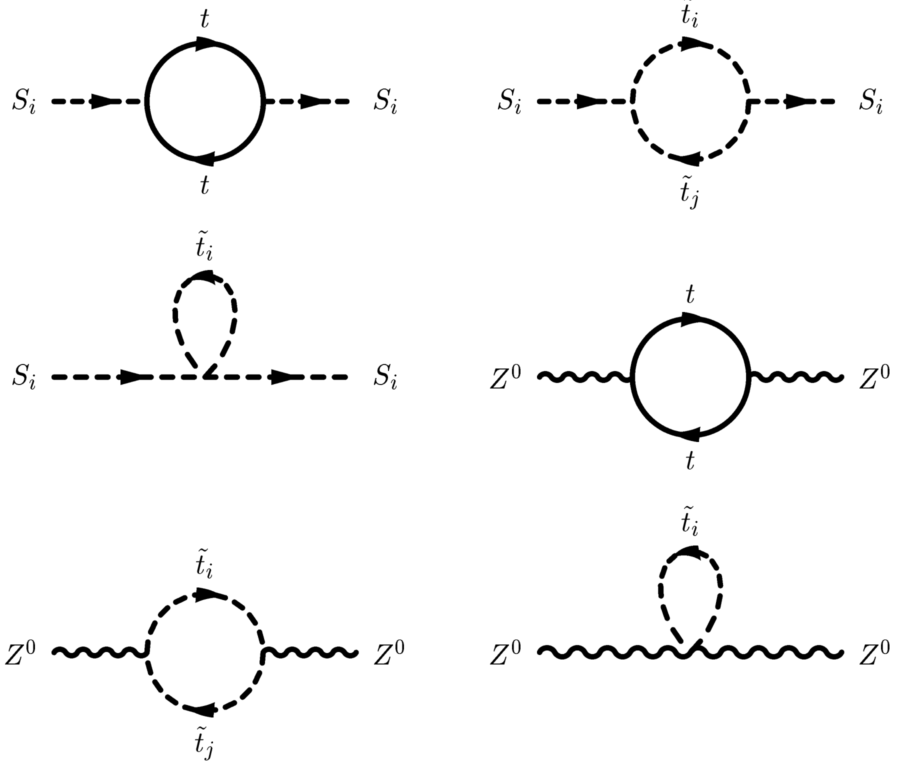


Figure 2.1: One-loop self-energy graphs contributing to Δ. S_i denotes one of the fields h, H, A, G.

The self-energy diagrams needed to compute Δ are shown in fig. 2.1. We compute the complete contributions from the third generation of quarks and squarks, i.e. top, stop, bottom and sbottom. As a matter of fact, only graphs with tops and stops running in the loops must be calculated. The diagrams with bottoms and sbottoms are obtained from the latter ones by simple substitutions which will be given below. The contributions to Δ from other particles of the MSSM are small, as has been argued above. The diagrams in fig. 2.1 do not depend on the choice of a specific R_ξ gauge, as the vertex factors are independent of ξ [24] and there are no gauge bosons running in the loops. We have taken the Feynman rules from refs. [25, 26] and crosschecked them with ref. [27]. The loop diagrams are regularized via dimensional reduction. The result of the regularization has an infinite part (plus the artifacts of dimensional regularization)

$$\frac{2}{\epsilon} - \gamma + \log 4\pi,$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant, and a finite part which is dependent on some renormalization scale Q. The renormalization scale Q may be chosen according to convenience [13]. As Δ is a difference of physical masses, it cannot depend on the renormalization scale.

The finite part of the correction Δ is a sum of integrals of the form

$$\int dx \log \Delta_{ij}, \quad \int dx\ x \log \Delta_{ij}, \quad \int dx\ x^2 \log \Delta_{ij}, \tag{2.50}$$

and Δ_{ij} is defined as

$$\Delta_{ij} \equiv \frac{-x(1-x)q^2 - x(m_i^2 - m_j^2) + m_i^2}{Q^2},$$

where m_i, m_j are the masses of the particles running in the loop, q^2 is the external momentum squared, and Q is the renormalization scale. To make loop calculations more feasible, a collection of formulae for scalar one-loop integrals has been devised and all other one-loop integrals are algebraically related to this set of expressions [28, 29]. We will express the finite part of our diagrams in terms of these one-loop functions, which are called Passarino-Veltman functions (short: PV functions). In appendix C we give the definitions of the PV functions and some useful relations.

There are good reasons to use PV functions. First, the integrals in eq. 2.50 can be explicitly solved, but the expressions are quite complicated. The argument of the logarithm Δ_{ij} can become negative in some region of the parameter space, and the the logarithm must be analytically continued to the complex plane. Naive algorithms for numerical integration do not work, because in most programming languages, the logarithm is only defined for real arguments. The symbolic computation engines Maple and Mathematica have extended the definition of the logarithm to complex values, but we have checked numerically that the results they give for the symbolic integration of the functions in eq. 2.50 do not agree with the explicit formulae as given in [29] where we have compensated for the discrepancies which could arise due to different conventions. Second, there are Fortran and C libraries for the numerical evaluation of PV functions. Algorithms which directly encode the explicit formulae are extremely unreliable due to numerical cancellations. Using a very large precision (120 digits and more) to avoid numerical instabilities makes the programs extremely slow. New algorithms have been devised which use standard precision and reliably evaluate the standard one-loop integrals [30, 31]. Third, calculating radiative corrections can be very tedious, and it is best to develop a general method for their numerical evaluation. For a given process, all contributing diagrams are calculated and expressed in terms of the standardized PV functions. Then all we need is a computer program which reliably implements the PV functions.

The actual calculation of the correction Δ is straightforward, but rather lengthy and tedious. There are six distinct classes of diagrams to be calculated, as can be seen from fig. 2.1. In each class, the analytic structure of the integral is exactly the same, and the diagrams differ only in the vertex factors. All in all, 26 different diagrams are to be calculated. For each diagram, the result of the calculation is listed in appendix F. Note that some diagrams are listed twice to make it easier for the reader to keep track of the long calculations. As we want to compare the result of our calculation to ref. [6], we adopted the notation of the latter article. In ref. [6], the finite part of the loop integrals are expressed in terms of three functions defined as

$$F(q^2, m_1^2, m_2^2) = \int dx \ \log \frac{x m_1^2 + (1-x) m_2^2 - x(1-x) q^2}{Q^2}, \tag{2.51}$$

$$G(q^2, m_1^2, m_2^2) = \int dx \ x(1-x) \ \log \frac{x m_1^2 + (1-x) m_2^2 - x(1-x) q^2}{Q^2}, \tag{2.52}$$

$$H(q^2, m_1^2, m_2^2) = \int dx \ x \ \log \frac{x m_1^2 + (1-x) m_2^2 - x(1-x) q^2}{Q^2}. \tag{2.53}$$

F, G, H can be related to the Passarino-Veltman functions, see eqs. C.20 -C.22. The details of the calculation of Δ are given in appendix D. The correction Δ is divided into 3 pieces, $\Delta = \Delta_4 + \Delta_2 + \Delta_0$, where Δ_n, $n = 0, 2, 4$, is the part of the correction in which the nth power of the top or stop mass parameters occur. The cancellation of divergences and the calculation of the finite part is then performed separately for each Δ_n. As already stated above, we calculated the Feynman diagrams with tops and stops in the loops. The result for bottoms and sbottoms is obtained by simply substituting [6]

$$\theta_t \to \theta_b, \quad m_t \to m_b, \quad m_{\tilde{t}_i} \to m_{\tilde{b}_i}, \quad e_u \to e_d, \quad T_3 = \frac{1}{2} \to T_3 = -\frac{1}{2},$$
$$\sin\beta \to \cos\beta, \quad \cos\beta \to \sin\beta, \quad \sin\alpha \to \cos\alpha, \quad \cos\alpha \to \sin\alpha,$$
$$\sin(\alpha+\beta) \to -\sin(\alpha+\beta), \quad \cos(\alpha+\beta) \to \cos(\alpha+\beta).$$

The final result of our calculation is listed in section D.4. We compared our result to ref. [6]. In two points, we disagree with the formulae given there[2]. First, the term

$$\frac{N_c g^2 m_t^2}{32\pi^2 c_w^2} \log Q^2 \tag{2.54}$$

in the sixth line of the expression for Δ_2 in ref. [6] should be replaced by

$$\frac{N_c g^2 m_t^2}{32\pi^2 c_w^2} F(m_Z^2, m_t^2, m_t^2). \tag{2.55}$$

As can easily be seen from the definition of F, the part of eq. 2.55 which is dependent on the renormalization scale is

$$-\frac{N_c g^2 m_t^2}{32\pi^2 c_w^2} \log Q^2,$$

i.e. the negative of eq. 2.54. This sign is important, because our result is independent from the renormalization scale up to terms of higher order in the perturbation series (arising e.g. from the dependence of $\tan\beta$ on the renormalization scale), whereas the result of ref. [6] is not. Second, the term

$$\frac{N_c g^2 m_Z^2}{16\pi^2 c_w^2} \cdot \frac{1}{12}(1 - 4e_u s_w^2 + 8e_u^2 s_w^4) \tag{2.56}$$

is absent in the expression for Δ_0 of ref. [6]. This term, however, is numerically small and does not affect the renormalization scale independence.

The final result for the correction Δ is rather complicated. In the limiting case where the quark and squark masses are very large, i.e. much larger than the electroweak scale and the masses of the Higgs bosons, we obtain the approximation [6]

$$\Delta = \frac{3g^2 m_t^4}{16\pi^2 m_W^2 \sin^2\beta} \log \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}. \tag{2.57}$$

We point out that in our conventions, we have the opposite sign for the coefficient μ of $H_1 H_2$ in the superpotential as compared to refs. [6, 7].

2.4 The MSSM Parameter Space

The numerical analysis of the correction Δ to the neutral Higgs boson mass sum rule is not a trivial task, because the parameter space of the MSSM is large and our knowledge of the numerical values of the parameters is very limited. We have the usual Standard Model parameters, the gauge couplings g', g, and g_s, and the Higgs-fermion Yukawa coupling matrices h_t, h_b, and h_τ. One additional parameter is the coefficient of $H_1 H_2$ in the superpotential, μ. Other parameters which could occur in the superpotential

[2] See the footnote on p. 12

have been excluded by R-parity invariance. The remaining parameters of the MSSM are due to soft supersymmetry breaking. We have the three gaugino Majorana mass parameters, M_1, M_2, M_3, squark and slepton squared mass matrices, $m_{\tilde{Q}}$, $m_{\tilde{U}}$, $m_{\tilde{D}}$, $m_{\tilde{L}}$, $m_{\tilde{E}}$, Higgs-squark-squark and Higgs-slepton-slepton trilinear interaction terms proportional to A_t, A_b, A_τ, and the Higgs sector mass parameters m_1, m_2, m_{12}. In the full three-generation model, this gives rise to more than 100 new parameters. The greatest part of these parameters are due to our ignorance of the SUSY breaking mechanism and the soft-supersymmetry breaking parameters.

To perform phenomenological analyses and to make predictions, we have to reduce the parameter freedom. Since low-energy supersymmetry is motivated by the belief in a more fundamental theory at higher energy scales, we can try to constrain the parameters by assumptions at the grand unification scale or at the Planck scale (or an intermediate energy scale). The low-energy SUSY parameters are then related to the high-energy input parameters via the renormalization group equations (RGE). One of these assumptions is the unification of the Majorana mass terms at the grand unification scale $M_{\mathrm{GUT}} \sim 10^{16}$ GeV, which reduces the three independent gaugino mass parameters to one. Other popular assumptions are universal scalar mass matrices, universal A-parameters, or unification of Higgs-fermion Yukawa couplings. All these assumptions are more or less motivated or at least favored by some grand unified theories or string theory models. For a comprehensive overview of the most common assumptions used to constrain the MSSM parameter space, the reader is referred to ref. [32]. For an introductory discussion and further references on prominent supersymmetry-breaking scenarios, we recommend ref. [33].

We take a different approach. The correction Δ does not depend on all parameters listed above. We evaluate the contributions due to the third generation of quarks and squarks. Only 13 parameters enter the expression for Δ, namely the SU(2) coupling constant, g (or alternatively the electric charge e), the masses of the vector bosons, m_Z, m_W, the mass of the CP-odd Higgs boson, m_A, the ratio of the vacuum expectation values, $\tan\beta$, the coefficient of $H_1 H_2$ in the superpotential, μ, the masses of the top and bottom quarks, m_t, m_b, the soft supersymmetry-breaking masses, $m_{\tilde{Q}}$, $m_{\tilde{U}}$, $m_{\tilde{D}}$, and the trilinear soft supersymmetry-breaking parameters, A_t, A_b. To simplify the analysis, we make two assumptions. First, we assume a common value for the soft supersymmetry-breaking masses, $m_{\tilde{Q}} = m_{\tilde{U}} = m_{\tilde{D}} = M_{\mathrm{SUSY}}$, and second, we assume a common value for the trilinear soft supersymmetry-breaking parameters, $A_t = A_b = A$. We checked numerically that these simplifying assumptions do not change the results qualitatively. Next, we fix the top, bottom, Z and W masses at the experimentally measured values. The pole masses of the top and bottom quarks are [34]

$$m_t = 174.3 \pm 5.1 \text{ GeV}, \quad m_b = 4.9 \pm 0.2 \text{ GeV}. \tag{2.58}$$

The pole masses of the vector bosons are [34]

$$m_Z = 91.1882 \text{ GeV}, \quad m_W = 80.419 \text{ GeV}.$$

We define the weak mixing angle on-shell,

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} = 0.22224.$$

We note in passing that the differences introduced by defining the parameters entering the one-loop corrections in different renormalization schemes and at different momentum transfers are of second order, so they will be ignored. We have not described the complete renormalization scheme for the Higgs sector as the renormalization of the mass sum rules does not rely on all renormalization conditions. In practice though, it is best to use the parameters which are defined in the scheme one is working in, and at momentum transfers which correspond to the external momenta of the loop graphs. This is why we prefer to take the fine-structure constant at the Z-pole,

$$\alpha = \frac{1}{127.938}.$$

The SU(2) coupling constant is then given by

$$g = \frac{\sqrt{4\pi\alpha}}{\sin\theta_W}.$$

We are now left with 5 parameters, namely m_A, $\tan\beta$, μ, M_{SUSY}, A. They are completely free parameters, yet to be determined from experimental data. Since we expect the MSSM to be embedded into some more fundamental framework, we have some theoretical arguments and biases which constrain this parameter freedom. Let us first consider the dimensionful parameters, m_A, μ, M_{SUSY}, A. We do not want the dimensionful parameters to be much larger than 1 TeV, because otherwise we lose the naturalness of the model [32, 35]. The main motivation for supersymmetry is the cancellation of quadratic divergences. This cancellation is exact only if supersymmetry is unbroken. Since the superpartners are not observed with masses of the normal particles, supersymmetry is broken. If supersymmetry is still to provide a solution to the naturalness problem, the supersymmetry-breaking mechanism must not re-introduce quadratic divergences into the theory. The so-called "soft" supersymmetry breaking mass terms and couplings violate supersymmetry, but maintain the desirable properties of the theory. With these soft supersymmetry breaking terms, the correction to the scalar masses goes like $\Delta m_S^2 \propto m_{\mathrm{soft}}^2 \log(\Lambda/m_{\mathrm{soft}})$, where we have denoted the largest scale associated with low-energy supersymmetry-breaking by m_{soft}. Large values for m_{soft} would give unnaturally large corrections to the scalar masses. Thus, we have taken the values of m_A, μ, M_{SUSY}, A from 100 GeV to 2000 GeV.

Let us now consider $\tan\beta$. We know from experiment [36] that $0.5 < \tan\beta < 2.4$ is excluded. We can immediately see from the Feynman rules [26] that for small values of $\tan\beta$ the top Yukawa coupling becomes large and for large values of $\tan\beta$ the bottom Yukawa coupling becomes large, thus leaving the region where perturbation theory is valid. Let us sketch an idea to obtain bounds on $\tan\beta$ [32]. The MSSM is believed to be the low-energy effective theory of a more fundamental high-energy theory. We assume that "new physics" enters at a scale Λ which could be the grand unification scale $\sim 10^{16}$ GeV or the Planck scale $\sim 10^{19}$ GeV. We now evolve our choice of parameters given at the electroweak scale to the scale Λ using renormalization group equations. By requiring that the Yukawa couplings h_t, h_b, h_τ remain finite at all energy scales up to Λ one obtains the allowed set of values of $\tan\beta$ as a function of Λ. From this analysis, we can conclude $0.5 \lesssim \tan\beta \lesssim 50$. The limits on $\tan\beta$ do not get substantially weaker for Λ as low as 100 TeV. Based on these theoretical considerations and the experimentally excluded values, we deem it appropriate to vary $\tan\beta$ from 3 to 50.

From experiment, we have lower bounds on the squark masses [34]:

$$m_{\tilde{t}_{1,2}} > 86.4 \text{ GeV}, \quad m_{\tilde{b}_{1,2}} > 91.0 \text{ GeV} \tag{2.59}$$

The squark masses (see eqs. B.23, B.25) are given in terms of the quark masses and the soft supersymmetry-breaking terms which we have constrained to values around 1 TeV by the considerations above. In the numerical evaluation of the correction Δ, we calculate the squark masses and dismiss the parameters if the experimental bounds in eq. 2.59 are violated. We also have lower bounds on the Higgs masses [34]:

$$m_{h,H} > 91.2 \text{ GeV}, \quad m_A > 91.6 \text{ GeV}, \quad M_{H^\pm} > 69.0 \text{ GeV}.$$

2.5 Numerical Analysis

Our result for the correction Δ is given in terms of the three functions F, G, H defined in eqs. 2.51-2.53. According to the general scheme proposed in section 2.3, we have to relate F, G, H to the PV functions. These relations are given in eqs. C.20-C.22:

$$F(q^2, m_1^2, m_2^2) = -B_0(q^2, m_1^2, m_2^2), \tag{2.60}$$

$$G(q^2, m_1^2, m_2^2) = B_1(q^2, m_1^2, m_2^2) + B_{21}(q^2, m_1^2, m_2^2), \tag{2.61}$$

$$H(q^2, m_1^2, m_2^2) = B_1(q^2, m_2^2, m_1^2). \tag{2.62}$$

For the numerical analysis of our result for Δ we decided to write a computer program in C [37]. The programming language C is especially well-fitting for our purposes, because the programs have a very high performance and there are many algorithms designed and published for C, so we can rely on solutions which have been successfully tested many times before. The use of computation engines such as Mathematica and Maple for the numerical analysis is not convenient. The performance is extremely slow and the user does not have full control over the program which can be relevant when dealing with numerically difficult evaluations. The PV functions are implemented in C by the LoopTools package [30, 31, 38]. The usage of this package is intuitive and straightforward. The correspondence between the PV functions and their implementation as C functions is given in eq. C.34. The expression for Δ as given in section D.4 is now coded in C. The result for each Feynman diagram depends on the renormalization scale Q, but this dependence should cancel in the final result for Δ. We have checked that this is indeed true. For the following analysis, we fixed the renormalization scale at the Z pole mass, i.e. $Q = m_Z$.

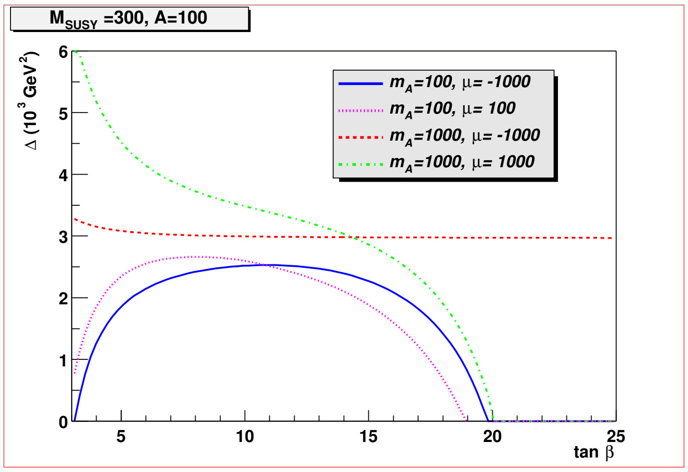


Figure 2.2: Δ as a function of $\tan\beta$

As discussed in section 2.4, the correction Δ to the neutral Higgs boson mass sum rule depends on five parameters, namely the mass of the CP-odd Higgs boson, m_A, the ratio of the vacuum expectation values, $\tan\beta$, the coefficient of $H_1 H_2$ in the superpotential, μ, the common soft supersymmetry-breaking mass, $M_{\rm SUSY}$, and the common trilinear soft supersymmetry-breaking parameter, A. To get an overview of the magnitude of $\widetilde{\Delta}$, we scanned the parameter space by giving the various parameters some representative values. Parameters which violate the experimentally set limits on the squark masses (see eq. 2.59) have been excluded from the analysis. We have varied in turn one of the five parameters $\tan\beta$, m_A, μ, $M_{\rm SUSY}$, A and plotted the functional dependence of the correction Δ on this parameter. We have taken representative values for the remaining four parameters, namely $\tan\beta = 3, 10, 50$, $m_A = 100, 1000, 2000$ GeV, $\mu = -1000, 100, 1000$ GeV, $M_{\rm SUSY} = 300, 1000, 2000$ GeV, $A = 100, 1000, 2000$ GeV. In each case, $3^4 = 81$ graphs were generated and analyzed. We think that in this way, we gained a good overview of the variation of Δ for the whole range of sensible parameter values. For the following analysis, one should note that the magnitude of the correction Δ is to be judged by comparing it to m_A. For small values of m_A, a correction $\sim 1 \times 10^3$ GeV^2 is appreciable, whereas for large values of m_A, the same correction might barely be noticeable. Let us begin with analyzing the dependence on $\tan\beta$. For $M_{\rm SUSY} \sim 300$ GeV, the correction is $\lesssim 3 \times 10^3$ GeV^2 for most values of the parameters. For $m_A = 1000$

GeV, $\mu = -1000$ GeV, however, the correction can be as large as 6×10^3 GeV^2, as is shown in the dashed curve of fig. 2.2. For values of $\tan\beta \geq 20$, the curves in fig. 2.2 are discontinued as the off-diagonal entry in the squared mass matrix of the sbottom, $X_b = m_b(A_b - \mu\tan\beta)$ gets large and the one of the eigenvalues thus falls below the experimental bounds. If we switch the sign of μ, the dashed curve changes to the dotted one. For low values of m_A, changing sign of μ does not have any noticeable effect. For $\mu \sim 100$ GeV, the correction approaches a constant straight line.

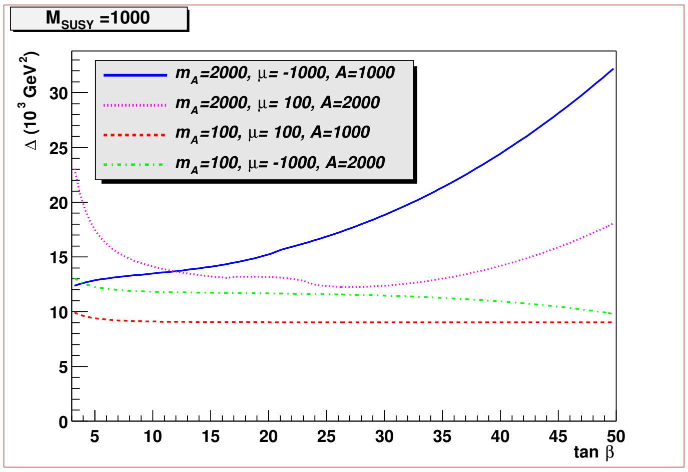


Figure 2.3: Δ as a function of $\tan\beta$

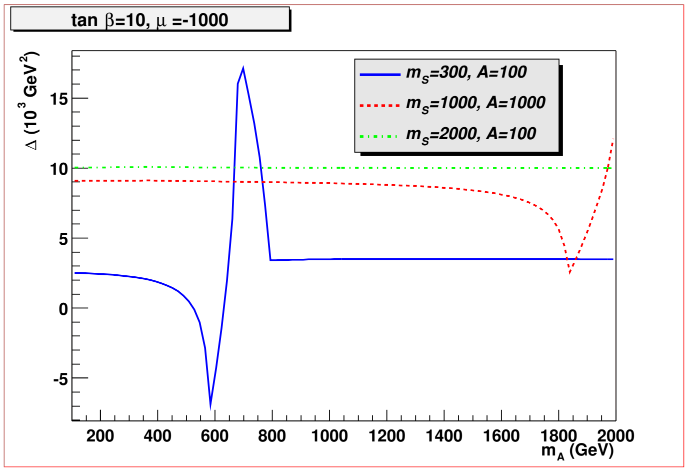


Figure 2.4: Δ as a function of m_A

In fig. 2.2, we present the curve for $m_A = 100$ GeV. Higher values of m_A display the same behavior, but the curve approaches the constant straight line from below. Now we turn our attention to the case where $M_{\mathrm{SUSY}} \sim 1000$ GeV. The correction is $\gtrsim 7 \times 10^3$ GeV^2 for all values of the parameters. For most values of the parameters, the graph is a flat curve which either rapidly approaches a constant value or decreases inappreciably for increasing $\tan\beta$. Examples are the dashed and dash-dotted curves in fig. 2.3. For large m_A and μ, $A \sim 1000$ GeV, however, the correction can grow very rapidly for increasing $\tan\beta$, as the solid curve in fig. 2.3 shows. For $m_A = 2000$ GeV, $\mu = -1000$ GeV, $M_{\mathrm{SUSY}} = 1000$ GeV, $A = 2000$ GeV, we found a curve which monotonously increases to values of 100×10^3 GeV^2. Let us now come to the dependence on m_A. Apart from threshold effects, the dependence of the correction Δ on m_A is very weak, and for all values of the parameters, the graph is a flat curve which is almost constant. This constant value is $\sim 5 \times 10^3$ GeV^2 for $M_{\mathrm{SUSY}} \sim 300$ GeV and $\sim 10 \times 10^3$ GeV^2 for $M_{\mathrm{SUSY}} \sim 1000$ GeV. For $m_A \sim 2M_{\mathrm{SUSY}}$, the graph for the correction Δ has cusps which correspond to threshold effects. These cusps are characteristic for diagrammatic calculations due to the external momentum dependence of the self-energies. In the effective potential approach or for the approximation formula, these cusps are absent. Let us investigate the origin of these cusps in some more detail. The self-energies are expressed in terms of

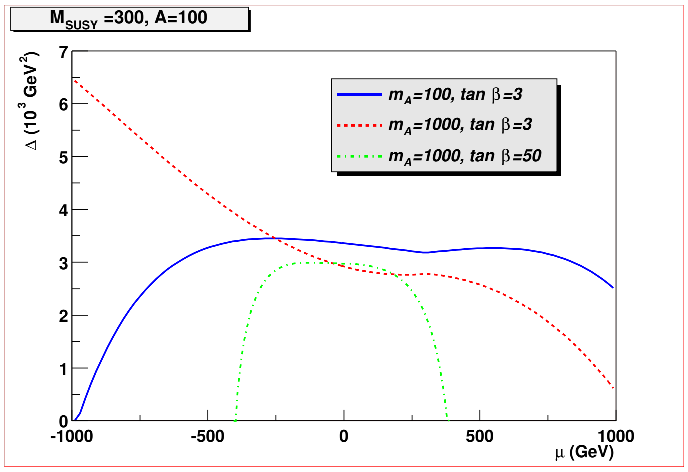


Figure 2.5: Δ as a function of μ

Passarino-Veltman functions, which are essentially integrals over a polynomial times the logarithm

$$\log\left[xm_1^2 + (1-x)m_2^2 - x(1-x)q^2\right],$$

where m_1, m_2 are the masses of the particles running in the loop and q^2 is the external momentum squared. Assume $m_1 = m_2 = m$ for simplicity. The external momentum squared is either 0, m_h, m_Z, m_H, or m_A. For $0 \le x \le 1$, we have $0 \le x(1-x) \le 1/2$. The mass m will be of order $M_{\rm SUSY}$, so for $q^2 = 0, m_h, m_Z$, the argument of the logarithm is always positive for $M_{\rm SUSY} \gtrsim 300$ GeV.

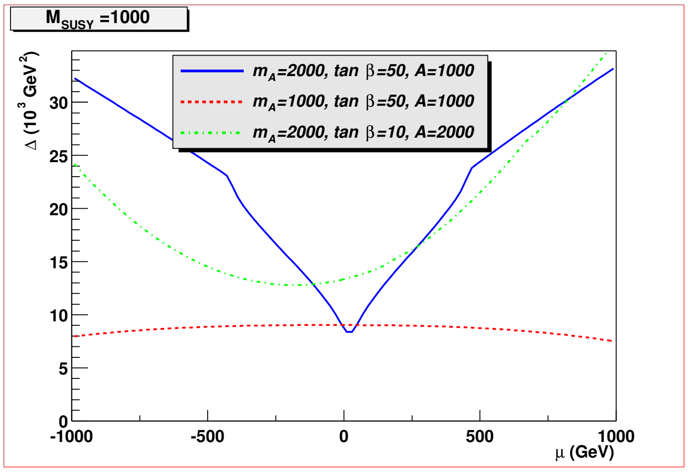


Figure 2.6: Δ as a function of μ

For m_A we do not have an upper bound, so when $m_A, m_H \sim 2M_{\rm SUSY}$, the argument of the logarithm changes sign. This exactly corresponds to the threshold for particle creation. As explained in section 2.3, we have to extend the definition of the logarithm by analytic continuation to the complex plane. When the argument changes sign, we change to the next sheet of the logarithm, and this discontinuity in the logarithm shows up in the integral as a cusp. In fig. 2.4,we present the graphs for some representative values of the parameters. In agreement with our arguments above, the cusps are shifted to the right with increasing values of $M_{\rm SUSY}$. Near the threshold, the correction can be as large as 15×10^3 GeV^2 even for small values of $M_{\rm SUSY}$. For large values of $M_{\rm SUSY}$ and $A \sim 2000$ GeV, Δ can take on values as large as $\sim 100 \times 10^3$ GeV^2. Let us now analyze the dependence of Δ on μ. In fig. 2.5 we present some plots for $M_{\rm SUSY} \sim 300$ GeV. The dash-dotted curve is discontinued as the experimental bounds are violated by the choice of parameters.

As we can learn from fig. 2.5, the correction can take on any value from 0 to 6.5×10^3 GeV^2. We should point out that for low values of $M_{\rm SUSY}$, the squark masses fall below their experimentally set lower bound when A takes on large values (X_t, X_b become large again). For large values of $M_{\rm SUSY}$, the situation is more manageable. In fig. 2.6, we present some graphs for $M_{\rm SUSY} \sim 1000$ GeV. The dashed curve is typical for high values of $M_{\rm SUSY}$. Only for $m_A \sim 2M_{\rm SUSY}$ we obtain curves which are not flat and take on values from 8×10^3 to 32×10^3 GeV^2 as can be verified considering the solid and dash-dotted curves in fig. 2.6. This is of course due to the threshold effects discussed in the previous paragraphs. The squark masses depend on μ, and near the threshold, varying μ has the effect of choosing squark masses which cross the threshold for particle creation. This is the reason for the strong dependence of Δ on μ for $m_A \sim 2M_{\rm SUSY}$. As a result, we can conclude that for high values of $M_{\rm SUSY}$ the μ-dependence of the correction is weak if we are not at a threshold. Let us now analyze the dependence of the correction Δ on $M_{\rm SUSY}$. In fig.

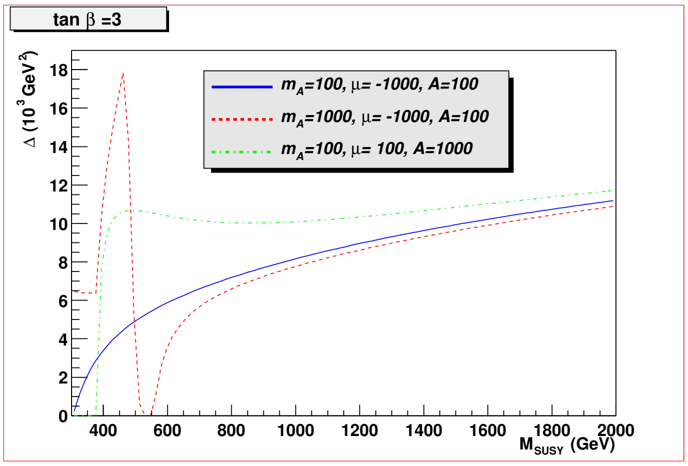


Figure 2.7: Δ as a function of $M_{\rm SUSY}$

2.7, we have plotted the graphs for the correction which are representative for all choices of parameter values. The graphs suggest that the dependence of Δ on M_{SUSY} is logarithmic. This is indeed true, as can easily be checked by considering the approximative formula, eq. 2.57. The correction is $\lesssim 12 \times 10^3$ GeV^2 apart from the cusps which are due to threshold effects. For increasing m_A, the cusps move to higher values of M_{SUSY}, as was already observed in the case when we varied m_A for fixed M_{SUSY}. For $A \gg |\mu|$, the graphs have a characteristic bump, as is shown in the dash-dotted curve in fig. 2.7. Let us now consider the dependence of the correction on A.

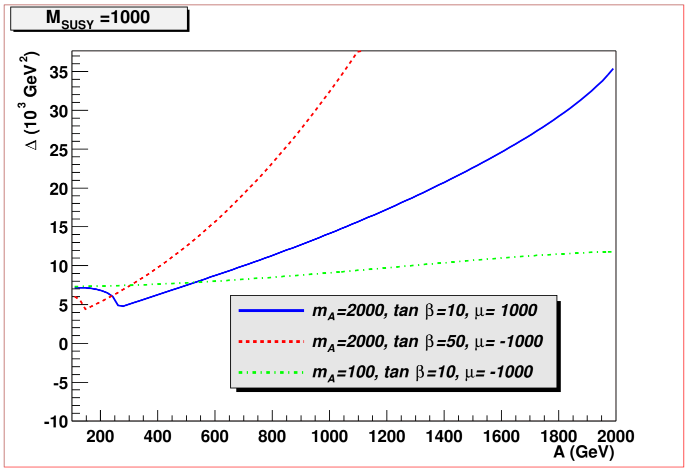


Figure 2.8: Δ as a function of A

The correction is for most values of the parameters a flat curve $\sim 10 \times 10^3$ GeV^2. The dash-dotted curve in fig. 2.8 is a typical example. Only for $m_A \sim 2M_{SUSY}$ the correction has a non-trivial dependence on A. For high values of $\tan\beta$ the correction grows very rapidly, as can be seen considering the dashed curve in fig. 2.8. This is due to threshold effects, as has been extensively discussed in the previous paragraphs. The squark masses depend on A, so what has been said about the μ-dependence and threshold effects is also valid in this case. From eq. 2.57 we see that the correction Δ to the neutral Higgs boson mass sum rule depends on the fourth power of m_t. Thus we expect the correction to be very sensitive to the errors on m_t.

In fig. 2.9, we present the graph for the correction Δ, where we have varied m_t and m_b within their experimentally measured limits, i.e. $m_t = 174.3 \pm 5.1$ GeV and $m_b = 4.9 \pm 0.2$ GeV. The correction changes in the order of 200 GeV^2 when we vary m_t. The dependence on m_b is much less pronounced and barely noticeable, as was to be expected, because the corrections arising from the b quark are known to be much smaller in a wide range of parameter space. We point out that this may change when the dependence on the quark masses is no longer quartic, but e.g. quadratic as in the case of the correction to the charged Higgs boson mass. We will investigate this question in section 3.4.

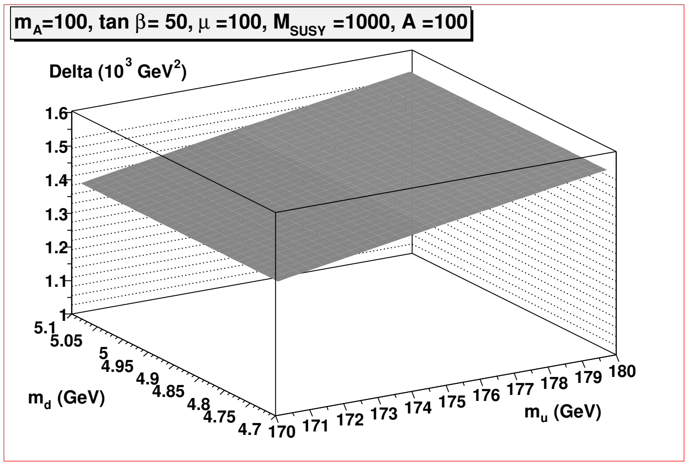


Figure 2.9: Δ as a function of m_t and m_b

Chapter 3

The Charged Higgs Boson Mass Sum Rule and its Renormalization

The mass sum rule involving the charged Higgs bosons in the Minimal Supersymmetric Standard Model is derived and renormalized at one-loop. The radiative corrections arising from top, stop, bottom, and sbottom loops are calculated, and the results are numerically analyzed. The renormalization and numerical analysis of the charged Higgs boson mass sum rule is similar to the neutral one. We accepted repetitions to make this chapter more self-contained and easy to read.

3.1 The Mass Sum Rule

As discussed in section 2.1, the Higgs sector of the MSSM gives rise to mass sum relations. The mass of the charged Higgs boson is given by

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{3.1}$$

as one can immediately verify by diagonalizing the squared mass matrix of the charged Higgs boson. The details of the calculation is given in section B.2.4. The relation in eq. 3.1 holds at tree level. The mass sum rule is subject to radiative corrections. We will describe the renormalization scheme in section 3.2. The renormalized version of eq. 3.1 is

$$m_{H^\pm}^2 = m_W^2 + m_A^2 + \widetilde{\Delta}. \tag{3.2}$$

The one-loop correction $\widetilde{\Delta}$ to the mass sum rule can be expressed entirely in terms of self energies:

$$\widetilde{\Delta} = \text{Re}\ \Pi_{AA}(m_A^2) - \text{Re}\ \Pi_{WW}(m_W^2) - \text{Re}\ \Pi_{H^\pm H^\pm}(m_{H^\pm}^2). \tag{3.3}$$

After discussing the dominant contributions, the diagrammatic calculation of the correction $\widetilde{\Delta}$ is performed in section 3.3 and our results are numerically analyzed in section 3.4.

3.2 The Renormalization Scheme

We now describe the renormalization scheme for the charged Higgs boson mass sum rule. We point out that the renormalization of the charged Higgs boson mass sum rule is less intricate than the renormalization of the neutral one, because we do not have the complications introduced by diagonalizing the CP-even squared mass matrix. Our renormalization scheme follows refs. [20].

We denote bare quantities with a subscript 'b', renormalized ones with a subscript 'r'. The calculations do not rely on a specific choice of gauge. The bare Higgs potential is given by eq. B.1:

$$\begin{aligned}V_{\mathrm{Higgs},b} &= m_{1b}^2 H_1^{i*}H_1^i + m_{2b}^2 H_2^{i*}H_2^i - m_{12b}^2(\epsilon_{ij}H_1^iH_1^j + \epsilon_{ij}H_1^{i*}H_1^{j*}) \\ &+ \frac{1}{8}(g_b^2 + g_b'^2)[H_1^{i*}H_1^i - H_2^{j*}H_2^j]^2 + \frac{1}{2}g_b^2|H_1^{i*}H_2^i|^2\end{aligned}$$

Note that we have absorbed the $|\mu|^2$ into the definition of m_1, m_2. Substituting

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\eta_1 + i\zeta_1 \\ \varphi_1^-\end{pmatrix} + \begin{pmatrix}v_1 \\ 0\end{pmatrix}, \qquad H_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2^+ \\ \eta_2 + i\zeta_2\end{pmatrix} + \begin{pmatrix}0 \\ v_2\end{pmatrix}$$

and rearranging the terms in powers of the fields gives:

$$\begin{aligned}V_{\mathrm{Higgs},b} = \mathrm{const} &+ \tau_{1b}\eta_1 + \tau_{2b}\eta_2 + \frac{1}{2}\eta_i(M_{\eta_i\eta_j})_b\eta_j + \frac{1}{2}\zeta_i(M_{\zeta_i\zeta_j})_b\zeta_j + \frac{1}{2}\varphi_i^{\pm}(M_{\varphi_i^{\pm}\varphi_j^{\pm}})_b\varphi_j^{\pm} \\ &+ \text{terms of higher order in } \eta_i, \zeta_i, \varphi^{\pm}\end{aligned}$$

The coefficients τ_{1b}, τ_{2b} of the terms linear in the fields are given in eqs. B.6, B.7. Note that the usual tree level minimization condition gives $\tau_{1b} = \tau_{2b} = 0$, so at tree level, terms linear in the fields are absent. At one-loop level, tadpole diagrams iT_1, iT_2 must be taken into account, as they shift the minimum. We will impose

$$T_1 = \tau_{1b}, \quad T_2 = \tau_{2b}$$

as a renormalization condition, which ensures that the renormalized vacua still minimize the one-loop potential. These two conditions constitute constraints among the parameters of the Higgs potential.

We define $m^2 \equiv m_1^2 + m_2^2$ and $v^2 \equiv v_1^2 + v_2^2$ to simplify the notation. The mass matrix $(M_{\varphi^{\pm}\varphi^{\pm}})_b$ is given by eq. B.16:

$$(M_{\varphi^{\pm}\varphi^{\pm}})_b = \begin{pmatrix} m_{12b}^2\frac{v_{2b}}{v_{1b}} + \frac{g_b^2}{2}v_{2b}^2 + \frac{g_b^2}{2}\xi v_{1b}^2 & m_{12b}^2 + \frac{g_b^2}{2}v_{1b}v_{2b} - \frac{g_b^2}{2}\xi v_{1b}v_{2b} \\ m_{12b}^2 + \frac{g_b^2}{2}v_{1b}v_{2b} - \frac{g_b^2}{2}\xi v_{1b}v_{2b} & m_{12b}^2\frac{v_{1b}}{v_{2b}} + \frac{g_b^2}{2}v_{1b}^2 + \frac{g_b^2}{2}\xi v_{2b}^2 \end{pmatrix}. \tag{3.4}$$

The eigenvalues are

$$m_{H^{\pm}b}^2 = m_{Ab}^2 + m_{Wb}^2, \qquad m_{G^{\pm}b}^2 = \xi m_{Wb}^2. \tag{3.5}$$

The first relation in eq. 3.5 is the tree level mass sum rule for the charged Higgs boson. The details of the derivation are given in section B.2.4.

We will now relate the bare masses in eq. 3.5 to the physical ones. The inverse propagators of A, $H^{\pm}$, and W are given by

$$\Gamma_{AA}(p^2) = p^2 - m_{Ab}^2 + \Pi_{AA}(p^2), \tag{3.6}$$

$$\Gamma_{H^{\pm}H^{\pm}}(p^2) = p^2 - m_{H^{\pm}b}^2 + \Pi_{H^{\pm}H^{\pm}}(p^2), \tag{3.7}$$

$$\Gamma_{WW}(p^2) = p^2 - m_{Wb}^2 - \Pi_{WW}(p^2). \tag{3.8}$$

For the W boson, only the transverse part of the self-energy plays a role. For the notation and sign convention of the self-energies, the reader is referred to section A.3. The physical mass is by definition the pole of the real part of the propagator. The physical masses will be denoted by m_A, $m_{H^{\pm}}$, m_W. Substituting $p^2 = m_A^2$ into eq. 3.6 and setting $\mathrm{Re}\,\Gamma_{AA}(m_A^2) = 0$ immediately yields

$$m_A^2 = m_{Ab}^2 - \mathrm{Re}\,\Pi_{AA}(m_A^2) \tag{3.9}$$

Analogously, from eqs. 3.7, 3.8 we conclude

$$m_{H^\pm}^2 = m_{H^\pm b}^2 - \text{Re } \Pi_{H^\pm H^\pm}(m_{H^\pm}^2), \tag{3.10}$$

$$m_W^2 = m_{Wb}^2 + \text{Re } \Pi_{WW}(m_W^2). \tag{3.11}$$

Substituting eqs. 3.9-3.11 into the tree level mass sum rule, eq. 3.5, gives the final result:

$$m_{H^\pm}^2 = m_A^2 + m_W^2 + \text{Re } \Pi_{AA}(m_A^2) - \text{Re } \Pi_{WW}(m_W^2) - \text{Re } \Pi_{H^\pm H^\pm}(m_{H^\pm}^2)$$

Comparing our result with eq. 3.2, we find

$$\widetilde{\Delta} = \text{Re } \Pi_{AA}(m_A^2) - \text{Re } \Pi_{WW}(m_W^2) - \text{Re } \Pi_{H^\pm H^\pm}(m_{H^\pm}^2). \tag{3.12}$$

We make some concluding remarks. First, our final result for the correction $\widetilde{\Delta}$ differs from the result in ref. [8] by the sign of the W self-energy, $\Pi_{WW}(m_W^2)$. This difference is due to conventions. We chose to denote the amputated vector boson self-energy diagrams by (see eq. A.3)

$$+ig_{\mu\nu}\Pi(q^2) + q_\mu q_\nu \ldots, \tag{3.13}$$

whereas ref. [8] decided to denote the same diagrams by

$$-ig_{\mu\nu}\Pi(q^2) + q_\mu q_\nu \ldots. \tag{3.14}$$

Second, ref. [6] gives an expression for the correction to the charged mass sum rule which does not agree with eq. 3.12:

$$\widetilde{\Delta} = \text{Re } \Pi_{AA}(m_A^2) - \text{Re } \Pi_{WW}(m_W^2) - \text{Re } \Pi_{H^\pm H^\pm}(m_{H^\pm}^2) + \text{Re } \Pi_{G^0 G^0}(0) - \text{Re } \Pi_{G^\pm G^\pm}(0)$$

We have proved by an explicit calculation that

$$\text{Re } \Pi_{G^0 G^0}(0) = \text{Re } \Pi_{G^\pm G^\pm}(0),$$

so there is no contradiction. The details of the calculation are given in section E.1.

3.3 Diagrammatic Calculation of the Correction

Now we turn our attention to the explicit diagrammatic calculation of the correction

$$\widetilde{\Delta} = \text{Re } \Pi_{AA}(m_A^2) - \text{Re } \Pi_{WW}(m_W^2) - \text{Re } \Pi_{H^\pm H^\pm}(m_{H^\pm}^2). \tag{3.15}$$

We will give an overview of the work which has been done so far by other authors before we proceed to give the details of the present calculation. Ref. [20] performs a diagrammatic calculation of the correction to the charged Higgs boson mass including contributions from gauge bosons, gauginos, Higgs bosons, Higgsinos in the loops. They conclude that even a large splitting between the gauge-Higgs and the gaugino-Higgsino sector and large supersymmetry breaking mass parameters lead to very small corrections. This statement is also true for the correction to the neutral Higgs boson mass sum rule. Ref. [39] gives a general proof that the mass sum rules predicted at tree level are screened against large one-loop corrections from any heavy sector, so long as there are no large trilinear Yukawa couplings. It is noted there that the Higgs-sfermion-sfermion couplings could be large and lead to large corrections to the mass sum rules. Ref. [40] performs a diagrammatic calculation and computes the leading one-loop correction to the charged Higgs boson mass arising from squark loops. All fermion masses have been neglected. The result is that the sum-rule corrections can be quite large when the Higgs-sfermion-sfermion couplings become substantially larger than the electroweak scale. Ref. [41] evaluates diagrammatically the one-loop contribution to the charged Higgs boson mass due to the third generations of quarks, fermions and their supersymmetric partners. Using renormalization group techniques, the leading logs are summed up to all orders, thereby improving the one-loop corrected charged Higgs boson mass. The result is that the charged Higgs boson mass shift is less than 10 GeV over nearly the entire range of the parameter space. Ref. [42] uses an effective potential approach for computing the charged Higgs boson mass. Loop contributions from tops, stops, bottoms, and sbottoms are included. The article draws the conclusion that the corrections to the charged Higgs boson mass are numerically small and at most a few GeV for generic values of the parameters, although they can be large in some special regions of parameter space. They expect their result to agree with the complete diagrammatic computation within a few GeV. Ref. [8] performs a full diagrammatic calculation including one-loop contributions from tops, stops, bottoms, and sbottoms. The corrections are found to be typically a few GeV or less, and larger corrections are stated to arise only if $\tan\beta \sim 40$ or $\tan\beta \lesssim 1$. The results are compared to the effective potential approach of ref. [42] and found to be in good agreement. At present, explicit formulae for the complete corrections to the Higgs boson masses in the MSSM are listed in the literature. One of the most complete works is ref. [43].

The diagrammatic calculation of the correction $\widetilde{\Delta}$ to the charged Higgs boson mass sum rule is completely analogous to the calculation of Δ as described in section 2.3, so we will only present the differences. The self-energy diagrams needed to compute $\widetilde{\Delta}$ are shown in fig. 3.1. There are eight distinct classes of diagrams to be calculated. We compute the contributions from the third generation of quarks and squarks. These diagrams constitute an R_ξ gauge independent subset, so we do not have to bother about the choice of gauge. We have taken the Feynman rules from refs. [25, 26]. The loop diagrams are regularized via dimensional reduction. The result for the calculation of the Feynman diagrams is listed in appendix F. Since $\widetilde{\Delta}$ is the correction to the mass sum rule, it is a finite quantity, and in the expression for $\widetilde{\Delta}$, the sum of the infinite parts of the self-energies must vanish. To compare the result of our calculation to ref. [8], we expressed the finite part of $\widetilde{\Delta}$ in terms of four functions defined as

$$F(q^2, m_1^2, m_2^2) = \int dx \ \log \frac{(1-x)m_1^2 + x m_2^2 - x(1-x)q^2}{Q^2}, \tag{3.16}$$

$$G(q^2, m_1^2, m_2^2) = \int dx \ x(1-x)q^2 \ \log \frac{(1-x)m_1^2 + x m_2^2 - x(1-x)q^2}{Q^2}, \tag{3.17}$$

$$H(q^2, m_1^2, m_2^2) = \int dx \ [(1-x)m_1^2 + x m_2^2] \log \frac{(1-x)m_1^2 + x m_2^2 - x(1-x)q^2}{Q^2}, \tag{3.18}$$

$$J(q^2, m_1^2, m_2^2) = 2H(q^2, m_1^2, m_2^2) - 2G(q^2, m_1^2, m_2^2) + \frac{1}{3}q^2. \tag{3.19}$$

The final result of our calculation is listed in section E.3 and is in perfect agreement with ref. [8]. For the numerical analysis of the correction $\widetilde{\Delta}$, we have to relate the functions in eqs. 3.16-3.19 to the PV functions. The according formulae are given in eqs. C.27-C.29.

It should be noted that the coefficient of $H_1 H_2$ in the superpotential, μ, has in our conventions the opposite sign as compared to ref. [8]. This difference does not show up in the expression for $\widetilde{\Delta}$ in terms of the self-energies, but is relevant when calculating the Feynman diagrams which contribute to the self-energies and comparing them to the explicit expressions listed in ref. [8].

Our final expression for $\widetilde{\Delta}$ is quite long and complicated. To understand the qualitative behavior of the correction, it is desirable to have an analytic formula at hand, which accounts for all dominant effects, but is still manageable. Ref. [44] presents approximation formulae for the charged Higgs boson mass.

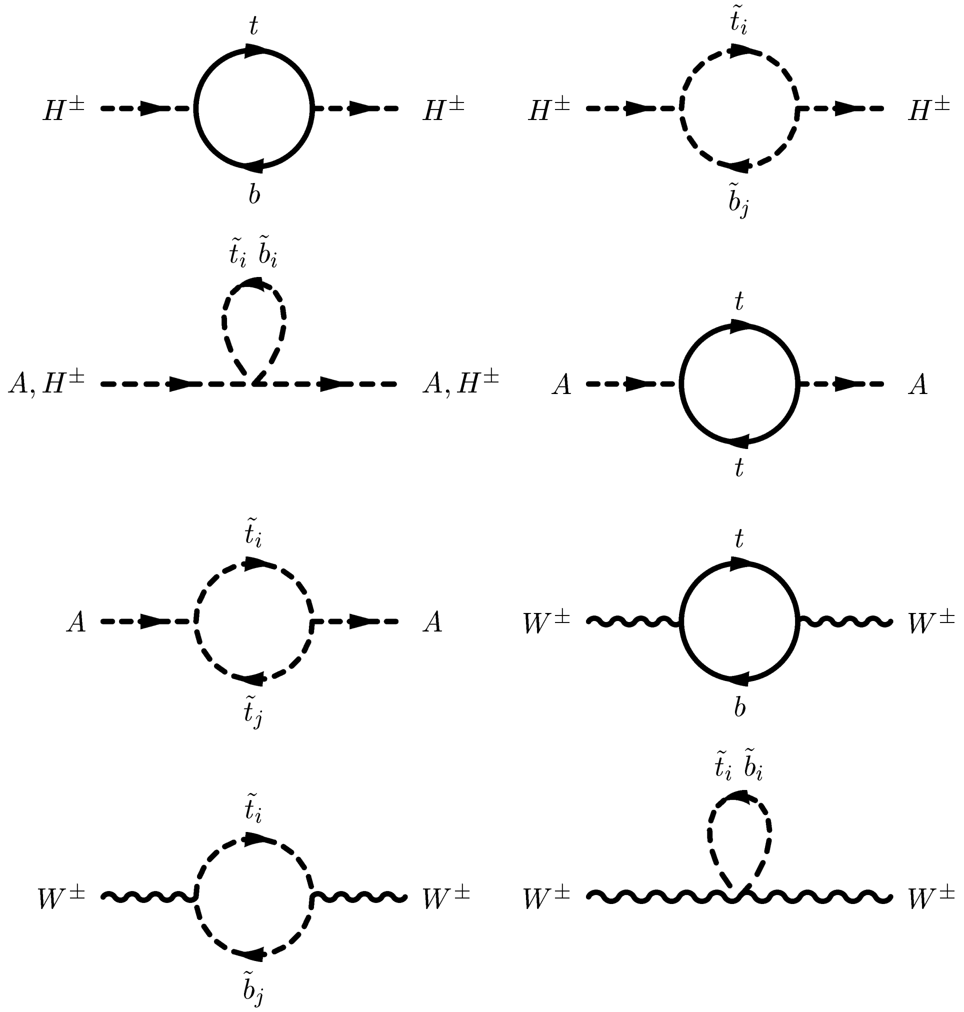


Figure 3.1: One-loop self-energy graphs contributing to $\widetilde{\Delta}$.

Under the assumptions that $m_A \lesssim \mathcal{O}(m_Z)$ and $M_{SUSY} \gg m_Z$, the mass of the charged Higgs boson can be approximated by

$$m_{H^\pm}^2 = m_A^2 + m_W^2 + \frac{N_c g^2}{32\pi^2 m_W^2}\left[\frac{2m_t^2 m_b^2}{\sin^2\beta\cos^2\beta} - m_W^2\left(\frac{m_t^2}{\sin^2\beta} + \frac{m_b^2}{\cos^2\beta} + \frac{2}{3}m_W^2\right)\right]\log\frac{M_{\text{SUSY}}^2}{m_t^2} + \frac{g^2 m_W^2}{48\pi^2}\left[N_c(N_g-1) + N_g - 9 + 15\tan^2\theta_w\right]\log\frac{M_{\text{SUSY}}^2}{m_W^2}, \tag{3.20}$$

where N_c is the color factor and N_g is the number of generations. The approximation formula in eq. 3.20 can be enhanced to take squark mixing and other effects into account. For our purposes, the most basic form as given in eq. 3.20 will suffice.

3.4 Numerical Analysis

The numerical analysis of the correction $\widetilde{\Delta}$ to the charged Higgs boson mass sum rule is completely analogous to the neutral case. The parameter space of the MSSM has been extensively discussed in section 2.4. We again scan the parameter space by giving the various parameters some representative values and exclude parameters which violate the experimentally set limits on the squark masses (see eq. 2.59). We have varied in turn one of the five parameters $\tan\beta$, m_A, μ, M_{SUSY}, A and plotted the functional dependence of the charged Higgs boson mass, $m_{H^\pm}$, on this parameter. We have taken representative values for the remaining four parameters, namely $\tan\beta = 3, 10, 50$, $m_A = 100, 1000, 2000$ GeV, $\mu = -1000, 100, 1000$ GeV, $M_{\text{SUSY}} = 300, 1000, 2000$ GeV, $A = 100, 1000, 2000$ GeV. In each case, $3^4 = 81$ graphs were generated and analyzed.

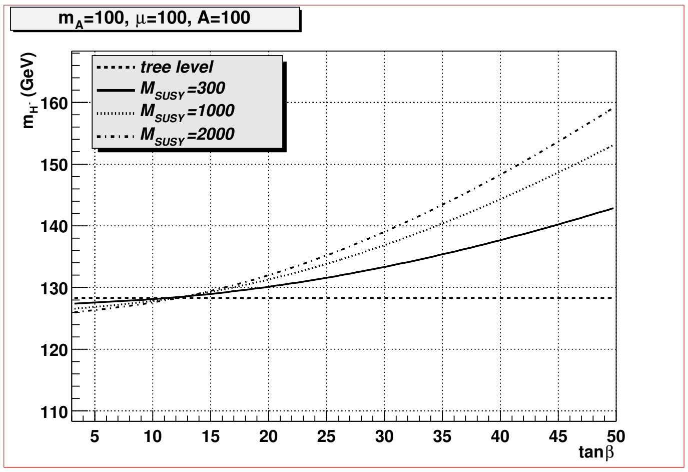


Figure 3.2: $m_{H^\pm}$ as a function of $\tan\beta$

Let us begin with analyzing the dependence on $\tan\beta$. We have varied $\tan\beta$ in the range from 3 to 50. For a light CP-odd Higgs boson A, the mass of the charged Higgs boson is small and the corrections can be significant. Fig. 3.2 shows the mass of the charged Higgs boson, $m_{H^\pm}$, as a function of $\tan\beta$ for three values of M_{SUSY}. The tree-level value for the mass of $H^\pm$ is included in the figure for comparison. The correction is larger for increasing $\tan\beta$ and M_{SUSY}. This statement is valid in full generality for all set of parameters. The form of the curves in fig. 3.2 suggests that $m_{H^\pm}$ depends quadratically on $\tan\beta$. This is indeed true, as can be easily checked by considering the approximate formula for $m_{H^\pm}$, eq. 3.20. For large $\tan\beta$, we have $\sin\beta \simeq 1$ and thus $1/\cos^2\beta \simeq \tan^2\beta$, and the terms with $\cos^2\beta$ in the denominator will dominate. For a heavy CP-odd Higgs boson, $m_A \sim 1000$ GeV, the corrections can be of the order 10 GeV, but they are in all cases $\lesssim 1.5\%$ of the tree level mass. The quadratic dependence on $\tan\beta$ is also present in this case, but the curves are flat and this dependence is less pronounced.

Let us now analyze the dependence of $m_{H^\pm}$ on m_A. In fig. 3.3 we present the graph for some specific choice of parameters. The functional dependence is to a very good approximation linear. The tree level mass of $H^\pm$ is given by $m^2_{H^\pm} = m^2_A + m^2_W$, and for $m_A \gg m_W$, the relation $m_{H^\pm} = m_A$ holds to a very great accuracy. Our analysis shows that for $m_A \gtrsim 400$ GeV, the graph for the one-loop corrected mass merges smoothly into the graph for the tree level value. This result is in agreement with the approximate formula for $m_{H^\pm}$ which reduces to the tree level value for $m_A \gg m_W$. For $m_A \lesssim 400$ GeV, the difference from the tree level value is appreciable only for high values of $\tan\beta$, and the curve is almost a straight line.

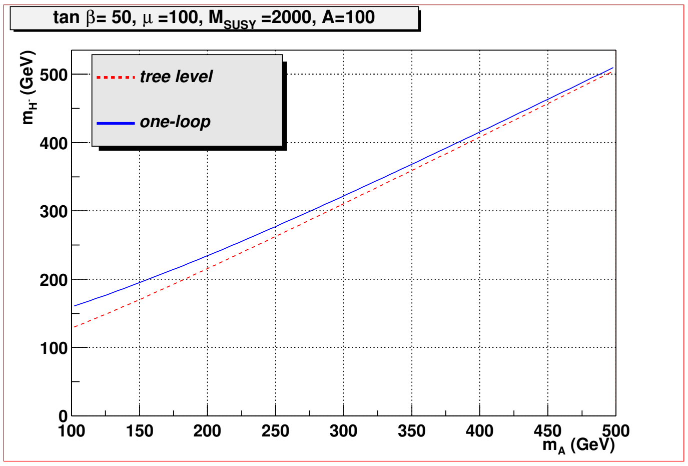


Figure 3.3: $m_{H^\pm}$ as a function of m_A

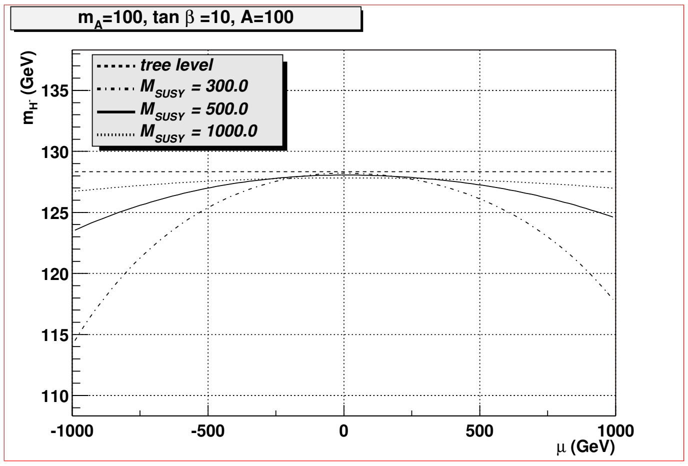


Figure 3.4: $m_{H^\pm}$ as a function of μ

We now vary μ in the range from -1000 to +1000 GeV. For all values of the parameters, the curve is almost symmetric, and becomes flat for large values of M_{SUSY}. For $m_A \sim 1000$ GeV, the variation of μ has only a negligible effect on $m_{H^\pm}$. The difference to its tree level value is $\lesssim 1.5\%$. For small values of m_A, the correction to the mass can be significant. Fig. 3.4 shows the one-loop corrected mass of the charged Higgs boson for different choices of M_{SUSY}. For increasing values of M_{SUSY}, the curve approaches a constant straight line. For large values of $\tan\beta$, the correction becomes positive, and the correction can be as large as $\sim$ 20 GeV. In this case, the one-loop corrected Higgs mass is an almost parallel constant straight line.

Now let us analyze the dependence of $m_{H^\pm}$ on M_{SUSY}. We have varied M_{SUSY} in the range from 400 to 2000 GeV. For $m_A \gg m_W$, the one-loop corrected mass of the charged Higgs boson $H^\pm$ is in very good approximation equal to its tree level value. For $m_A \sim m_W$, the correction is negative for low values of $\tan\beta$ and positive for high values of $\tan\beta$. Fig. 3.5 shows the dependence of $m_{H^\pm}$ on M_{SUSY} for different choices of $\tan\beta$. Increasing $\tan\beta$ shifts the curves to higher values of $m_{H^\pm}$. The correction seems to approach a constant value for large M_{SUSY}. Inspecting the approximative formula for $m_{H^\pm}$ (see eq. 3.20), one notes that the dependence on M_{SUSY} is in fact logarithmic.

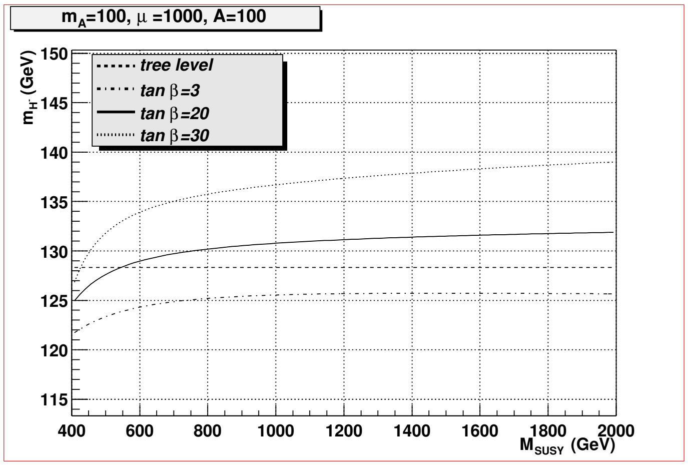


Figure 3.5: $m_{H^\pm}$ as a function of M_{SUSY}

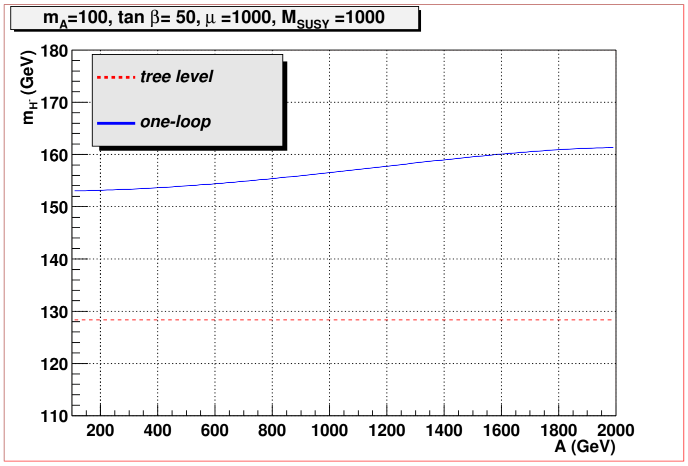


Figure 3.6: $m_{H^\pm}$ as a function of A

Let us now come to the dependence of $m_{H^\pm}$ on the trilinear soft supersymmetry-breaking parameter A. We have varied A in the range from 100 to 2000 GeV. The dependence of $m_{H^\pm}$ on A is very weak. Fig. 3.6 shows the mass of the charged Higgs boson, $m_{H^\pm}$, as a function of A. We have checked that the graph is typical also for other values of the parameters. For small values of m_A, varying A gives a flat curve which is constant within ± 5 GeV. For large m_A, the correction is $\lesssim 1.5\%$, and we have abstained from presenting the graphs for this case. For the A-dependency to become noticeable, we have checked that values of order $A \sim 5000$ GeV are needed. The charged Higgs boson mass $m_{H^\pm}$ grows very rapidly for these very large A values, which are excluded from our considerations on naturalness grounds.

Considering the approximate expression for $m_{H^\pm}$, eq. 3.20, we see that the Higgs boson mass depends quadratically on the quark masses, m_t and m_b. In contrast, the correction Δ to the neutral Higgs boson mass sum rule depends on the fourth power of m_t, as can be seen in eq. 2.57. We thus expect the dependence of $m_{H^\pm}$ on m_t to be less pronounced. On the other hand, one should notice that even though $m_t \gg m_b$, the relative error of m_b is greater. In fig. 3.7, we present the mass of the charged Higgs boson, $m_{H^\pm}$, as a function of the squark masses, m_t and m_b, where we have varied the quark masses within their experimental uncertainty, $m_t = 174.3 \pm 5.1$ GeV and $m_b = 4.9 \pm 0.2$ GeV. We can see from fig. 3.7 that $m_{H^\pm}$ varies in the range from 158 to 166 GeV. The errors on the experimentally measured values of m_t and m_b can thus lead to an uncertainty in the charged Higgs mass of the order 10 GeV. By plotting the functional dependence of $m_{H^\pm}$ on m_t, m_b for other values of the parameters, we have checked that these conclusions are true in general for a wide range of the parameter space.

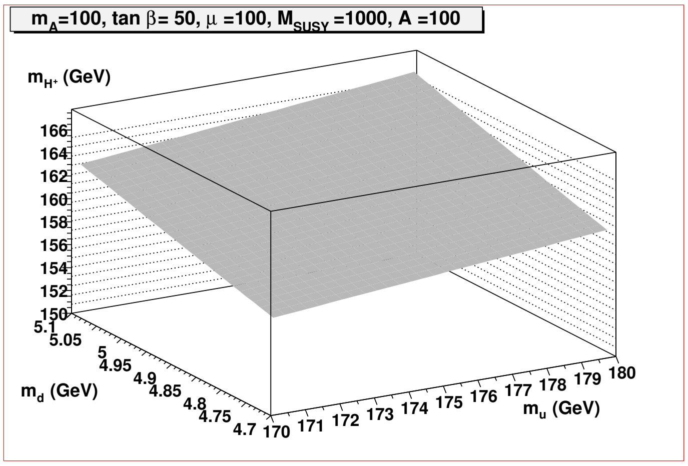


Figure 3.7: $m_{H^\pm}$ as a function of m_t and m_b

Appendix A

Notations and Conventions

A.1 Units, Constants, Metric

We will work in natural units, where

$$\hbar = c = 1.$$

In this system,

$$[\text{length}] = [\text{time}] = [\text{energy}]^{-1} = [\text{mass}]^{-1}.$$

The conversion constants [34]

$$\hbar c = 197.32 \text{ MeVfm}, \quad (\hbar c)^2 = 0.389 \text{ GeV}^2\text{mbarn},$$

allow us to express our final results in physical units again.

We use Heaviside-Lorentz units, in which the factors of 4π do not appear in Maxwell's equations, but do appear in the Coulomb potential and in the fine-structure constant:

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

In these units, the $e^-e^-\gamma$ vertex is just $ie\gamma^\mu$, where e denotes the charge of the positron. We use the Minkowski metric with signature (+ - - -),

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

with Greek indices running over 0,1,2,3. We define the totally antisymmetric tensor $\epsilon^{\mu\nu\rho\sigma}$ as

$$\epsilon^{0123} = +1.$$

Repeated indices are summed in all cases. The Pauli matrices are given by

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A.2 Symbols and Abbreviations

We list the most often used symbols and abbreviations for convenience. We denote by

$$\tan\beta = \frac{v_2}{v_1}$$

the ratio of the vacuum expectation values, see section B.2. We define the diagonalizing angle α by the relation (see section B.2.3)

$$\sin 2\alpha = -\sin 2\beta \frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}.$$

The light and heavy CP-even Higgs bosons are denoted by h and H, respectively, and the CP-odd Higgs boson is denoted by A. The gauge bosons are denoted by Z and W. The charged Higgs bosons are denoted by $H^\pm$.

We denote the coupling of the $U(1)_Y$ and $\mathrm{SU}(2)_L$ gauge groups by g' and g, respectively. The electric charge in Heaviside-Lorentz units is denoted by e, and in these units the relation $g\sin\theta_w = e$ is valid. The charge of the up-type and down-type quarks in units of e is given by

$$e_u = +\frac{2}{3}, \quad e_d = -\frac{1}{3}.$$

The weak mixing-angle (or Weinberg angle) is denoted by θ_w. The trigonometric functions will frequently occur in our calculations, so we introduce the abbreviations

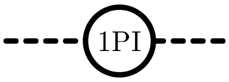
$$c_w \equiv \cos\theta_w \text{ and } s_w \equiv \sin\theta_w$$

for the cosine and sine of the weak mixing-angle. Analogously, we define

$$c_\alpha \equiv \cos\alpha, \quad s_\alpha \equiv \sin\alpha, \quad c_\beta \equiv \cos\beta, \quad s_\beta \equiv \sin\beta.$$

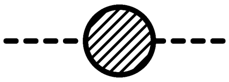
A.3 Self-energies of Scalars and Vector Bosons

We define $+i\Pi(q^2)$ as the sum of all one-particle-irreducible insertions into the propagator for a scalar particle:



$$= \quad +i\Pi_{SS}(q^2) \qquad \text{(A.1)}$$

The full two-point function is given by the geometric series,



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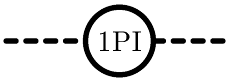

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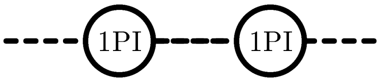


$+ \quad \ldots$

$$= \quad \frac{i}{q^2 - m_b^2 + \Pi_{SS}(q^2)},$$

where m_b is the bare mass. Defining the physical mass as the pole of the real part of the propagator gives the relation [12]

$$m_{\text{phys}}^2 = m_b^2 - \mathrm{Re}\,\Pi_{SS}(m_{\text{phys}}^2) \qquad \text{(A.2)}$$

between the physical and bare mass for a scalar particle. Analogously, for a vector boson we define

$$+ig_{\mu\nu}\Pi_{VV}(q^2) + q_\mu q_\nu\text{-terms} \tag{A.3}$$

as the sum of all one-particle-irreducible insertions into the propagator. Considering the full two-point function and defining the physical mass as the pole of the real part of the propagator gives the relation [12]

$$m^2_{\text{phys}} = m_b^2 + \text{Re}\;\Pi_{VV}(m^2_{\text{phys}}) \tag{A.4}$$

for vector bosons.

A.4 Loop Integrals and Dimensional Regularization

We list some useful formulae for reference purposes and to fix the notation. For an introduction to dimensional regularization, the reader is referred to the standard literature [12, 13].

To combine the denominators of the propagators, we use Feynman's trick:

$$\frac{1}{AB} = \int_0^1 dx\; \frac{1}{[xA + (1-x)B]^2}$$

The integral over the so-called Feynman parameter x will occur very frequently. If no bounds are explicitly given, it will be understood that the integration over x will run from 0 to 1:

$$\boxed{\int_0^1 dx \quad \to \quad \int dx}$$

Next, we complete the square and shift the integration variables. For a one-loop integral, there is a single integration momentum p_μ, which is shifted to a momentum variable ℓ_μ. After this shift, the denominator takes the form $(\ell^2 - \Delta_{ij})^n$, where we have defined

$$\boxed{\Delta_{ij} \equiv \frac{-x(1-x)q^2 - x(m_i^2 - m_j^2) + m_i^2}{Q^2},}$$

where m_i, m_j are the masses of the particles running in the loop, q^2 is the external momentum squared, and Q is the renormalization scale. Note that in the appendices D, E, F, we will omit Q for the sake of brevity, but restore it in the final results to test the independence of the renormalization scale.

In the numerator, terms with an odd number of powers of ℓ vanish. Symmetry allows us to replace

$$\ell^\mu \ell^\nu \to \frac{1}{d}\ell^2 g^{\mu\nu},$$

where d is the dimension of space-time. The integration over ℓ is can be performed, the result is given in terms of the Γ function:

$$\int \frac{d^d\ell}{(2\pi)^d}\,\frac{1}{(\ell^2 - \Delta_{ij})^n} = \frac{(-1)^n i}{(4\pi)^{d/2}}\,\frac{\Gamma(n-d/2)}{\Gamma(n)}\left(\frac{1}{\Delta_{ij}}\right)^{n-d/2}$$

$$\int \frac{d^d\ell}{(2\pi)^d}\,\frac{\ell^2}{(\ell^2 - \Delta_{ij})^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}}\,\frac{d}{2}\,\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\left(\frac{1}{\Delta_{ij}}\right)^{n-d/2-1}$$

In principle, the Γ function has to be expanded around $d = 4$ to extract the divergent part of integral. In practice, the following combination of terms often appears in calculations and can be replaced immediately:

$$\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(\frac{1}{\Delta}\right)^{2-d/2} = \frac{1}{(4\pi)^2}\left(\frac{2}{\epsilon} - \log\Delta_{ij} - \gamma + \log 4\pi + \mathcal{O}(\epsilon)\right),$$

where $\epsilon \equiv 4 - d$ and $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. The pole and the artifacts of the dimensional regularization are conveniently abbreviated as

$$\boxed{\frac{2}{\bar{\epsilon}} \equiv \frac{2}{\epsilon} - \gamma + \log 4\pi.}$$

The gamma matrix algebra is performed in 4 dimensions [15, 16], explicitly preserving supersymmetry. The formulae for the γ^μ contraction identities in 4 dimensions can be found in the standard literature. One should pay attention to how ϵ^{0123} is normalized before using these formulae.

Appendix B

Definition of the MSSM and the Higgs Sector

As there are slight differences in the way different authors define the MSSM, the main goal is to fix the notation and conventions, and to give a precise definition of the MSSM. Many important aspects have been dismissed from our considerations as we concentrate on the Higgs sector. For the Higgs sector, we have given some details for reference purposes and in order to make the the main chapters more readable and self-contained. Our notation and conventions follow ref. [32]. For a detailed discussion of the MSSM, the reader is referred to ref. [32].

B.1 The Definition of the MSSM

B.1.1 The Particle Spectrum

To extend the Standard Model to a supersymmetric theory, we first have to specify the particle content. Supersymmetry implies a bosonic partner to each chiral fermion, i.e. to q_L and q_R we associate $\tilde{q}_L$ and $\tilde{q}_R$, which are spin-0 particles.

Names		spin 0	spin 1/2	$SU(3)$, $SU(2)$, $U(1)$
(s)quarks	Q	$(\widetilde{u}_L \ \ \widetilde{d}_L)$	$(u_L \ \ d_L)$	$(\,\mathbf{3},\,\mathbf{2}\,,\,\frac{1}{3})$
(×3 families)	U	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\,\mathbf{1},\,-\frac{4}{3})$
	D	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\,\mathbf{1},\,\frac{2}{3})$
(s)leptons	L	$(\widetilde{\nu}\ \widetilde{e}_L)$	$(\nu \ \ e_L)$	$(\,\mathbf{1},\,\mathbf{2}\,,\,-1)$
(×3 families)	E	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\,\mathbf{1},\,2)$
Higgs(ino)s	H_1	$(H_1^0 \ \ H_1^-)$	$(\widetilde{H}_1^0 \ \ \widetilde{H}_1^-)$	$(\,\mathbf{1},\,\mathbf{2}\,,\,-1)$
	H_2	$(H_2^+ \ \ H_2^0)$	$(\widetilde{H}_2^+ \ \ \widetilde{H}_2^0)$	$(\,\mathbf{1},\,\mathbf{2}\,,\,+1)$

Table B.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

Each gauge boson has a spin-1/2 superpartner, the gaugino. They are not the mass eigenstates of the theory. The neutral gauginos and the Higgsinos mix to form the neutralinos, and the charged gauginos and Higgsinos mix to form the charginos.

Names	spin 1/2	spin 1	$SU(3)$, $SU(2)$, $U(1)$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\widetilde{W}^{\pm}\ \ \widetilde{W}^{0}$	$W^{\pm}\ \ W^{0}$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\mathbf{1}, \mathbf{1}, 0)$

Table B.2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

In the Standard Model, the gauge anomalies exactly cancel. The superpartners of the Higgs boson are chiral fermions which contribute to the $SU(2) \times U(1)$ gauge anomaly. In order to cancel this anomaly, we must add a second Higgs doublet with opposite hypercharge.
The particle spectrum is summarized in tables B.1, B.2. For a detailed discussion of the particle spectrum of the MSSM, see ref. [32].

B.1.2 The Superpotential

Our definition of the MSSM and our conventions follow ref. [32]. The supersymmetric interactions are generated by the superpotential. The most general gauge-invariant and R-parity conserving superpotential has the following form:

$$W = \epsilon_{ij}\left[h_e H_1^i L^j E + h_d H_1^i Q^j D - h_u H_2^i Q^j U - \mu H_1^i H_2^j\right], \qquad (\epsilon_{12} = +1).$$

Generation indices are suppressed. In general, $(h_e)_{\alpha\beta}$, $(h_u)_{\alpha\beta}$, $(h_d)_{\alpha\beta}$ are 3×3 matrices in generation space, and L^α, E^α, D^α, Q^α denote the particles of the α-th generation. We assume no intergenerational mixing. In this case, the matrices are diagonal, and the matrix multiplication in generation space amounts to writing down the superpotential once for every generation.
For the radiative corrections to the mass sum rules, only the third generation of quarks is relevant. Then, the superpotential has just the form above, and L, E, D, Q denote the particles of the 3rd generation. In this case, h_e, h_u, h_d are just proportional to the corresponding fermion masses:

$$h_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos\beta}, \quad h_b = \frac{g m_b}{\sqrt{2} m_W \cos\beta}, \quad h_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta}.$$

B.1.3 The Soft-Supersymmetry Breaking Terms

The superpartners of the Standard Model particles are not observed in nature, so supersymmetry must be broken. Different mechanisms of supersymmetry-breaking are discussed in ref. [32]. The origin of supersymmetry breaking is not yet fully understood. The most convenient way to handle this aspect of the theory is to admit all possible types of terms which explicitly break supersymmetry.

One of the major reasons why we believe that supersymmetry is realized in nature is that quadratic divergences arising from Standard Model spin-0 particles running in loops are canceled by loop diagrams with their corresponding superpartners. The explicit supersymmetry-breaking terms could reintroduce quadratic divergences and thus spoil the nice properties inherent to supersymmetric theories. Explicit supersymmetry-breaking terms which do not reintroduce quadratic divergences are called “soft”. They have been first classified in ref. [45].

We have included all soft supersymmetry-breaking terms which respect R-parity. The Lagrangian arising from these terms reads:

$$
\begin{aligned}
-L_{\text{soft}} =& m_1^2 H_1^{i*} H_1^i + m_2^2 H_2^{i*} H_2^i - m_{12}^2(\epsilon_{ij} H_1^i H_2^j + \mathbf{c.c.}) + \frac{1}{2} M_1 \overline{\widetilde{B}} \widetilde{B} + \frac{1}{2} M_2 \overline{\widetilde{W}}^a \widetilde{W}^a + \frac{1}{2} M_3 \overline{\widetilde{G}}^a \widetilde{G}^a \\
&+ m_{\widetilde{L}}^2 [\tilde{\nu}_L^* \tilde{\nu}_L + \tilde{\tau}_L^* \tilde{\tau}_L] + m_{\widetilde{E}}^2 \tilde{\tau}_R^* \tilde{\tau}_R + m_{\widetilde{Q}}^2 [\tilde{t}_L^* \tilde{t}_L + \tilde{b}_L^* \tilde{b}_L] + m_{\widetilde{U}}^2 \tilde{t}_R^* \tilde{t}_R + m_{\widetilde{D}}^2 \tilde{b}_R^* \tilde{b}_R \\
&+ \epsilon_{ij} h_\tau A_\tau H_1^i \widetilde{L}^j \tilde{e}_R^* + \epsilon_{ij} h_b A_b H_1^i \widetilde{Q}^j \tilde{b}_R^* - \epsilon_{ij} h_t A_t H_2^i \widetilde{Q}^j \tilde{t}_R^*
\end{aligned}
$$

B.2 The Higgs Sector of the MSSM

B.2.1 The Higgs Potential

The MSSM contains two complex doublets of Higgs fields H_1 with hypercharge $Y = -1$ and H_2 with hypercharge $Y = 1$. Two Higgs doublets are necessary to give masses to both u-type and d-type quarks, and to guarantee the cancellation of gauge anomalies.

The Higgs potential follows from the Lagrangian we derived following refs. [32], [35]:

$$
\begin{aligned}
V_{\text{Higgs}} =& (m_1^2 + |\mu|^2) H_1^{i*} H_1^i + (m_2^2 + |\mu|^2) H_2^{i*} H_2^i - m_{12}^2(\epsilon_{ij} H_1^i H_1^j + \epsilon_{ij} H_1^{i*} H_1^{j*}) \\
&+ \frac{1}{8}(g^2 + g'^2)[H_1^{i*} H_1^i - H_2^{j*} H_2^j]^2 + \frac{1}{2} g^2 |H_1^{i*} H_2^i|^2 \\
&+ \frac{g'^2}{8} \xi (\phi^+ \phi_0 - \phi_0^+ \phi - \psi^+ \psi_0 + \psi_0^+ \psi)^2 + \frac{g^2}{8} \xi (\phi^+ \tau^a \phi_0 - \phi_0^+ \tau^a \phi + \psi^+ \tau^a \psi_0 - \psi_0^+ \tau^a \psi)^2
\end{aligned}
\tag{B.1}
$$

For the definition of ϕ and ψ, see equation B.2 below. Our computations are performed in R_ξ gauge. The gauge-fixing terms have been chosen so as to cancel the Goldstone boson - vector boson mixing terms. For the renormalization procedure, we will specialize to $\xi = 0$ (Landau gauge). This choice of gauge will become clear, when we describe the renormalization procedure. m_1, m_2, and m_{12} are soft supersymmetry-breaking terms, and μ is the coefficient of $H_1 H_2$ in the superpotential. Apart from the gauge fixing terms, the expression for the Higgs potential has been checked against ref. [32], and we have full agreement.

The neutral components acquire non-vanishing vacuum expectation values v_1 and v_2 so that spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_Q$ occurs. The doublets are re-expressed in terms of real and imaginary degrees of freedom by substituting

$$
H_1 = \phi + \phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + i\zeta_1 \\ \varphi_1^- \end{pmatrix} + \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad H_2 = \psi + \psi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_2^+ \\ \eta_2 + i\zeta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \tag{B.2}
$$

in V_{Higgs} above. The minimization conditions are

$$
\left. \frac{\partial}{\partial \lambda_i} V_{\text{Higgs}} \right|_{\lambda_i = 0} = 0,
$$

where λ_i is one of the fields η_i, ζ_i, $\varphi_i^\pm$, respectively. Inserting the fields and taking the derivative, we arrive at explicit expressions for these minimization conditions:

$$
\begin{aligned}
(m_1^2 + |\mu|^2) + \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2) &= m_{12}^2 \frac{v_2}{v_1} \\
(m_2^2 + |\mu|^2) - \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2) &= m_{12}^2 \frac{v_1}{v_2}
\end{aligned}
\tag{B.3}
$$

Note that in the above potential, μ only enters in the combinations $m_1^2 + |\mu|^2$ and $m_2^2 + |\mu|^2$, so there are three independent parameters. The minimization conditions impose relations among the vacuum expectations values v_1, v_2 and the parameters in the Higgs potential. We can eliminate two parameters in the Higgs potential in favor of v_1, v_2. Since the combination

$$\frac{g^2}{2}(v_1^2 + v_2^2) = m_W^2 \tag{B.4}$$

is determined by the W mass, only one vacuum expectation value can be chosen freely, and we are left with two independent parameters for the Higgs potential. We choose these two independent parameters to be

$$\tan\beta \equiv \frac{v_2}{v_1} \quad \text{and} \quad m_A. \tag{B.5}$$

For the definition of m_A, see eq. B.10 below. v_1 and v_2 can be chosen to be real and positive, so we have $0 \leq \beta \leq \pi/2$. This choice is not unique. We can fix any two independent parameters.

Note that we redefine $m_1^2 \equiv m_1^2 + |\mu|^2$ and $m_2^2 \equiv m_2^2 + |\mu|^2$ for convenience.

B.2.2 Terms linear in the Fields

Substituting the decomposition of the Higgs doublets in real and imaginary degrees of freedom (see eq. B.2) into the Higgs potential eq. B.1 and collecting the terms linear in η_1, η_2 immediately gives

$$V_{\text{Higgs}} = \text{const} + \tau_1\eta_1 + \tau_2\eta_2 + (\ldots)$$

with

$$\tau_1 = \sqrt{2}\left[m_1^2 v_1 - m_{12}^2 v_2 + \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2)v_1\right], \tag{B.6}$$

$$\tau_2 = \sqrt{2}\left[m_2^2 v_2 - m_{12}^2 v_1 - \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2)v_2\right]. \tag{B.7}$$

B.2.3 The Mass Eigenstates

The matrix for the squares of the Higgs boson masses is given by

$$M_{ij} = \frac{1}{2}\frac{\partial^2}{\partial\lambda_i\partial\lambda_j}V_{\text{Higgs}},$$

where λ_i is one of the fields η_i, ζ_i, $\varphi_i^{\pm}$, respectively.

The CP-odd Higgs boson

The mass matrix of the CP-odd Higgs in the basis ζ_1, ζ_2 is given by

$$M_{\zeta_i\zeta_j} = \begin{pmatrix} m_1^2 + \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2) + \frac{v_1^2}{2}\xi(g^2 + g'^2) & m_{12}^2 - \frac{g'^2}{2}\xi v_1 v_2 - \frac{g^2}{2}\xi v_1 v_2 \\ m_{12}^2 - \frac{g'^2}{2}\xi v_1 v_2 - \frac{g^2}{2}\xi v_1 v_2 & m_2^2 - \frac{1}{4}(g^2 + g'^2)(v_1^2 - v_2^2) + \frac{v_2^2}{2}\xi(g^2 + g'^2) \end{pmatrix}. \tag{B.8}$$

Using the minimization conditions B.3 the mass matrix can be simplified:

$$M_{\zeta_i\zeta_j} = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \frac{g'^2}{2}\xi v_1^2 + \frac{g^2}{2}\xi v_1^2 & m_{12}^2 - \frac{g'^2}{2}\xi v_1 v_2 - \frac{g^2}{2}\xi v_1 v_2 \\ m_{12}^2 - \frac{g'^2}{2}\xi v_1 v_2 - \frac{g^2}{2}\xi v_1 v_2 & m_{12}^2\frac{v_1}{v_2} + \frac{g'^2}{2}\xi v_2^2 + \frac{g^2}{2}\xi v_2^2 \end{pmatrix} \tag{B.9}$$

The matrix is diagonalized by

$$U M_{\zeta_i\zeta_j} U^{-1} = \begin{pmatrix} \xi m_Z^2 & 0 \\ 0 & m_A^2 \end{pmatrix}, \qquad U^{-1} = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix}.$$

The eigenvalues are

$$m_A^2 = \frac{m_{12}^2}{\sin\beta\cos\beta}, \qquad m_{G^0}^2 = \xi m_Z^2, \tag{B.10}$$

and the mass eigenstates are given by

$$\begin{aligned} \tfrac{1}{\sqrt{2}} G^0 &= -(\mathrm{Im}\ H_1^0)\cos\beta + (\mathrm{Im}\ H_2^0)\sin\beta, \\ \tfrac{1}{\sqrt{2}} A &= (\mathrm{Im}\ H_1^0)\sin\beta + (\mathrm{Im}\ H_2^0)\cos\beta. \end{aligned}$$

For the renormalization of the neutral Higgs boson mass sum rule which we described in section 2.2, it is convenient to express the mass matrix eq. B.8 in Landau gauge ($\xi = 0$) as

$$M_{\zeta_i\zeta_j} = \begin{pmatrix} m^2 s_\beta^2 + \frac{1}{\sqrt{2}v}(c_\beta\tau_1 - s_\beta\tau_2) & m^2 s_\beta c_\beta - \frac{1}{\sqrt{2}v}(s_\beta\tau_1 + c_\beta\tau_2) \\ m^2 s_\beta c_\beta - \frac{1}{\sqrt{2}v}(s_\beta\tau_1 + c_\beta\tau_2) & m^2 c_\beta^2 - \frac{1}{\sqrt{2}v}(c_\beta\tau_1 - s_\beta\tau_2) \end{pmatrix}, \tag{B.11}$$

where τ_1, τ_2 are the coefficients of η_1, η_2 in the Higgs potential (see eqs. B.6, B.7) and we have defined $v \equiv v_1^2 + v_2^2$, $m^2 \equiv m_1^2 + m_2^2$. Substituting eqs. B.6, B.7 for τ_1, τ_2 in eq. B.11 and noting that $c_\beta = v_1/v$, $s_\beta = v_2/v$ immediately gives eq. B.8 with $\xi = 0$.

The CP-even Higgs bosons

The mass matrix in basis η_1, η_2

$$M_{\eta_i\eta_j} = \frac{1}{v_1^2 + v_2^2} \begin{pmatrix} m_A^2 v_2^2 + m_Z^2 v_1^2 & -(m_A^2 + m_Z^2) v_1 v_2 \\ -(m_A^2 + m_Z^2) v_1 v_2 & m_A^2 v_1^2 + m_Z^2 v_2^2 \end{pmatrix} \tag{B.12}$$

is diagonalized by

$$U M_{\eta_i\eta_j} U^{-1} = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}, \qquad U^{-1} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad \sin 2\alpha = -\sin 2\beta \frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}.$$

Note that the above relation implies $-\pi/2 \le \alpha \le 0$. The eigenvalues are

$$m_H^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 + \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta} \right], \tag{B.13}$$

$$m_h^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos^2 2\beta} \right], \tag{B.14}$$

and the mass eigenstates are given by

$$\begin{aligned} \tfrac{1}{\sqrt{2}} H &= (\mathrm{Re}\ H_1^0 - v_1)\cos\alpha + (\mathrm{Re}\ H_2^0 - v_2)\sin\alpha, \\ \tfrac{1}{\sqrt{2}} h &= -(\mathrm{Re}\ H_1^0 - v_1)\sin\alpha + (\mathrm{Re}\ H_2^0 - v_2)\cos\alpha. \end{aligned}$$

For the renormalization of the neutral Higgs boson mass sum rule which we described in section 2.2, it is convenient to express the mass matrix eq. B.12 as

$$M_{\eta_i\eta_j} = \begin{pmatrix} M_{\zeta_1\zeta_1} + m_Z^2 c_\beta^2 & -M_{\zeta_1\zeta_2} - m_Z^2 s_\beta c_\beta \\ -M_{\zeta_2\zeta_1} - m_Z^2 s_\beta c_\beta & M_{\zeta_2\zeta_2} + m_Z^2 s_\beta^2 \end{pmatrix}. \tag{B.15}$$

Substituting the expressions for $M_{\zeta_i\zeta_j}$ for $\xi = 0$ as given in eq. B.9 into eq. B.15 and using $c_\beta = v_1/v$, $s_\beta = v_2/v$, $v \equiv v_1^2 + v_2^2$ gives after a straightforward calculation eq. B.12.

The charged Higgs bosons

The mass matrix in basis $\varphi_1^\pm$, $\varphi_2^\pm$

$$M_{\varphi_i^\pm\varphi_j^\pm} = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \frac{g^2}{2}v_2^2 + \frac{g^2}{2}\xi v_1^2 & m_{12}^2 + \frac{g^2}{2}v_1v_2 - \frac{g^2}{2}\xi v_1 v_2 \\ m_{12}^2 + \frac{g^2}{2}v_1v_2 - \frac{g^2}{2}\xi v_1 v_2 & m_{12}^2\frac{v_1}{v_2} + \frac{g^2}{2}v_1^2 + \frac{g^2}{2}\xi v_2^2 \end{pmatrix} \tag{B.16}$$

is diagonalized by

$$UM_{\varphi_i^\pm\varphi_j^\pm}U^{-1} = \begin{pmatrix} \xi m_W^2 & 0 \\ 0 & m_{H^\pm}^2 \end{pmatrix}, \qquad U^{-1} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}. \tag{B.17}$$

The eigenvalues are

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \qquad m_{G^\pm}^2 = \xi m_W^2, \tag{B.18}$$

and the mass eigenstates are given by

$$\begin{aligned} \tfrac{1}{\sqrt{2}}H^\pm &= -\varphi_1^\pm\ \sin\beta + \varphi_2^\pm\ \cos\beta, \\ \tfrac{1}{\sqrt{2}}G^\pm &= \varphi_1^\pm\ \cos\beta + \varphi_2^\pm\ \sin\beta. \end{aligned}$$

The above eq. B.18 is the tree level mass sum rule for the charged Higgs. As one of the main subjects of this diploma thesis is the calculation of radiative corrections to this mass sum rule, we will give the explicit derivation of the tree level formula.

B.2.4 Derivation of the Tree Level Mass Sum Rule

The derivation of eq. B.18 amounts to calculating the eigenvalues of the mass matrix in eq. B.16. This task is greatly simplified by noting that for any 2×2 matrix

$$M = \begin{pmatrix} A & B \\ B & C \end{pmatrix}$$

the eigenvalues are given by

$$\lambda_{1/2} = \frac{1}{2}\left[A + C \pm \sqrt{(A-C)^2 + 4B^2}\right]. \tag{B.19}$$

The diagonalizing matrix is

$$U^{-1} = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix},$$

and the diagonalizing angle is given by

$$\tan\delta = \frac{2B}{A-C}.$$

To simplify the algebra, we will write the mass matrix of the charged Higgs as

$$M_{H^\pm} = \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \frac{g^2}{2}v_2^2 & m_{12}^2 + \frac{g^2}{2}v_1v_2 \\ m_{12}^2 + \frac{g^2}{2}v_1v_2 & m_{12}^2\frac{v_1}{v_2} + \frac{g^2}{2}v_1^2 \end{pmatrix} + \begin{pmatrix} \frac{g^2}{2}\xi v_1^2 & -\frac{g^2}{2}\xi v_1v_2 \\ -\frac{g^2}{2}\xi v_1 v_2 & \frac{g^2}{2}\xi v_2^2 \end{pmatrix}.$$

Now note that both matrices are diagonalized by the same angle. For the first matrix, we have

$$\tan\delta_1 = \frac{2B}{A-C} = \frac{2(m_{12}^2 + g^2v_1v_2/2)}{m_{12}^2\,(v_2/v_1 - v_1/v_2) + g^2(v_2^2 - v_1^2)/2} = \frac{2(m_{12}^2 + g^2v_1v_2/2)}{(v_2^2 - v_1^2)(m_{12}^2/(v_1v_2) + g^2/2)} = \frac{2v_1v_2}{v_2^2 - v_1^2},$$

and for the second matrix

$$\tan\delta_2 = \frac{2B}{A-C} = \frac{-g^2\xi v_1 v_2}{g^2\xi v_1^2/2 - g^2\xi v_2^2/2} = \frac{2v_1v_2}{v_2^2 - v_1^2}.$$

So we can calculate the eigenvalues for each matrix separately. Let us start with the first matrix:

$$\begin{aligned}
\lambda_{1/2} &= \frac{1}{2}\left[A + C \pm \sqrt{(A-C)^2 + 4B^2}\right] \\
&= \frac{1}{2}\left[\frac{m_{12}^2}{v_1v_2}(v_1^2+v_2^2) + \frac{g^2}{2}(v_1^2+v_2^2) \pm \left(\left[\frac{m_{12}^2}{v_1v_2}(v_2^2-v_1^2) + \frac{g^2}{2}(v_2^2-v_1^2)\right]^2 + 4\left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2\right)^{1/2}\right] \\
&= \frac{1}{2}\left[\frac{m_{12}^2}{v_1v_2}(v_1^2+v_2^2) + \frac{g^2}{2}(v_1^2+v_2^2) \pm \left(\left[\frac{m_{12}^2}{v_1v_2}(v_2^2+v_1^2) + \frac{g^2}{2}(v_2^2+v_1^2)\right]^2\right)^{1/2}\right] \\
\rightarrow \quad & \lambda_1 = m_{H^\pm}^2 = \frac{m_{12}^2}{v_1v_2}(v_1^2+v_2^2) + \frac{g^2}{2}(v_1^2+v_2^2), \qquad \lambda_2 = 0
\end{aligned}$$

From the second line to the third, we use the result of the auxiliary calculation below.

Auxiliary Calculation

$$\begin{aligned}
&\left[\frac{m_{12}^2}{v_1v_2}(v_2^2-v_1^2) + \frac{g^2}{2}(v_2^2-v_1^2)\right]^2 + 4\left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2 = \frac{(v_2^2-v_1^2)^2}{v_1^2v_2^2}\left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2 + 4\left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2 \\
&\quad = \left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2\left(\frac{(v_2^2-v_1^2)^2}{v_1^2v_2^2} + 4\right) = \left[m_{12}^2 + \frac{g^2}{2}v_1v_2\right]^2\left(\frac{(v_2^2+v_1^2)^2}{v_1^2v_2^2}\right) \\
&\quad = \left[\frac{m_{12}^2}{v_1v_2}(v_2^2+v_1^2) + \frac{g^2}{2}(v_2^2+v_1^2)\right]^2
\end{aligned}$$

The expression for $m_{H^\pm}^2$ can be cast in a more familiar form:

$$\begin{aligned}
m_{H^\pm}^2 &= \frac{m_{12}^2}{v_1v_2}(v_1^2+v_2^2) + \frac{g^2}{2}(v_1^2+v_2^2) = m_{12}^2\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right) + m_W^2 = m_{12}^2(\cot\beta + \tan\beta) + m_W^2 \\
&= m_{12}^2\left(\frac{\cos\beta}{\sin\beta} + \frac{\sin\beta}{\cos\beta}\right) + m_W^2 = m_{12}^2\left(\frac{\sin^2\beta + \cos^2\beta}{\sin\beta\cos\beta}\right) + m_W^2 = \frac{m_{12}^2}{\sin\beta\cos\beta} + m_W^2 \\
&= m_A^2 + m_W^2
\end{aligned}$$

This is the mass sum rule for the charged Higgs we are seeking for.

The calculations for the second matrix yield:

$$\begin{aligned}
\lambda_{1/2} &= \frac{1}{2}\left[A + C \pm \sqrt{(A-C)^2 + 4B^2}\right] = \frac{1}{2}\left[\frac{g^2}{2}\xi(v_1^2+v_2^2) \pm \left(\left[\frac{g^2}{2}\xi(v_1^2-v_2^2)\right]^2 + 4\left[-\frac{g^2}{2}\xi v_1v_2\right]^2\right)^{1/2}\right] \\
&= \frac{g^2\xi}{4}\left[(v_1^2+v_2^2) \pm \left((v_1^2-v_2^2)^2 + 4v_1^2v_2^2\right)^{1/2}\right] = \frac{g^2\xi}{4}\left[(v_1^2+v_2^2) \pm (v_1^2+v_2^2)\right], \\
\rightarrow \quad & \lambda_1 = 0, \qquad \lambda_2 = m_{G^\pm}^2 = \frac{g^2\xi}{2}(v_1^2+v_2^2) = \xi m_W^2
\end{aligned}$$

Remember that for both matrices, λ_i is the eigenvalue which corresponds to the i-th eigenvector and the eigenvectors are the columns of the diagonalizing matrix U^{-1}, so diagonalization yields

$$UM_{H^\pm}U^{-1} = \begin{pmatrix} m_{H^\pm}^2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \xi m_W^2 \end{pmatrix} = \begin{pmatrix} m_{H^\pm}^2 & 0 \\ 0 & \xi m_W^2 \end{pmatrix},$$

as stated in eq. B.17 above.

B.2.5 Some Useful Relations

In the calculation of the infinite and finite part of the correction Δ to the neutral mass sum rule we used the tree level relation

$$m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_A^2 c_\beta^2 - m_Z^2 s_\beta^2 = 0. \tag{B.20}$$

In the calculation of the infinite and finite part of the correction $\widetilde{\Delta}$ to the charged mass sum rule we used the tree level relation

$$\cos^2\theta_t m_{\tilde{t}_1}^2 + \sin^2\theta_t m_{\tilde{t}_2}^2 - \cos^2\theta_b m_{\tilde{b}_1}^2 - \sin^2\theta_b m_{\tilde{b}_2}^2 = m_t^2 - m_b^2 + m_W^2 \cos 2\beta. \tag{B.21}$$

The interaction eigenstates $\tilde{t}_L$, $\tilde{t}_R$ mix to mass eigenstates $\tilde{t}_1$, $\tilde{t}_2$. The mixing angle is given by

$$\cos 2\theta_t = \frac{m_{\tilde{Q}}^2 - m_{\tilde{U}}^2 + \frac{1}{6}(8m_W^2 - 5m_Z^2)\cos 2\beta}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \tag{B.22}$$

and the masses in terms of the top quark mass and the soft supersymmetry breaking mass parameters are

$$\begin{aligned} m_{\tilde{t}_{1,2}}^2 &= m_t^2 + \frac{1}{2}(m_{\tilde{Q}}^2 + m_{\tilde{U}}^2) + \frac{1}{4}m_Z^2 \cos 2\beta \\ &\pm \left(\left[\frac{1}{2}(m_{\tilde{Q}}^2 - m_{\tilde{U}}^2) + \frac{1}{12}(8m_W^2 - 5m_Z^2)\cos 2\beta \right]^2 + m_t^2 (A_t - \mu \cot\beta)^2 \right)^{1/2}. \end{aligned} \tag{B.23}$$

Analogously, the interaction eigenstates $\tilde{b}_L$, $\tilde{b}_R$ mix to $\tilde{b}_1$, $\tilde{b}_2$. The mixing angle is given by

$$\cos 2\theta_b = \frac{m_{\tilde{Q}}^2 - m_{\tilde{D}}^2 - \frac{1}{6}(4m_W^2 - m_Z^2)\cos 2\beta}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2} \tag{B.24}$$

and the masses are

$$\begin{aligned} m_{\tilde{b}_{1,2}}^2 &= m_b^2 + \frac{1}{2}(m_{\tilde{Q}}^2 + m_{\tilde{D}}^2) - \frac{1}{4}m_Z^2 \cos 2\beta \\ &\pm \left(\left[\frac{1}{2}(m_{\tilde{Q}}^2 - m_{\tilde{D}}^2) - \frac{1}{12}(4m_W^2 - m_Z^2)\cos 2\beta \right]^2 + m_b^2 (A_b - \mu \tan\beta)^2 \right)^{1/2}. \end{aligned} \tag{B.25}$$

In doing the calculations, we found the relations

$$A_t - \mu\cot\beta = \frac{\sin 2\theta_t}{2m_t}(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2), \qquad A_b - \mu\tan\beta = \frac{\sin 2\theta_b}{2m_b}(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2) \tag{B.26}$$

to be useful.

Appendix C

Passarino-Veltman-Functions

Definitions and Basic Relations

There are a number of integrals which appear at one-loop in a self-energy calculation [29]. These integrals are standardized and are known as Passarino-Veltman functions. We point out that our definitions and relations will differ from those of ref. [29] since we use the Minkowski metric. Q is the renormalization scale. The one-point integral is given by:

$$A(m^2) = \frac{Q^{4-d}}{i\pi^2} \int d^d k \, \frac{1}{k^2 - m^2} \tag{C.1}$$

The two-point integrals occurring are:

$$B_0(q^2, m_1^2, m_2^2) = \frac{Q^{4-d}}{i\pi^2} \int d^d k \, \frac{1}{(k^2 - m_1^2)[(k+q)^2 - m_2^2]} \tag{C.2}$$

$$B_\mu(q^2, m_1^2, m_2^2) = \frac{Q^{4-d}}{i\pi^2} \int d^d k \, \frac{k_\mu}{(k^2 - m_1^2)[(k+q)^2 - m_2^2]} \tag{C.3}$$

$$B_{\mu\nu}(q^2, m_1^2, m_2^2) = \frac{Q^{4-d}}{i\pi^2} \int d^d k \, \frac{k_\mu k_\nu}{(k^2 - m_1^2)[(k+q)^2 - m_2^2]} \tag{C.4}$$

The functions B_1, B_{21}, and B_{22} are defined as follows:

$$B_\mu(q^2, m_1^2, m_2^2) = q_\mu B_1(q^2, m_1^2, m_2^2), \quad B_{\mu\nu}(q^2, m_1^2, m_2^2) = q_\mu q_\nu B_{21} + g_{\mu\nu} B_{22} \tag{C.5}$$

The functions B_1, B_{21}, and B_{22} are algebraically related to A and B_0. We will derive some of the relations which we need in our calculations. Compensating for the different metric conventions, eq. (D.8) in ref. [29] reads:

$$q^2 B_{21}(q^2, m_1^2, m_2^2) + 4B_{22}(q^2, m_1^2, m_2^2) = A(m_2^2) + m_1^2 B_0(q^2, m_1^2, m_2^2) + \frac{1}{2}\left(m_1^2 + m_2^2 - \frac{1}{3}q^2\right) \tag{C.6}$$

For the special case $q = 0$, eq. C.6 gives:

$$4B_{22}(q^2, m_1^2, m_2^2) = A(m_2^2) + m_1^2 B_0(q^2, m_1^2, m_2^2) + \frac{1}{2}(m_1^2 + m_2^2) \tag{C.7}$$

B_0, B_{22} are symmetric in m_1, m_2:

$$4B_{22}(q^2,m_1^2,m_2^2) = A(m_1^2) + m_2^2 B_0(q^2,m_1^2,m_2^2) + \frac{1}{2}(m_1^2+m_2^2) \tag{C.8}$$

After changing to the Minkowski metric, eq. D.6 in ref. [29] reads:

$$q^2 B_1(q^2,m_1^2,m_2^2) = \frac{1}{2}\left[A(m_1^2) - A(m_2^2) - (q^2+m_1^2-m_2^2)B_0(q^2,m_1^2,m_2^2)\right] \tag{C.9}$$

For the special case $q=0$, eq. C.9 gives:

$$(m_1^2-m_2^2)B_0(0,m_1^2,m_2^2) = A(m_1^2) - A(m_2^2) \tag{C.10}$$

By an abuse of notation, we will denote the finite part of the PV functions by the same symbols. We render the PV functions finite by subtracting

$$\frac{2}{\epsilon} - \gamma + \log 4\pi.$$

Our Loop Integrals in Terms of PV-Functions

The finite part of the loop integrals evaluated in appendix F is a sum of integrals of the form

$$\int dx \log\Delta_{ij}, \quad \int dx\; x\log\Delta_{ij}, \quad \int dx\; x^2\log\Delta_{ij}, \tag{C.11}$$

and Δ_{ij} is defined as

$$\Delta_{ij} \equiv \frac{-x(1-x)q^2 - x(m_i^2-m_j^2) + m_i^2}{Q^2},$$

where m_i, m_j are the masses of the particles running in the loop, q^2 is the external momentum squared, and Q is the renormalization scale. For numerical evaluation, we relate these integrals to the PV functions. Regularizing the loop integrals in eqs. C.1-C.5 and expressing the result in terms of the integrals given in eq. C.11 immediately yields the desired relations:

$$A(m^2) = m^2 - m^2\log m^2 \tag{C.12}$$

$$B_0(q^2,m_1^2,m_2^2) = -\int dx\;\log\Delta_{12} \tag{C.13}$$

$$B_1(q^2,m_1^2,m_2^2) = \int dx\; x\;\log\Delta_{12} \tag{C.14}$$

$$B_{21}(q^2,m_1^2,m_2^2) = -\int dx\; x^2\;\log\Delta_{12} \tag{C.15}$$

$$B_{22}(q^2,m_1^2,m_2^2) = -\frac{1}{2}\int dx\;\left[-x(1-x)q^2 - x(m_1^2-m_2^2) + m_1^2\right](\log\Delta_{12} - 1) \tag{C.16}$$

Correction to the Neutral Mass Sum Rule in Terms of PV Functions

As we want to compare our expression for the correction to the neutral Higgs boson mass sum rule Δ to that of ref. [6], we have to express our results in terms of 3 functions defined as

$$F(q^2,m_1^2,m_2^2) = \int dx \;\; \log \frac{xm_1^2 + (1-x)m_2^2 - x(1-x)q^2}{Q^2}, \tag{C.17}$$

$$G(q^2,m_1^2,m_2^2) = \int dx \;\, x(1-x) \;\log \frac{xm_1^2 + (1-x)m_2^2 - x(1-x)q^2}{Q^2}, \tag{C.18}$$

$$H(q^2,m_1^2,m_2^2) = \int dx \;\, x \;\log \frac{xm_1^2 + (1-x)m_2^2 - x(1-x)q^2}{Q^2}. \tag{C.19}$$

These functions can be related to the PV functions, as can be immediately checked using eqs. C.12-C.16:

$$F(q^2,m_1^2,m_2^2) = -B_0(q^2,m_1^2,m_2^2) \tag{C.20}$$

$$G(q^2,m_1^2,m_2^2) = B_1(q^2,m_1^2,m_2^2) + B_{21}(q^2,m_1^2,m_2^2) \tag{C.21}$$

$$H(q^2,m_1^2,m_2^2) = B_1(q^2,m_2^2,m_1^2) \tag{C.22}$$

Correction to the Charged Mass Sum Rule in Terms of PV Functions

Ref. [8] expresses the result for the correction to the charged Higgs boson mass sum rule $\widetilde{\Delta}$ in terms of 4 functions defined as

$$F(q^2,m_1^2,m_2^2) = \int dx \;\; \log \frac{(1-x)m_1^2 + xm_2^2 - x(1-x)q^2}{Q^2}, \tag{C.23}$$

$$G(q^2,m_1^2,m_2^2) = \int dx \;\, x(1-x)q^2 \;\log \frac{(1-x)m_1^2 + xm_2^2 - x(1-x)q^2}{Q^2}, \tag{C.24}$$

$$H(q^2,m_1^2,m_2^2) = \int dx \;\, [(1-x)m_1^2 + xm_2^2] \;\log \frac{(1-x)m_1^2 + xm_2^2 - x(1-x)q^2}{Q^2}, \tag{C.25}$$

$$J(q^2,m_1^2,m_2^2) = 2H(q^2,m_1^2,m_2^2) - 2G(q^2,m_1^2,m_2^2) + \frac{1}{3}q^2. \tag{C.26}$$

These functions can be related to the PV functions, as can be immediately checked using eqs. C.12-C.16:

$$F(q^2,m_1^2,m_2^2) = -B_0(q^2,m_1^2,m_2^2) \tag{C.27}$$

$$G(q^2,m_1^2,m_2^2) = q^2B_1(q^2,m_1^2,m_2^2) + q^2B_{21}(q^2,m_1^2,m_2^2) \tag{C.28}$$

$$H(q^2,m_1^2,m_2^2) = -m_1^2B_0(q^2,m_2^2,m_1^2) - (m_1^2 - m_2^2)B_1(q^2,m_2^2,m_1^2) \tag{C.29}$$

In my notation, the integrals are given by:

$$F(q^2,m_1^2,m_2^2) = \int dx \log \Delta_{12}, \tag{C.30}$$

$$G(q^2,m_1^2,m_2^2) = \int dx x(1-x)q^2 \log \Delta_{12}, \tag{C.31}$$

$$H(q^2,m_1^2,m_2^2) = \int dx \big[-x(m_1^2 - m_2^2) + m_1^2 \big] \log \Delta_{12}, \tag{C.32}$$

$$J(q^2,m_1^2,m_2^2) = 2\int dx \big[-x(1-x)q^2 - x(m_1^2 - m_2^2) + m_1^2 \big] \log \Delta_{12} + \frac{1}{3}q^2. \tag{C.33}$$

Implementation of the PV Functions

For the numerical analysis we need implementations of the PV functions as computer routines. We decided to use the LoopTools package [30, 31, 38] which provides implementations of the PV functions for Fortran, C, and Mathematica. The correspondence between the PV functions and their implementations as C functions is given by

$$\begin{aligned} A_0(m^2) &\quad\leftrightarrow\quad A0(ms), \\ B_0(q^2, m_1^2, m_2^2) &\quad\leftrightarrow\quad B0(ps, m1s, m2s), \\ B_1(q^2, m_1^2, m_2^2) &\quad\leftrightarrow\quad B1(ps, m1s, m2s), \\ B_{22}(q^2, m_1^2, m_2^2) &\quad\leftrightarrow\quad B00(ps, m1s, m2s), \\ B_{21}(q^2, m_1^2, m_2^2) &\quad\leftrightarrow\quad B11(ps, m1s, m2s), \end{aligned} \tag{C.34}$$

as is immediately checked by comparing the definitions of the PV functions in eqs. C.1-C.5 to those in ref. [38].

Appendix D

Details of the Calculation: The Neutral Higgs Boson Mass Sum Rule

In this appendix, we consequently use first family representation. For the particle spectrum and naming conventions, the reader is referred to section B.1.1. The abbreviations and symbols we used in the calculations are explained in section A.

We will express the correction

$$\Delta = -\mathrm{Re}\ \Pi_{HH}(m_H^2) - \mathrm{Re}\ \Pi_{hh}(m_h^2) + \mathrm{Re}\ \Pi_{AA}(m_A^2) + \mathrm{Re}\ \Pi_{GG}(0) - \mathrm{Re}\ \Pi_{ZZ}(m_Z^2)$$

as a sum of three quantities,

$$\Delta = \Delta_4 + \Delta_2 + \Delta_0,$$

where the subscript $i = 0, 2, 4$ denotes the power of the up-quark or up-squark mass parameters which occur in the expression for Δ_i. To be specific, these mass parameters are m_u, A_u, μ, $m_{\tilde{u}_1}$, $m_{\tilde{u}_2}$. We caution the reader that the coefficient μ of $H_1 H_2$ in the superpotential has opposite sign in our conventions as compared to ref. [6]. The finite part of the correction will be written in terms of three functions F, G, and H defined in eqs. C.17-C.19. We recommend the reader to review the appendices A and C.

D.1 Correction Δ_4

Let us calculate the part of the correction to the neutral Higgs boson mass sum rule in which the quark and squark mass parameters occur in 4th order. Drop the i to get the self-energy. For the Z-boson, take the coefficient of $g^{\mu\nu}$ (the transverse part of the self-energy). Note that contributions from H^0, h^0, Z enter the sum with a minus, those from A and G^0 with a plus (see eq. 2.5). Remember that the self-energies are to be calculated at $q^2 = M^2$, where M is the mass of the respective particle.

$$\begin{aligned}
\Delta_4 = &+ \frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^4}{m_W^2 s_\beta^2} s_\alpha^2 - \frac{m_u^2}{m_W^2 s_\beta^2} c_u^2 s_u^2 (\mu c_\alpha - A_u s_\alpha)^2 \\
&+ 2\frac{m_u^3}{m_W^2 s_\beta^2} s_\alpha c_u s_u (\mu c_\alpha - A_u s_\alpha)\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_1}\right) \quad \text{from fig. F.12} \\
&+ \frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^4}{m_W^2 s_\beta^2} s_\alpha^2 - \frac{m_u^2}{m_W^2 s_\beta^2} c_u^2 s_u^2 (\mu c_\alpha - A_u s_\alpha)^2 \\
&- 2\frac{m_u^3}{m_W^2 s_\beta^2} s_\alpha c_u s_u (\mu c_\alpha - A_u s_\alpha)\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_2 \tilde u_2}\right) \quad \text{from fig. F.13} \\
&+ 2\cdot\frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^2}{4 m_W^2 s_\beta^2} (c_u^2 - s_u^2)^2 (\mu c_\alpha - A_u s_\alpha)^2\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.14, F.11} \\
&+ \frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^4}{m_W^2 s_\beta^2} c_\alpha^2 - \frac{m_u^2}{m_W^2 s_\beta^2} c_u^2 s_u^2 (\mu s_\alpha + A_u c_\alpha)^2 \\
&- 2\frac{m_u^3}{m_W^2 s_\beta^2} c_\alpha c_u s_u (\mu s_\alpha + A_u c_\alpha)\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_1}\right) \quad \text{from fig. F.18} \\
&+ \frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^4}{m_W^2 s_\beta^2} c_\alpha^2 - \frac{m_u^2}{m_W^2 s_\beta^2} c_u^2 s_u^2 (\mu s_\alpha + A_u c_\alpha)^2 \\
&+ 2\frac{m_u^3}{m_W^2 s_\beta^2} c_\alpha c_u s_u (\mu s_\alpha + A_u c_\alpha)\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_2 \tilde u_2}\right) \quad \text{from fig. F.19} \\
&+ 2\cdot\frac{g^2}{16\pi^2}\Bigg[- \frac{m_u^2}{4 m_W^2 s_\beta^2} (c_u^2 - s_u^2)^2 (\mu s_\alpha + A_u c_\alpha)^2\Bigg] \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.20, F.21} \\
&+ 2\cdot\frac{1}{16\pi^2}\frac{m_u^2}{4m_W^2}(\mu + A_u \cot\beta)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.26, F.27} \\
&+ 2\cdot\frac{1}{16\pi^2}\frac{m_u^2}{4m_W^2}(\mu \cot\beta - A_u)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.61, F.62} \\
&+ \frac{3g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{uu}\right) + \frac{g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \quad \text{from fig. F.10} \\
&+ \frac{3g^2 m_u^4 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{uu}\right) + \frac{g^2 m_u^4 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \quad \text{from fig. F.17} \\
&- \frac{g^2 m_u^4 \cot^2\beta}{16\pi^2 m_W^2} \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{uu}\right) - \frac{g^2 m_u^4 \cot^2\beta}{16\pi^2 m_W^2} \quad \text{from fig. F.24} \\
&- \frac{g^2 m_u^4}{16\pi^2 m_W^2} \int dx \left(\frac{2}{\bar\epsilon} - \log \Delta_{uu}\right) - \frac{g^2 m_u^4}{16\pi^2 m_W^2} \quad \text{from fig. F.59}
\end{aligned}$$

D.1.1 Infinite Part

Collect the coefficients of $2/\bar\epsilon$ in the expression for Δ_4 and denote the result by Δ_4^I. The following calculation shows that the infinite part is zero.

$$
\begin{aligned}
\Delta_4^I = &+ \frac{g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[- m_u^4 s_\alpha^2 - m_u^2 c_u^2 s_u^2 (\mu c_\alpha - A_u s_\alpha)^2 + 2 m_u^3 s_\alpha c_u s_u (\mu c_\alpha - A_u s_\alpha)\Big] \\
&+ \frac{g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[- m_u^4 s_\alpha^2 - m_u^2 c_u^2 s_u^2 (\mu c_\alpha - A_u s_\alpha)^2 - 2 m_u^3 s_\alpha c_u s_u (\mu c_\alpha - A_u s_\alpha)\Big] \\
&- \frac{g^2}{32\pi^2 m_W^2 s_\beta^2}\Big[m_u^2 (c_u^2 - s_u^2)^2 (\mu c_\alpha - A_u s_\alpha)^2\Big] \\
&- \frac{g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[m_u^4 c_\alpha^2 + m_u^2 c_u^2 s_u^2 (\mu s_\alpha + A_u c_\alpha)^2 + 2 m_u^3 c_\alpha c_u s_u (\mu s_\alpha + A_u c_\alpha)\Big] \\
&- \frac{g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[m_u^4 c_\alpha^2 + m_u^2 c_u^2 s_u^2 (\mu s_\alpha + A_u c_\alpha)^2 - 2 m_u^3 c_\alpha c_u s_u (\mu s_\alpha + A_u c_\alpha)\Big] \\
&- \frac{g^2}{32\pi^2 m_W^2 s_\beta^2}\Big[m_u^2 (c_u^2 - s_u^2)^2 (\mu s_\alpha + A_u c_\alpha)^2\Big] \\
&+ \frac{m_u^2}{32\pi^2 m_W^2} (\mu + A_u \cot\beta)^2 + \frac{m_u^2}{32\pi^2 m_W^2} (\mu \cot\beta - A_u)^2 \\
&+ \frac{3 g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} + \frac{3 g^2 m_u^4 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} - \frac{g^2 m_u^4 \cot^2\beta}{16\pi^2 m_W^2} - \frac{g^2 m_u^4}{16\pi^2 m_W^2} \\
= &+ \frac{2g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[- m_u^4 s_\alpha^2 - m_u^2 c_u^2 s_u^2 (\mu c_\alpha - A_u s_\alpha)^2\Big] \\
&- \frac{2g^2}{16\pi^2 m_W^2 s_\beta^2}\Big[m_u^4 c_\alpha^2 + m_u^2 c_u^2 s_u^2 (\mu s_\alpha + A_u c_\alpha)^2\Big] \\
&- \frac{g^2}{32\pi^2 m_W^2 s_\beta^2} m_u^2 (c_u^2 - s_u^2)^2 \Big[(\mu c_\alpha - A_u s_\alpha)^2 + (\mu s_\alpha + A_u c_\alpha)^2\Big] \\
&+ \frac{m_u^2}{32\pi^2 m_W^2 s_\beta^2} (\mu s_\beta + A_u c_\beta)^2 + \frac{m_u^2}{32\pi^2 m_W^2 s_\beta} (\mu c_\beta - A_u s_\beta)^2 \\
&+ \frac{2 g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \\
= &- \frac{g^2}{8\pi^2 m_W^2 s_\beta^2}\Big[m_u^4 + m_u^2 c_u^2 s_u^2 (\mu^2 + A_u^2)\Big] - \frac{g^2}{32\pi^2 m_W^2 s_\beta^2} m_u^2 (c_u^2 - s_u^2)^2 (\mu^2 + A_u^2) \\
&+ \frac{m_u^2}{32\pi^2 m_W^2 s_\beta^2} (\mu^2 + A_u^2) + \frac{g^2 m_u^4 s_\alpha^2}{8\pi^2 m_W^2 s_\beta^2} \\
= &- \frac{g^2}{32\pi^2 m_W^2 s_\beta^2} (\mu^2 + A_u^2) m_u^2 \Big[4 c_u^2 s_u^2 + (c_u^2 - s_u^2)^2 - 1\Big] \\
= &\;\; 0
\end{aligned}
$$

D.1.2 Finite Part

Let us denote the finite part of Δ_4 by Δ_4^F. We will rewrite the integrals in terms of the functions defined in eqs. C.17-C.19.

$$
\begin{aligned}
\Delta_4^F = &+\frac{g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\left[m_u^4 s_\alpha^2 + m_u^2 \frac{s_{2u}^2}{4}(\mu c_\alpha - A_u s_\alpha)^2 - m_u^3 s_\alpha s_{2u}(\mu c_\alpha - A_u s_\alpha)\right] \\
&+ \frac{g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\left[m_u^4 s_\alpha^2 + m_u^2 \frac{s_{2u}^2}{4}(\mu c_\alpha - A_u s_\alpha)^2 + m_u^3 s_\alpha s_{2u}(\mu c_\alpha - A_u s_\alpha)\right] \\
&+ \frac{2g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\left[m_u^2 \frac{c_{2u}^2}{4}(\mu c_\alpha - A_u s_\alpha)^2\right] \\
&+ \frac{g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\left[m_u^4 c_\alpha^2 + m_u^2 \frac{s_{2u}^2}{4}(\mu s_\alpha + A_u c_\alpha)^2 + m_u^3 c_\alpha s_{2u}(\mu s_\alpha + A_u c_\alpha)\right] \\
&+ \frac{g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\left[m_u^4 c_\alpha^2 + m_u^2 \frac{s_{2u}^2}{4}(\mu s_\alpha + A_u c_\alpha)^2 - m_u^3 c_\alpha s_{2u}(\mu s_\alpha + A_u c_\alpha)\right] \\
&+ \frac{2g^2}{16\pi^2 m_W^2 s_\beta^2} F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\left[m_u^2 \frac{c_{2u}^2}{4}(\mu s_\alpha + A_u c_\alpha)^2\right] \\
&- 2\cdot\frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2} F(m_A^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\beta + A_u c_\beta)^2 \\
&- 2\cdot\frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2} F(m_G^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\beta - A_u s_\beta)^2 \\
&- \frac{3g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} F(m_H^2, m_u^2, m_u^2) + \frac{g^2 m_u^4 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \\
&- \frac{3g^2 m_u^4 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} F(m_h^2, m_u^2, m_u^2) + \frac{g^2 m_u^4 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2} \\
&+ \frac{g^2 m_u^4 c_\beta^2}{16\pi^2 m_W^2 s_\beta^2} F(m_A^2, m_u^2, m_u^2) - \frac{g^2 m_u^4 c_\beta^2}{16\pi^2 m_W^2 s_\beta^2} \\
&+ \frac{g^2 m_u^4 s_\beta^2}{16\pi^2 m_W^2 s_\beta^2} F(m_G^2, m_u^2, m_u^2) - \frac{g^2 m_u^4 s_\beta^2}{16\pi^2 m_W^2 s_\beta^2} \\
= &\frac{g^2 m_u^4}{16\pi^2 m_W^2 s_\beta^2}\Big[s_\alpha^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + s_\alpha^2 F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - 3 s_\alpha^2 F(m_H^2, m_u^2, m_u^2) + c_\alpha^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ c_\alpha^2 F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - 3 c_\alpha^2 F(m_h^2, m_u^2, m_u^2) + c_\beta^2 F(m_A^2, m_u^2, m_u^2) + s_\beta^2 F(0, m_u^2, m_u^2)\Big] \\
&+ \frac{g^2 m_u^3 c_{2u} s_\alpha}{16\pi^2 m_W^2 s_\beta^2}(\mu c_\alpha - A_u s_\alpha)\left[-F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right] \\
&+ \frac{g^2 m_u^3 c_{2u} c_\alpha}{16\pi^2 m_W^2 s_\beta^2}(\mu s_\alpha + A_u c_\alpha)\left[F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right] \\
&+ \frac{g^2 m_u^2}{64\pi^2 m_W^2 s_\beta^2}\Big\{s_{2u}^2\Big[F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)(\mu c_\alpha - A_u s_\alpha)^2 + F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)(\mu c_\alpha - A_u s_\alpha)^2 \\
&+ F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)(\mu s_\alpha + A_u c_\alpha)^2 + F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)(\mu s_\alpha + A_u c_\alpha)^2\Big] \\
&+ c_{2u}^2\Big[F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\alpha - A_u s_\alpha)^2 + F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\alpha - A_u s_\alpha)^2 \\
&+ F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\alpha + A_u c_\alpha)^2 + F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\alpha + A_u c_\alpha)^2\Big] \\
&+ \Big[-F(m_A^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\beta + A_u c_\beta)^2 - F(m_A^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\beta + A_u c_\beta)^2 \\
&- F(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\beta - A_u s_\beta)^2 - F(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\beta - A_u s_\beta)^2\Big]\Big\}
\end{aligned}
$$

$$= \frac{g^2 m_u^4}{16\pi^2 m_W^2 s_\beta^2}\Bigg[s_\alpha^2\Big\{F(m_H^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)+F(m_H^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)-3F(m_H^2,m_u^2,m_u^2)\Big\}+c_\alpha^2\Big\{F(m_h^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)$$
$$+F(m_h^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)-3F(m_h^2,m_u^2,m_u^2)\Big\}+c_\beta^2 F(m_A^2,m_u^2,m_u^2)+s_\beta^2 F(0,m_u^2,m_u^2)\Bigg]$$
$$+\frac{g^2 m_u^3 c_{2u} s_\alpha}{16\pi^2 m_W^2 s_\beta^2}(\mu c_\alpha - A_u s_\alpha)\big[-F(m_H^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)+F(m_H^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)\big]$$
$$+\frac{g^2 m_u^3 c_{2u} c_\alpha}{16\pi^2 m_W^2 s_\beta^2}(\mu s_\alpha + A_u c_\alpha)\big[F(m_h^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)-F(m_h^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)\big]$$
$$+\frac{g^2 m_u^2}{64\pi^2 m_W^2 s_\beta^2}\Bigg\{s_{2u}^2\Big[(\mu c_\alpha - A_u s_\alpha)^2\big[F(m_H^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)+F(m_H^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)\big]$$
$$+(\mu s_\alpha + A_u c_\alpha)^2\big[F(m_h^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_1}^2)+F(m_h^2,m_{\tilde{u}_2}^2,m_{\tilde{u}_2}^2)\big]\Big]$$
$$+c_{2u}^2\Big[2F(m_H^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_2}^2)(\mu c_\alpha - A_u s_\alpha)^2+2F(m_h^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_2}^2)(\mu s_\alpha + A_u c_\alpha)^2\Big]$$
$$+\Big[-2F(m_A^2,m_{\tilde{u}_1}^2,m_{\tilde{u}_2}^2)(\mu s_\beta + A_u c_\beta)^2-2F(0,m_{\tilde{u}_1}^2,m_{\tilde{u}_2}^2)(\mu c_\beta - A_u s_\beta)^2\Big]\Bigg\}$$

Our expression for Δ_4^F is in full agreement with ref. [6].

D.2 Correction Δ_2

Let us calculate the part of the correction to the neutral Higgs boson mass sum rule in which the quark and squark mass parameters occur in 2nd order. Drop the i to get the self-energy. For the Z-boson, take the coefficient of $g^{\mu\nu}$ (the transverse part of the self-energy). Note that contributions from H^0, h^0, Z enter the sum with a minus, those from A and G^0 with a plus (see eq. 2.5). Remember that the self-energies are to be calculated at $q^2 = M^2$, where M is the mass of the respective particle.

$$\Delta_2 = +\frac{g^2 m_u^2 s_\alpha^2}{16\pi^2 m_W^2 s_\beta^2}\int dx\left[3\left(\frac{2}{\bar\epsilon}-\log\Delta_{uu}\right)+1\right][-x(1-x)q^2] \quad \text{from fig. F.10}$$
$$+\frac{g^2 m_u^2 c_\alpha^2}{16\pi^2 m_W^2 s_\beta^2}\int dx\left[3\left(\frac{2}{\bar\epsilon}-\log\Delta_{uu}\right)+1\right][-x(1-x)q^2] \quad \text{from fig. F.17}$$
$$-\frac{g^2 m_u^2 c_\beta^2}{16\pi^2 m_W^2 s_\beta^2}\int dx\left[3\left(\frac{2}{\bar\epsilon}-\log\Delta_{uu}\right)+1\right][-x(1-x)q^2] \quad \text{from fig. F.24}$$
$$-\frac{g^2 m_u^2 s_\beta^2}{16\pi^2 m_W^2 s_\beta^2}\int dx\left[3\left(\frac{2}{\bar\epsilon}-\log\Delta_{uu}\right)+1\right][-x(1-x)q^2] \quad \text{from fig. F.59}$$
$$-\frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta}s_\alpha c_{\alpha+\beta}\left[\frac{1}{2}c_u^2 - c_{2u}e_u s_w^2\right]\int dx\left(\frac{2}{\bar\epsilon}-\log\Delta_{\tilde{u}_1\tilde{u}_1}\right) \quad \text{from fig. F.12}$$
$$+\frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta}c_u s_u c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha)\left[\frac{1}{2}c_u^2 - c_{2u}e_u s_w^2\right]\int dx\left(\frac{2}{\bar\epsilon}-\log\Delta_{\tilde{u}_1\tilde{u}_1}\right) \quad \text{from fig. F.12}$$

$$
\begin{aligned}
&- \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} s_\alpha c_{\alpha+\beta} \left[\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from fig. F.13} \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{\alpha+\beta} (\mu c_\alpha - A_u s_\alpha) \left[\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from fig. F.13} \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{2u} c_{\alpha+\beta} \left[\frac{1}{2} - 2 e_u s_w^2\right] (\mu c_\alpha - A_u s_\alpha) \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from figs. F.14, F.11} \\
&- \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} c_\alpha s_{\alpha+\beta} \left[\frac{1}{2} c_u^2 - c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1\tilde{u}_1}\right) \quad \text{from fig. F.18} \\
&+ \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u s_{\alpha+\beta} (\mu s_\alpha + A_u c_\alpha) \left[\frac{1}{2} c_u^2 - c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1\tilde{u}_1}\right) \quad \text{from fig. F.18} \\
&+ \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} c_\alpha s_{\alpha+\beta} \left[\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from fig. F.19} \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u s_{\alpha+\beta} (\mu s_\alpha + A_u c_\alpha) \left[\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from fig. F.19} \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{2u} s_{\alpha+\beta} \left[\frac{1}{2} - 2 e_u s_w^2\right] (\mu s_\alpha + A_u c_\alpha) \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \quad \text{from figs. F.20, F.21} \\
&- \frac{g^2 m_u^2}{32\pi^2 c_w^2} \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{uu}\right) \quad \text{from fig. F.34} \\
&- \frac{g^2}{64\pi^2 c_w^2} [c_u^2 - 2 e_u s_w^2]^2 \int dx \left(\frac{2}{\bar{\epsilon}} \cdot 2 m_{\tilde{u}_1}^2 - 2 m_{\tilde{u}_1}^2 \log \Delta_{\tilde{u}_1\tilde{u}_1} + 2 m_{\tilde{u}_1}^2\right) \quad \text{from F.35} \\
&- \frac{g^2}{64\pi^2 c_w^2} [s_u^2 - 2 e_u s_w^2]^2 \int dx \left(\frac{2}{\bar{\epsilon}} \cdot 2 m_{\tilde{u}_2}^2 - 2 m_{\tilde{u}_2}^2 \log \Delta_{\tilde{u}_2\tilde{u}_2} + 2 m_{\tilde{u}_2}^2\right) \quad \text{from F.36} \\
&- \frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2 \int dx \Big(\frac{2}{\bar{\epsilon}} \cdot 2(-x(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) + m_{\tilde{u}_1}^2) + 2x(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \log \Delta_{\tilde{u}_1\tilde{u}_2} \\
&- 2 m_{\tilde{u}_1}^2 \log \Delta_{\tilde{u}_1\tilde{u}_2} + m_{\tilde{u}_1}^2 + m_{\tilde{u}_2}^2\Big) \quad \text{from F.37} \\
&- \frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2 \int dx \Big(\frac{2}{\bar{\epsilon}} \cdot 2(x(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) + m_{\tilde{u}_2}^2) - 2x(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \log \Delta_{\tilde{u}_2\tilde{u}_1} \\
&- 2 m_{\tilde{u}_2}^2 \log \Delta_{\tilde{u}_2\tilde{u}_1} + m_{\tilde{u}_1}^2 + m_{\tilde{u}_2}^2\Big) \quad \text{from F.38} \\
&+ \frac{g^2}{8\pi^2 c_w^2} \left[\frac{1}{4} c_u^2 - \frac{2}{3} s_w^2 c_u^2 + \frac{4}{9} s_w^4\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right) m_{\tilde{u}_1}^2 \quad \text{from F.39} \\
&+ \frac{g^2}{8\pi^2 c_w^2} \left[\frac{1}{4} s_u^2 - \frac{2}{3} s_w^2 s_u^2 + \frac{4}{9} s_w^4\right] \int dx \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right) m_{\tilde{u}_2}^2 \quad \text{from F.40}
\end{aligned}
$$

D.2.1 Infinite Part

Collect the coefficients of $2/\bar{\epsilon}$ in the expression for Δ_2 and denote the result by Δ_2^I. The following calculation shows that the infinite part is zero. To cancel the divergences, we used the tree level relation

$$m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_A^2 c_\beta^2 - m_Z^2 s_\beta^2 = 0, \tag{D.1}$$

which is given in appendix B.2, see eq. B.20. To understand the following calculation, it is also useful to keep in mind the relations $c_{2u} = c_u^2 - s_u^2$ and $m_W = m_Z c_w$.

$$
\begin{aligned}
\Delta_2^I = &- \frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2}\left[s_\alpha^2 \cdot 3 \cdot \frac{1}{6} m_H^2 - c_\alpha^2 \cdot 3 \cdot \frac{1}{6} m_h^2 + c_\beta^2 \cdot 3 \cdot \frac{1}{6} m_A^2 + s_\beta^2 \cdot 3 \cdot \frac{1}{6} m_G^2\right] \\
&- \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} s_\alpha c_{\alpha+\beta}\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] - \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} s_\alpha c_{\alpha+\beta}\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right] \\
&+ \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha)\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha)\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right] \\
&+ \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} c_\alpha s_{\alpha+\beta}\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] + \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} c_\alpha s_{\alpha+\beta}\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right] \\
&+ \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha)\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha)\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right] \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{2u} c_{\alpha+\beta}\left[\frac{1}{2} - 2 e_u s_w^2\right](\mu c_\alpha - A_u s_\alpha) \\
&- \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u c_{2u} s_{\alpha+\beta}\left[\frac{1}{2} - 2 e_u s_w^2\right](\mu s_\alpha + A_u c_\alpha) \\
&- \frac{g^2 m_u^2}{32\pi^2 c_w^2} - \frac{2g^2}{64\pi^2 c_w^2}[c_u^2 - 2 e_u s_w^2]^2 m_{\tilde{u}_1}^2 - \frac{2g^2}{64\pi^2 c_w^2}[s_u^2 - 2 e_u s_w^2]^2 m_{\tilde{u}_2}^2 \\
&- \frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2 2(-\frac{1}{2}(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) + m_{\tilde{u}_1}^2) - \frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2 2(\frac{1}{2}(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) + m_{\tilde{u}_2}^2) \\
&+ \frac{g^2}{8\pi^2 c_w^2}\left[\frac{1}{4}c_u^2 - \frac{2}{3}s_w^2 c_u^2 + \frac{4}{9}s_w^4\right] m_{\tilde{u}_1}^2 + \frac{g^2}{8\pi^2 c_w^2}\left[\frac{1}{4}s_u^2 - \frac{2}{3}s_w^2 s_u^2 + \frac{4}{9}s_w^4\right] m_{\tilde{u}_2}^2 \\
= &- \frac{g^2 m_u^2}{32\pi^2 m_W^2 s_\beta^2}\left[s_\alpha^2 m_H^2 - c_\alpha^2 m_h^2 + c_\beta^2 m_A^2 + s_\beta^2 m_G^2\right] - \frac{g^2 m_u^2}{32\pi^2 c_w^2} \\
&+ \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta}\left[-\frac{1}{2}s_\alpha c_{\alpha+\beta} + \frac{1}{2}c_\alpha s_{\alpha+\beta}\right] \\
&+ \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u \left\{ c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha)\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] - c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha)\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right]\right. \\
&+ s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha)\left[\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right] - s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha)\left[\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right] \\
&\left. - c_{2u} c_{\alpha+\beta}\left[\frac{1}{2} - 2 e_u s_w^2\right](\mu c_\alpha - A_u s_\alpha) - c_{2u} s_{\alpha+\beta}\left[\frac{1}{2} - 2 e_u s_w^2\right](\mu s_\alpha + A_u c_\alpha)\right\} \\
&- \frac{g^2}{32\pi^2 c_w^2} m_{\tilde{u}_1}^2 \left\{[c_u^2 - 2 e_u s_w^2]^2 + c_u^2 s_u^2 - \left[c_u^2 - \frac{8}{3}s_w^2 c_u^2 + \frac{16}{9}s_w^4\right]\right\} \\
&- \frac{g^2}{32\pi^2 c_w^2} m_{\tilde{u}_2}^2 \left\{[s_u^2 - 2 e_u s_w^2]^2 + c_u^2 s_u^2 - \left[s_u^2 - \frac{8}{3}s_w^2 s_u^2 + \frac{16}{9}s_w^4\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= - \frac{g^2 m_u^2}{32\pi^2 m_W^2 s_\beta^2} s_\beta^2 m_Z^2 - \frac{g^2 m_u^2}{32\pi^2 c_w^2} \\
&+ \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} \left[-\frac{1}{2} s_\alpha c_{\alpha+\beta} + \frac{1}{2} c_\alpha s_{\alpha+\beta} \right] \\
&+ \frac{g^2 m_u m_Z}{8\pi^2 m_W c_w s_\beta} c_u s_u \Big\{ c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha) \left[\frac{1}{2} c_u^2 - \frac{1}{2} s_u^2\right] - 2 c_{\alpha+\beta}(\mu c_\alpha - A_u s_\alpha) c_{2u} e_u s_w^2 \\
&+ s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha) \left[\frac{1}{2} c_u^2 - \frac{1}{2}\right] - 2 s_{\alpha+\beta}(\mu s_\alpha + A_u c_\alpha) c_{2u} e_u s_w^2 \\
&- c_{2u} c_{\alpha+\beta} \left[\frac{1}{2} - 2 e_u s_w^2\right] (\mu c_\alpha - A_u s_\alpha) - c_{2u} s_{\alpha+\beta} \left[\frac{1}{2} - 2 e_u s_w^2\right] (\mu s_\alpha + A_u c_\alpha) \Big\} \\
&= - \frac{g^2 m_u^2}{32\pi^2 m_W^2 s_\beta^2} s_\beta^2 m_Z^2 - \frac{g^2 m_u^2}{32\pi^2 c_w^2} + \frac{g^2 m_u^2 m_Z}{8\pi^2 m_W c_w s_\beta} \left[-\frac{1}{2} s_\alpha c_{\alpha+\beta} + \frac{1}{2} c_\alpha s_{\alpha+\beta} \right] \\
&= - \frac{g^2 m_u^2}{32\pi^2 c_w^2} - \frac{g^2 m_u^2}{32\pi^2 c_w^2} + \frac{g^2 m_u^2}{8\pi^2 c_w^2 s_\beta} \frac{1}{2} s_\beta \\
&= \; 0
\end{aligned}
$$

D.2.2 Finite Part

Let us denote the finite part of Δ_2 by Δ_2^F. We will rewrite the integrals in terms of the functions defined in eqs. C.17-C.17.

$$
\begin{aligned}
\Delta_2^F = &+ \frac{g^2 m_u^2}{8\pi^2 c_w^2 s_\beta} \Bigg[s_\alpha c_{\alpha+\beta} \left(\frac{1}{2} c_u^2 - c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + s_\alpha c_{\alpha+\beta} \left(\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&- c_\alpha s_{\alpha+\beta} \left(\frac{1}{2} c_u^2 - c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - c_\alpha s_{\alpha+\beta} \left(\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \Bigg] \\
&+ \frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2} \Bigg[3 s_\alpha^2 G(m_H^2, m_u^2, m_u^2) m_H^2 - s_\alpha^2 m_H^2 \frac{1}{6} + 3 c_\alpha^2 G(m_h^2, m_u^2, m_u^2) m_h^2 - c_\alpha^2 m_h^2 \frac{1}{6} \\
&- 3 c_\beta^2 G(m_A^2, m_u^2, m_u^2) m_A^2 + c_\beta^2 m_A^2 \frac{1}{6} - 3 s_\beta^2 G(0, m_u^2, m_u^2) \cdot 0^2 + s_\beta^2 \cdot 0^2 \cdot \frac{1}{6} \Bigg] \\
&+ \frac{g^2 m_u}{16\pi^2 c_w^2 s_\beta} (\mu c_\alpha - A_u s_\alpha) c_{\alpha+\beta} s_{2u} \Bigg[- \left(\frac{1}{2} c_u^2 - c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \left(\frac{1}{2} s_u^2 + c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + \frac{1}{2} c_{2u} \left(\frac{1}{2} - e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{1}{2} c_{2u} \left(\frac{1}{2} - e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{g^2 m_u}{16\pi^2 c_w^2 s_\beta}(\mu s_\alpha + A_u c_\alpha) s_{\alpha+\beta} s_{2u}\Bigg[- \left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + \frac{1}{2}c_{2u}\left(\frac{1}{2} - e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{1}{2}c_{2u}\left(\frac{1}{2} - e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2)\Bigg] \\
&+ \frac{g^2 m_{\tilde{u}_1}^2}{8\pi^2 c_w^2}\Bigg[\frac{1}{4}(c_u^2 - 2e_u s_w^2)^2 F(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - \frac{1}{4}(c_u^2 - 2e_u s_w^2)^2 - \frac{1}{4}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2) \\
&+ \frac{1}{4}s_u^2 c_u^2 F(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - \frac{1}{8}s_u^2 c_u^2 + \frac{1}{4}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) - \frac{1}{8}s_u^2 c_u^2 \\
&- \left(\frac{1}{4}c_u^2 - \frac{2}{3}c_u^2 s_w^2 + \frac{4}{9}s_w^4\right)\log m_{\tilde{u}_1}^2 + \left(\frac{1}{4}c_u^2 - \frac{2}{3}c_u^2 s_w^2 + \frac{4}{9}s_w^4\right)\Bigg] \\
&+ \frac{g^2 m_{\tilde{u}_2}^2}{8\pi^2 c_w^2}\Bigg[\frac{1}{4}(s_u^2 - 2e_u s_w^2)^2 F(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - \frac{1}{4}(s_u^2 - 2e_u s_w^2)^2 + \frac{1}{4}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2) \\
&+ \frac{1}{4}s_u^2 c_u^2 F(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - \frac{1}{8}s_u^2 c_u^2 - \frac{1}{4}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) - \frac{1}{8}s_u^2 c_u^2 \\
&- \left(\frac{1}{4}s_u^2 - \frac{2}{3}s_u^2 s_w^2 + \frac{4}{9}s_w^4\right)\log m_{\tilde{u}_2}^2 + \frac{1}{2}\left(\frac{1}{4}s_u^2 - \frac{2}{3}s_u^2 s_w^2 + \frac{4}{9}s_w^4\right)\Bigg] \\
&+ \frac{g^2 m_u^2}{32\pi^2 c_w^2} F(m_Z^2, m_u^2, m_u^2)
\end{aligned}
$$

The above sum can be simplified if one notes that

$$
\begin{aligned}
&-\frac{1}{4}(c_u^2 - 2e_u s_w^2)^2 - \frac{1}{8}c_u^2 s_u^2 - \frac{1}{8}c_u^2 s_u^2 + \left(\frac{1}{4}c_u^2 - \frac{2}{3}c_u^2 s_w^2 + \frac{4}{9}s_w^4\right) \\
&\quad = -\frac{1}{4}\left[c_u^4 + 4e_u^2 s_w^4 - 4c_u^2 e_u s_w^2 + c_u^2 s_u^2 - c_u^2 + \frac{8}{3}c_u^2 s_w^2 - \frac{16}{9}s_w^4\right] = -\frac{1}{4}\left[c_u^2(c_u^2 + s_u^2) - c_u^2\right] = 0.
\end{aligned}
$$

The same result holds if one replaces c_u by s_u.

To compare our result with ref. [6], it is necessary to switch arguments in $H(q^2, m_2^2, m_1^2)$. Note that this function is not symmetric in m_1, m_2:

$$
\begin{aligned}
H(q^2, m_2^2, m_1^2) &= \int_0^1 dx x \log[x m_2^2 + (1-x) m_1^2 - x(1-x) q^2] \\
&= -\int_1^0 dx (1-x) \log[(1-x) m_2^2 + x m_1^2 - x(1-x) q^2] \\
&= \int_0^1 dx (1-x) \log[x m_1^2 + (1-x) m_2^2 - x(1-x) q^2] = F(q^2, m_1^2, m_2^2) - H(q^2, m_1^2, m_2^2)
\end{aligned}
$$

As in the infinite part, we need the tree level relation

$$
m_H^2 s_\alpha^2 + m_h^2 c_\alpha^2 - m_A^2 c_\beta^2 - m_Z^2 s_\beta^2 = 0
$$

to rewrite

$$
\frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2}\left[- s_\alpha^2 m_H^2 \frac{1}{6} - c_\alpha^2 m_h^2 \frac{1}{6} + c_\beta^2 m_A^2 \frac{1}{6}\right] = -\frac{g^2 m_u^2}{96\pi^2 m_W^2 s_\beta^2} s_\beta^2 m_Z^2 = -\frac{g^2 m_u^2}{96\pi^2 c_w^2}.
$$

With these steps in mind, the finite part reads:

$$
\begin{aligned}
\Delta_2^F =& + \frac{g^2 m_u^2}{8\pi^2 c_w^2 s_\beta}\Bigg[\left(\frac{1}{2}c_u^2 - c_{2u}e_u s_w^2\right)\left(s_\alpha c_{\alpha+\beta}F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - c_\alpha s_{\alpha+\beta}F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\right) \\
&+ \left(\frac{1}{2}s_u^2 + c_{2u}e_u s_w^2\right)\left(s_\alpha c_{\alpha+\beta}F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - c_\alpha s_{\alpha+\beta}F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right)\Bigg] \\
&+ \frac{g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2}\Big[3s_\alpha^2 G(m_H^2, m_u^2, m_u^2)m_H^2 + 3c_\alpha^2 G(m_h^2, m_u^2, m_u^2)m_h^2 - 3c_\beta^2 G(m_A^2, m_u^2, m_u^2)m_A^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{g^2 m_u c_{\alpha+\beta} s_{2u}}{16\pi^2 c_w^2 s_\beta}(\mu c_\alpha - A_u s_\alpha)\Bigg[-\left(\frac{1}{2}c_u^2 - c_{2u}e_u s_w^2\right)F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \left(\frac{1}{2}s_u^2 + c_{2u}e_u s_w^2\right)F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + c_{2u}\left(\frac{1}{2} - e_u s_w^2\right)F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg] \\
&+ \frac{g^2 m_u s_{\alpha+\beta} s_{2u}}{16\pi^2 c_w^2 s_\beta}(\mu s_\alpha + A_u c_\alpha)\Bigg[-\left(\frac{1}{2}c_u^2 - c_{2u}e_u s_w^2\right)F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \left(\frac{1}{2}s_u^2 + c_{2u}e_u s_w^2\right)F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + c_{2u}\left(\frac{1}{2} - e_u s_w^2\right)F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg] \\
&+ \frac{g^2 m_{\tilde{u}_1}^2}{8\pi^2 c_w^2}\Bigg[\left(\frac{1}{2}c_u^2 - e_u s_w^2\right)^2 F(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - \left(\frac{1}{4}c_u^2 - \frac{2}{3}c_u^2 s_w^2 + \frac{4}{9}s_w^4\right)F(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \frac{1}{2}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg] \\
&+ \frac{g^2 m_{\tilde{u}_2}^2}{8\pi^2 c_w^2}\Bigg[\left(\frac{1}{2}s_u^2 - e_u s_w^2\right)^2 F(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - \left(\frac{1}{4}s_u^2 - \frac{2}{3}s_u^2 s_w^2 + \frac{4}{9}s_w^4\right)F(0, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{1}{2}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2)\Bigg] \\
&- \frac{g^2 m_u^2}{96\pi^2 c_w^2} + \frac{g^2 m_u^2}{32\pi^2 c_w^2}F(m_Z^2, m_u^2, m_u^2)
\end{aligned}
$$

Our result for Δ_2^F differs from the expression given in ref. [6]. The term

$$\frac{g^2 m_u^2}{32\pi^2 c_w^2}F(m_Z^2, m_u^2, m_u^2)$$

is absent in ref. [6]. See our discussion at the end of section 2.3.

D.3 Correction Δ_0

Let us calculate the part of the correction to the neutral Higgs boson mass sum rule in which the quark and squark mass parameters occur in 0th order. Drop the i to get the self-energy. For the Z-boson, take the coefficient of $g^{\mu\nu}$ (the transverse part of the self-energy). Note that contributions from H^0, h^0, Z enter the sum with a minus, those from A and G^0 with a plus (see eq. 2.5). Remember that the self-energies are to be calculated at $q^2 = M^2$, where M is the mass of the respective particle.

$$
\begin{aligned}
\Delta_0 = &-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2)e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_1}\right) \quad \text{from fig. F.12} \\
&-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2)e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_2 \tilde u_2}\right) \quad \text{from fig. F.13} \\
&-2\cdot\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_u^2 c_u^2 c_{\alpha+\beta}^2 \left(\frac{1}{2} - 2e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.14, F.11} \\
&-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2)e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_1}\right) \quad \text{from fig. F.18} \\
&-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2)e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_2 \tilde u_2}\right) \quad \text{from fig. F.19} \\
&-2\cdot\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_u^2 c_u^2 s_{\alpha+\beta}^2 \left(\frac{1}{2} - 2e_u s_w^2\right)^2 \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_2}\right) \quad \text{from figs. F.20, F.21} \\
&-\frac{g^2}{16\pi^2 c_w^2} \int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_2}\right)\left(-2\left[\frac{1}{2} - \frac{4}{3}s_w^2 + \frac{16}{9}s_w^4\right] x(1-x)q^2\right) \quad \text{from fig. F.34} \\
&-\frac{g^2}{64\pi^2 c_w^2}(c_u^2 - 2e_u s_w^2)\left[\int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_1}\right)(-2x(1-x)q^2) - \frac{1}{3}m_Z^2\right] \quad \text{from fig. F.35} \\
&-\frac{g^2}{64\pi^2 c_w^2}(s_u^2 - 2e_u s_w^2)\left[\int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_2 \tilde u_2}\right)(-2x(1-x)q^2) - \frac{1}{3}m_Z^2\right] \quad \text{from fig. F.36} \\
&-2\cdot\frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2\left[\int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1 \tilde u_2}\right)(-2x(1-x)q^2) - \frac{1}{3}m_Z^2\right] \quad \text{from figs. F.37, F.38}
\end{aligned}
$$

D.3.1 Infinite Part

Collect the coefficients of $2/\bar\epsilon$ in the expression for Δ_2 and denote the result by Δ_2^I. The following calculation shows that the infinite part is zero.

$$
\begin{aligned}
\Delta_0^I = &-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2)e_u s_w^2\right)^2 - \frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2)e_u s_w^2\right)^2 \\
&-\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2)e_u s_w^2\right)^2 - \frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2)e_u s_w^2\right)^2 \\
&-2\cdot\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_u^2 c_u^2 c_{\alpha+\beta}^2 \left(\frac{1}{2} - 2e_u s_w^2\right)^2 - 2\cdot\frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_u^2 c_u^2 s_{\alpha+\beta}^2 \left(\frac{1}{2} - 2e_u s_w^2\right)^2 \\
&-\frac{g^2}{64\pi^2 c_w^2}(c_u^2 - 2e_u s_w^2)\left(-2\cdot\frac{1}{6}\cdot m_Z^2\right) - \frac{g^2}{64\pi^2 c_w^2}(s_u^2 - 2e_u s_w^2)\left(-2\cdot\frac{1}{6}\cdot m_Z^2\right) \\
&-\frac{g^2}{16\pi^2 c_w^2}\left(-2\left[\frac{1}{2} - \frac{4}{3}s_w^2 + \frac{16}{9}s_w^4\right]\cdot\frac{1}{6}\cdot m_Z^2\right) - 2\cdot\frac{g^2}{64\pi^2 c_w^2} c_u^2 s_u^2\left(-2\cdot\frac{1}{6}\cdot m_Z^2\right)
\end{aligned}
$$

$$
\begin{aligned}
= &- \frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \Bigg\{ \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2) e_u s_w^2\right)^2 + \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2) e_u s_w^2\right)^2 + 2 s_u^2 c_u^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 \Bigg\} \\
&- \frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \Bigg\{ \left(\frac{1}{2}c_u^2 - (c_u^2 - s_u^2) e_u s_w^2\right)^2 + \left(\frac{1}{2}s_u^2 + (c_u^2 - s_u^2) e_u s_w^2\right)^2 + 2 s_u^2 c_u^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 \Bigg\} \\
&+ \frac{g^2}{64\pi^2 c_w^2} \frac{1}{3} m_Z^2 \Big\{ (c_u^2 - 2 e_u s_w^2)^2 + (s_u^2 - 2 e_u s_w^2)^2 + 2 c_u^2 s_u^2 \Big\} \\
&+ \frac{g^2}{16\pi^2 c_w^2} \frac{1}{3} m_Z^2 \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] \\
= &- \frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \frac{1}{2} \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] - \frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \frac{1}{2} \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] \\
&+ \frac{g^2}{64\pi^2 c_w^2} \frac{1}{3} m_Z^2 2 \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] + \frac{g^2}{16\pi^2 c_w^2} \frac{1}{3} m_Z^2 \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] \\
= \;& 0
\end{aligned}
$$

D.3.2 Finite Part

Let us denote the finite part of Δ_0 by Δ_0^F. We will rewrite the integrals in terms of the functions defined in eqs. C.17-C.17.

$$
\begin{aligned}
\Delta_0^F = &+ \frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right)^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \frac{g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right)^2 F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{2g^2}{16\pi^2 c_w^2} m_Z^2 c_{\alpha+\beta}^2 c_u^2 s_u^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right)^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \frac{g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right)^2 F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&+ \frac{2g^2}{16\pi^2 c_w^2} m_Z^2 s_{\alpha+\beta}^2 c_u^2 s_u^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) \\
&- \frac{2g^2}{16\pi^2 c_w^2} m_Z^2 \left[\frac{1}{2} - \frac{4}{3} s_w^2 + \frac{16}{9} s_w^4\right] G(m_Z^2, m_u^2, m_u^2) \\
&- \frac{g^2}{64\pi^2 c_w^2} (c_u^2 - 2 e_u s_w^2)^2 \left[2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) m_Z^2 - \frac{1}{3} m_Z^2\right] \\
&- \frac{g^2}{64\pi^2 c_w^2} (s_u^2 - 2 e_u s_w^2)^2 \left[2 G(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) m_Z^2 - \frac{1}{3} m_Z^2\right] \\
&- \frac{2g^2}{64\pi^2 c_w^2} c_u^2 s_u^2 \left[2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) m_Z^2 - \frac{1}{3} m_Z^2\right]
\end{aligned}
$$

$$
\begin{aligned}
&= + \frac{g^2 m_Z^2}{16\pi^2 c_w^2} \Bigg[\left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\right) \\
&\quad + \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right) \\
&\quad + s_{2u}^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\right) \Bigg] \\
&\quad - \frac{g^2 m_Z^2}{16\pi^2 c_w^2} \Bigg[2\left(\frac{1}{2} - \frac{4}{3}s_w^2 + \frac{16}{9}s_w^4\right) G(m_Z^2, m_u^2, m_u^2) + \frac{1}{2}(c_u^2 - 2 e_u s_w^2)^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&\quad + \frac{1}{2}(s_u^2 - 2 e_u s_w^2)^2 G(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + c_u^2 s_u^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) \\
&\quad + \left(-\frac{1}{12}(c_u^2 - 2 e_u s_w^2)^2 - \frac{1}{12}(s_u^2 - 2 e_u s_w^2)^2 - \frac{1}{12}c_u^2 s_u^2 - \frac{1}{12}c_u^2 s_u^2\right) \Bigg]
\end{aligned}
$$

To compare our result with the literature, use the relations

$$
\begin{aligned}
c_{2u} &= c_u^2 - s_u^2, \\
\frac{1}{4}(c_u^2 - 2 e_u s_w^2)^2 &= \left(\frac{1}{2}c_u^2 - e_u s_w^2 c_u^2 - e_u s_w^2 s_u^2\right)^2 = \left(c_u^2\left(-\frac{1}{2} + e_u s_w^2\right) + e_u s_w^2 s_u^2\right)^2, \\
(c_u^2 - 2 e_u s_w^2)^2 + (s_u^2 - 2 e_u s_w^2)^2 + 2 c_u^2 s_u^2 &= (c_u^2 + s_u^2)^2 - 4 e_u s_w^2 (c_u^2 + s_u^2) + 8 e_u^2 s_w^4 = 1 - 4 e_u s_w^2 + 8 e_u^2 s_w^4.
\end{aligned}
$$

With these equations in mind, Δ_0^F can be cast into the form:

$$
\begin{aligned}
\Delta_0^F &= + \frac{g^2 m_Z^2}{16\pi^2 c_w^2} \Bigg[\left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\right) \\
&\quad + \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right) \\
&\quad + s_{2u}^2 \left(\frac{1}{2} - 2 e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\right) \Bigg] \\
&\quad - \frac{g^2 m_Z^2}{16\pi^2 c_w^2} \Bigg[4\left(\left(-\frac{1}{2} - e_u s_w^2\right)^2 + (e_u s_w^2)^2\right) G(m_Z^2, m_u^2, m_u^2) \\
&\quad + \left(c_u^2\left(-\frac{1}{2} + e_u s_w^2\right) + e_u s_w^2 s_u^2\right)^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&\quad + \left(s_u^2\left(-\frac{1}{2} + e_u s_w^2\right) + e_u s_w^2 c_u^2\right)^2 G(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&\quad + c_u^2 s_u^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) - \frac{1}{12}(1 - 4 e_u s_w^2 + 8 e_u^2 s_w^4) \Bigg]
\end{aligned}
$$

Our result for Δ_0^F differs from the expression given in ref. [6]. The term

$$
-\frac{1}{12}(1 - 4 e_u s_w^2 + 8 e_u^2 s_w^4)
$$

is absent in ref. [6]. See our discussion at the end of section 2.3.

D.4 Final Result

We summarize the result of the previous calculations for convenience. The correction Δ to the neutral Higgs boson mass sum rule is divided into 3 pieces

$$\Delta = \Delta_4 + \Delta_2 + \Delta_0,$$

where Δ_n, $n = 0, 2, 4$, are given by:

$$\begin{aligned}
\Delta_4 = {} & \frac{N_c g^2 m_u^4}{16\pi^2 m_W^2 s_\beta^2}\Bigg[s_\alpha^2 \Big\{ F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - 3F(m_H^2, m_u^2, m_u^2)\Big\} + c_\alpha^2 \Big\{ F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
& + F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - 3F(m_h^2, m_u^2, m_u^2)\Big\} + c_\beta^2 F(m_A^2, m_u^2, m_u^2) + s_\beta^2 F(0, m_u^2, m_u^2)\Bigg] \\
& + \frac{N_c g^2 m_u^3 c_{2u} s_\alpha}{16\pi^2 m_W^2 s_\beta^2} (\mu c_\alpha - A_u s_\alpha) \big[- F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\big] \\
& + \frac{N_c g^2 m_u^3 c_{2u} c_\alpha}{16\pi^2 m_W^2 s_\beta^2} (\mu s_\alpha + A_u c_\alpha) \big[F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\big] \\
& + \frac{N_c g^2 m_u^2}{64\pi^2 m_W^2 s_\beta^2} \Bigg\{ s_{2u}^2 \Bigg[(\mu c_\alpha - A_u s_\alpha)^2 \big[F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\big] \\
& + (\mu s_\alpha + A_u c_\alpha)^2 \big[F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\big]\Bigg] \\
& + c_{2u}^2 \Bigg[2F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\alpha - A_u s_\alpha)^2 + 2F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\alpha + A_u c_\alpha)^2\Bigg] \\
& + \Bigg[- 2F(m_A^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu s_\beta + A_u c_\beta)^2 - 2F(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)(\mu c_\beta - A_u s_\beta)^2\Bigg]\Bigg\}
\end{aligned}$$

$$\begin{aligned}
\Delta_2 = {} & + \frac{N_c g^2 m_u^2}{8\pi^2 c_w^2 s_\beta}\Bigg[\left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right) \Big(s_\alpha c_{\alpha+\beta} F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - c_\alpha s_{\alpha+\beta} F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\Big) \\
& + \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right) \Big(s_\alpha c_{\alpha+\beta} F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - c_\alpha s_{\alpha+\beta} F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\Big)\Bigg] \\
& + \frac{N_c g^2 m_u^2}{16\pi^2 m_W^2 s_\beta^2}\Big[3 s_\alpha^2 G(m_H^2, m_u^2, m_u^2) m_H^2 + 3 c_\alpha^2 G(m_h^2, m_u^2, m_u^2) m_h^2 - 3 c_\beta^2 G(m_A^2, m_u^2, m_u^2) m_A^2\Big] \\
& + \frac{N_c g^2 m_u c_{\alpha+\beta} s_{2u}}{16\pi^2 c_w^2 s_\beta} (\mu c_\alpha - A_u s_\alpha) \Bigg[- \left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
& + \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + c_{2u}\left(\frac{1}{2} - e_u s_w^2\right) F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg] \\
& + \frac{N_c g^2 m_u s_{\alpha+\beta} s_{2u}}{16\pi^2 c_w^2 s_\beta} (\mu s_\alpha + A_u c_\alpha) \Bigg[- \left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
& + \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + c_{2u}\left(\frac{1}{2} - e_u s_w^2\right) F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{N_c g^2 m_{\tilde{u}_1}^2}{8\pi^2 c_w^2}\Bigg[\left(\frac{1}{2}c_u^2 - e_u s_w^2\right)^2 F(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) - \left(\frac{1}{4}c_u^2 - \frac{2}{3}c_u^2 s_w^2 + \frac{4}{9}s_w^4\right) F(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&\quad + \frac{1}{2}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\Bigg] \\
&+\frac{N_c g^2 m_{\tilde{u}_2}^2}{8\pi^2 c_w^2}\Bigg[\left(\frac{1}{2}s_u^2 - e_u s_w^2\right)^2 F(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) - \left(\frac{1}{4}s_u^2 - \frac{2}{3}s_u^2 s_w^2 + \frac{4}{9}s_w^4\right) F(0, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&\quad + \frac{1}{2}s_u^2 c_u^2 H(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2)\Bigg] \\
&- \frac{N_c g^2 m_u^2}{96\pi^2 c_w^2} + \frac{N_c g^2 m_u^2}{32\pi^2 c_w^2} F(m_Z^2, m_u^2, m_u^2)
\end{aligned}
$$

$$
\begin{aligned}
\Delta_0 = &+\frac{N_c g^2 m_Z^2}{16\pi^2 c_w^2}\Bigg[\left(\frac{1}{2}c_u^2 - c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2)\right) \\
&+ \left(\frac{1}{2}s_u^2 + c_{2u} e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2)\right) \\
&+ s_{2u}^2 \left(\frac{1}{2} - 2e_u s_w^2\right)^2 \left(c_{\alpha+\beta}^2 F(m_H^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) + s_{\alpha+\beta}^2 F(m_h^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2)\right)\Bigg] \\
&- \frac{N_c g^2 m_Z^2}{16\pi^2 c_w^2}\Bigg[4\left(\left(-\frac{1}{2} - e_u s_w^2\right)^2 + (e_u s_w^2)^2\right) G(m_Z^2, m_u^2, m_u^2) \\
&+ \left(c_u^2\left(-\frac{1}{2} + e_u s_w^2\right) + e_u s_w^2 s_u^2\right)^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_1}^2) \\
&+ \left(s_u^2\left(-\frac{1}{2} + e_u s_w^2\right) + e_u s_w^2 c_u^2\right)^2 G(m_Z^2, m_{\tilde{u}_2}^2, m_{\tilde{u}_2}^2) \\
&+ c_u^2 s_u^2 G(m_Z^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) - \frac{1}{12}(1 - 4e_u s_w^2 + 8e_u^2 s_w^4)\Bigg]
\end{aligned}
$$

The factor $N_c = 3$ in the expressions for Δ_n above is for color. The result for down quarks and down squarks is obtained by substituting

$$
\begin{gathered}
\theta_u \to \theta_d, \quad m_u \to m_d, \quad m_{\tilde{u}_i} \to m_{\tilde{d}_i}, \quad e_u \to e_d, \quad T_3 = \frac{1}{2} \to T_3 = -\frac{1}{2}, \\
\sin\beta \to \cos\beta, \quad \cos\beta \to \sin\beta, \quad \sin\alpha \to \cos\alpha, \quad \cos\alpha \to \sin\alpha, \\
\sin(\alpha+\beta) \to -\sin(\alpha+\beta), \quad \cos(\alpha+\beta) \to \cos(\alpha+\beta)
\end{gathered}
$$

in the expressions for Δ_4, Δ_2, and Δ_0.

Appendix E

Details of the Calculation: The Charged Higgs Boson Mass Sum Rule

In this appendix, we consequently use first family representation. For the particle spectrum and naming conventions, the reader is referred to section B.1.1. The abbreviations and symbols we used in the calculations are explained in section A.

E.1 Cancellation of the Feynman Graphs involving Goldstone Bosons

We will show by an explicit calculation that the Goldstone boson self-energy graphs do not contribute to the correction to the charged Higgs mass sum rule, because

$$\Pi_{G^\pm G^\pm}(0) - \Pi_{G^0 G^0}(0) = 0. \tag{E.1}$$

The calculations are divided into two parts. We will show that eq. E.1 is valid separately for quarks running in the loops and for squarks running in the loops. Note that we calculate the self-energy for $q^2 = 0$, so

$$\Delta_{ud} = -x(1-x)q^2 - x(m_u^2 - m_d^2) + m_u^2$$

simplifies to

$$\Delta_{ud} = -x(m_u^2 - m_d^2) + m_u^2, \qquad \Delta_{uu} = m_u^2, \qquad \Delta_{dd} = m_d^2.$$

E.1.1 Quarks running in the Loops

Let $\mathbb{X}$ denote the part of eq. E.1 originating from quarks in the loops. Consider the graphs of figs. F.50, F.59, F.60. Drop the i to get the self-energy and cancel the factor $g^2/(16\pi^2 m_W^2)$ which is irrelevant, because we just want to show that $\mathbb{X}$ is zero. Note that contributions from $G^\pm$-graphs enter $\mathbb{X}$ with a plus, those from G^0-graphs with a minus.

$$
\begin{aligned}
\mathbb{X} = &-\frac{1}{2}\int dx\,\left\{\frac{2}{\bar\epsilon}\left[4(m_u^2+m_d^2)\Delta_{ud}-2(m_u^2+m_d^2)x(1-x)q^2+m_um_d(-4m_um_d)\right]\right.\\
&\left.+2(m_u^2+m_d^2)(1-2\log\Delta_{ud})\Delta_{ud}+\left[2(m_u^2+m_d^2)x(1-x)q^2-m_um_d(-4m_um_d)\right]\log\Delta_{ud}\right\}\\
&+m_u^2\int dx\left\{\left(\frac{2}{\bar\epsilon}-\log\Delta_{uu}\right)(3\Delta_{uu}-2m_u^2)+\Delta_{uu}\right\}\\
&+m_d^2\int dx\left\{\left(\frac{2}{\bar\epsilon}-\log\Delta_{dd}\right)(3\Delta_{dd}-2m_d^2)+\Delta_{dd}\right\}\\
=&-\int dx\left\{\frac{2}{\bar\epsilon}\Big[2(m_u^2+m_d^2)\Delta_{ud}-2m_u^2m_d^2\Big]+(m_u^2+m_d^2)(1-2\log\Delta_{ud})\Delta_{ud}+2m_u^2m_d^2\log\Delta_{ud}\right.\\
&-\frac{2}{\bar\epsilon}\Big(3\Delta_{uu}-2m_u^2\Big)m_u^2+\log\Delta_{uu}\Big(3\Delta_{uu}-2m_u^2\Big)m_u^2-\Delta_{uu}m_u^2\\
&\left.-\frac{2}{\bar\epsilon}\Big(3\Delta_{dd}-2m_d^2\Big)m_d^2+\log\Delta_{dd}\Big(3\Delta_{dd}-2m_d^2\Big)m_d^2-\Delta_{dd}m_d^2\right\}
\end{aligned}
$$

We show that the finite and the infinite part of $\mathbb{X}$ is zero. Let's consider the infinite part first.

Infinite Part

$$
\begin{aligned}
&\int dx\frac{2}{\bar\epsilon}\Big\{2(m_u^2+m_d^2)\Delta_{ud}-2m_u^2m_d^2-(3\Delta_{uu}-2m_u^2)m_u^2-(3\Delta_{dd}-2m_d^2)m_d^2\Big\}\\
&=\int dx\frac{2}{\bar\epsilon}\Big\{2(m_u^2+m_d^2)[-x(m_u^2-m_d^2)+m_u^2]-2m_u^2m_d^2-(3m_u^2-2m_u^2)m_u^2-(3m_d^2-2m_d^2)m_d^2\Big\}\\
&=\frac{2}{\bar\epsilon}\Big\{2(m_u^2+m_d^2)\left(-\frac{1}{2}(m_u^2-m_d^2)+m_u^2\right)-2m_u^2m_d^2-m_u^4-m_d^4\Big\}\\
&=\frac{2}{\bar\epsilon}\Big\{-(m_u^2+m_d^2)(m_u^2-m_d^2)+2(m_u^2+m_d^2)m_u^2-2m_u^2m_d^2-m_u^4-m_d^4\Big\}\\
&=\frac{2}{\bar\epsilon}\Big\{-(m_u^4-m_d^4)+2m_u^4+2m_u^2m_d^2-2m_u^2m_d^2-m_u^4-m_d^4\Big\}\\
&=0
\end{aligned}
$$

Next, consider the finite part of $\mathbb{X}$.

Finite Part

$$
\begin{aligned}
&\int dx\Big\{(m_u^2+m_d^2)(1-2\log\Delta_{ud})\Delta_{ud}+2m_u^2m_d^2\log\Delta_{ud}+\log\Delta_{uu}(3\Delta_{uu}-2m_u^2)m_u^2-\Delta_{uu}m_u^2\\
&\quad+\log\Delta_{dd}(3\Delta_{dd}-2m_d^2)m_d^2-\Delta_{dd}m_d^2\Big\}\\
&=\int dx\Big\{(m_u^2+m_d^2)\Delta_{ud}-2(m_u^2+m_d^2)\Delta_{ud}\log\Delta_{ud}+2m_u^2m_d^2\log\Delta_{ud}+m_u^4\log m_u^2-m_u^4\\
&\quad+m_d^4\log m_d^2-m_d^4\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&= (m_u^2 + m_d^2) \int dx[-x(m_u^2 - m_d^2) + m_u^2] - 2(m_u^2 + m_d^2) \int dx \Delta_{ud} \log \Delta_{ud} + 2m_u^2 m_d^2 \int dx \log \Delta_{ud} \\
&\quad + m_u^4 \log m_u^2 - m_u^4 + m_d^4 \log m_d^2 - m_d^4 \\
&= (m_u^2 + m_d^2) \left(-\frac{1}{2}(m_u^2 - m_d^2) + m_u^2 \right) - 2(m_u^2 + m_d^2) \int dx \Delta_{ud} (\log \Delta_{ud} - 1) \\
&\quad - 2(m_u^2 + m_d^2) \int dx \Delta_{ud} + 2m_u^2 m_d^2 \int dx \log \Delta_{ud} - m_u^2(m_u^2 - m_u^2 \log m_u^2) - m_d^2(m_d^2 - m_d^2 \log m_d^2) \\
&= (m_u^2 + m_d^2) \left(\frac{1}{2}(m_u^2 + m_d^2) \right) + 4(m_u^2 + m_d^2) B_{22}(0, m_u^2, m_d^2) - 2(m_u^2 + m_d^2) \left(-\frac{1}{2}(m_u^2 - m_d^2) + m_u^2 \right) \\
&\quad - 2m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) - m_u^2 A(m_u^2) - m_d^2 A(m_d^2) \qquad \text{(see eqs. C.12, C.13, C.16)} \\
&= \frac{1}{2}(m_u^2 + m_d^2)^2 + 4m_u^2 B_{22}(0, m_u^2, m_d^2) + 4m_d^2 B_{22}(0, m_u^2, m_d^2) - (m_u^2 + m_d^2)^2 - 2m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) \\
&\quad - m_u^2 A(m_u^2) - m_d^2 A(m_d^2) \\
&= \frac{1}{2}(m_u^2 + m_d^2)^2 + m_u^2 \Big\{ A(m_u^2) + m_d^2 B_0(0, m_u^2, m_d^2) + \frac{1}{2}(m_u^2 + m_d^2) \Big\} \\
&\quad + m_d^2 \Big\{ A(m_d^2) + m_u^2 B_0(0, m_u^2, m_d^2) + \frac{1}{2}(m_u^2 + m_d^2) \Big\} \\
&\quad - (m_u^2 + m_d^2)^2 - 2m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) - m_u^2 A(m_u^2) - m_d^2 A(m_d^2) \qquad \text{(see eqs. C.7, C.8)} \\
&= \frac{1}{2}(m_u^2 + m_d^2)^2 + m_u^2 A(m_u^2) + m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) + \frac{1}{2} m_u^2 (m_u^2 + m_d^2) + m_d^2 A(m_d^2) + m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) \\
&\quad + \frac{1}{2} m_d^2 (m_u^2 + m_d^2) - (m_u^2 + m_d^2)^2 - 2m_u^2 m_d^2 B_0(0, m_u^2, m_d^2) - m_u^2 A(m_u^2) - m_d^2 A(m_d^2) \\
&= \frac{1}{2}(m_u^2 + m_d^2)^2 + \frac{1}{2} m_u^2 (m_u^2 + m_d^2) + \frac{1}{2} m_d^2 (m_u^2 + m_d^2) - (m_u^2 + m_d^2)^2 \\
&= \frac{1}{2}(m_u^2 + m_d^2)^2 + \frac{1}{2}(m_u^2 + m_d^2)(m_u^2 + m_d^2) - (m_u^2 + m_d^2)^2 \\
&= 0
\end{aligned}
$$

E.1.2 Squarks running in the Loop

Let $\mathbb{X}$ denote the part of eq. E.1 originating from squarks in the loops. To make the calculations more transparent, I will sum over the seagull-type graphs and the rest separately.
To show that the result is zero, I use another approach now. The infinities are "absorbed" into the definitions of the Passarino-Veltman functions (see chapter C). Once I have shown that the result is zero, it is clear that this is true both for the finite and the infinite part, because to do so, I only use algebraic relations which relate the Passarino-Veltman functions to each other, and these relations are valid for the loop integrals in which the divergence has not yet been removed.

Sum over the Seagull Graphs

Let $\mathbb{Y}$ denote the sum over the seagull graphs, where terms arising from $G^{\pm}$-loops enter the sum with a plus, those from G^0-loops with a minus. Drop the factor $ig^2/(32\pi^2 m_W^2)$ to simplify the algebra. By inspection one sees that there are strong cancellations between the graphs of figures F.55, F.56, F.57, F.58 and those of figures F.65, F.66, F.67, F.68. We have written down only the terms which do not cancel at one glance.

$$
\begin{aligned}
\mathbb{Y} = &-m_W^2 \left\{ \frac{m_u^2}{m_W^2} \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1 \right) m_{\tilde{u}_1}^2 + \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1 \right) m_{\tilde{u}_2}^2 + \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1 \right) m_{\tilde{d}_1}^2 \right. \\
&+ \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1 \right) m_{\tilde{d}_2}^2 + \left(c_u^2 c_{2\beta} - \frac{m_d^2}{m_W^2} c_u^2 - \frac{m_u^2}{m_W^2} s_u^2 \right) \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1 \right) m_{\tilde{u}_1}^2 \\
&+ \left(s_u^2 c_{2\beta} - \frac{m_d^2}{m_W^2} s_u^2 - \frac{m_u^2}{m_W^2} c_u^2 \right) \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1 \right) m_{\tilde{u}_2}^2 \\
&+ \left(-c_d^2 c_{2\beta} - \frac{m_u^2}{m_W^2} c_d^2 - \frac{m_d^2}{m_W^2} s_d^2 \right) \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1 \right) m_{\tilde{d}_1}^2 \\
&\left. + \left(-s_d^2 c_{2\beta} - \frac{m_u^2}{m_W^2} s_d^2 - \frac{m_d^2}{m_W^2} c_d^2 \right) \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1 \right) m_{\tilde{d}_2}^2 \right\} \\
= &-m_W^2 \left\{ A(m_{\tilde{u}_1}^2) \left[\frac{m_u^2}{m_W^2} - \frac{m_d^2}{m_W^2} c_u^2 - \frac{m_u^2}{m_W^2} s_u^2 + c_u^2 c_{2\beta} \right] + A(m_{\tilde{u}_2}^2) \left[\frac{m_u^2}{m_W^2} - \frac{m_d^2}{m_W^2} s_u^2 - \frac{m_u^2}{m_W^2} c_u^2 + s_u^2 c_{2\beta} \right] \right. \\
&\left. + A(m_{\tilde{d}_1}^2) \left[\frac{m_d^2}{m_W^2} - \frac{m_u^2}{m_W^2} c_d^2 - \frac{m_d^2}{m_W^2} s_d^2 - c_d^2 c_{2\beta} \right] + A(m_{\tilde{d}_2}^2) \left[\frac{m_d^2}{m_W^2} - \frac{m_u^2}{m_W^2} s_d^2 - \frac{m_d^2}{m_W^2} c_d^2 - s_d^2 c_{2\beta} \right] \right\} \\
= &-A(m_{\tilde{u}_1}^2) \left[m_u^2 - m_d^2 c_u^2 - m_u^2 s_u^2 + c_u^2 m_W^2 c_{2\beta} \right] - A(m_{\tilde{u}_2}^2) \left[m_u^2 - m_d^2 s_u^2 - m_u^2 c_u^2 + s_u^2 m_W^2 c_{2\beta} \right] \\
&- A(m_{\tilde{d}_1}^2) \left[m_d^2 - m_u^2 c_d^2 - m_d^2 s_d^2 - c_d^2 m_W^2 c_{2\beta} \right] - A(m_{\tilde{d}_2}^2) \left[m_d^2 - m_u^2 s_d^2 - m_d^2 c_d^2 - s_d^2 m_W^2 c_{2\beta} \right]
\end{aligned}
$$

From the first line to the second, we used eq. C.12.

Sum over the other Graphs

Let $\mathbb{Z}$ denote the sum over the graphs of figures F.51, F.52, F.53, F.54, F.61, F.62, F.63, F.64, where terms arising from $G^{\pm}$-loops enter the sum with a plus, those from G^0-loops with a minus. Drop the factor $ig^2/(32\pi^2 m_W^2)$ to simplify the algebra and to be consistent with the previous calculation.

$$
\begin{aligned}
\mathbb{Z} = &\Big[+ c_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2 \right] + c_u s_d m_d (\mu \tan\beta - A_d) - s_u c_d m_u (\mu \cot\beta - A_u) \Big]^2 B_0(0, m_{\tilde{u}_1}^2, m_{\tilde{d}_1}^2) \\
+ &\Big[- c_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2 \right] + c_u c_d m_d (\mu \tan\beta - A_d) + s_u s_d m_u (\mu \cot\beta - A_u) \Big]^2 B_0(0, m_{\tilde{u}_1}^2, m_{\tilde{d}_2}^2) \\
+ &\Big[- s_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2 \right] - s_u s_d m_d (\mu \tan\beta - A_d) - c_u c_d m_u (\mu \cot\beta - A_u) \Big]^2 B_0(0, m_{\tilde{u}_2}^2, m_{\tilde{d}_1}^2) \\
+ &\Big[+ s_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2 \right] - s_u c_d m_d (\mu \tan\beta - A_d) + c_u s_d m_u (\mu \cot\beta - A_u) \Big]^2 B_0(0, m_{\tilde{u}_2}^2, m_{\tilde{d}_2}^2) \\
- &\frac{1}{2} m_u^2 (\mu \cot\beta - A_u)^2 B_0(0, m_{\tilde{u}_1}^2, m_{\tilde{u}_2}^2) - \frac{1}{2} m_u^2 (\mu \cot\beta - A_u)^2 B_0(0, m_{\tilde{u}_2}^2, m_{\tilde{u}_1}^2) \\
- &\frac{1}{2} m_d^2 (\mu \tan\beta - A_d)^2 B_0(0, m_{\tilde{d}_1}^2, m_{\tilde{d}_2}^2) - \frac{1}{2} m_d^2 (\mu \tan\beta - A_d)^2 B_0(0, m_{\tilde{d}_2}^2, m_{\tilde{d}_1}^2)
\end{aligned}
$$

The coefficients can be simplified. Let's do the calculation for the first one. From the first line to the second, we used eq. B.21.

$$
\begin{aligned}
&\Big[c_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] + c_u s_d m_d(\mu \tan\beta - A_d) - s_u c_d m_u(\mu \cot\beta - A_u)\Big]^2 \\
&\quad = \Bigg(c_u c_d \left[c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2\right] - c_u s_d m_d \left[\frac{s_{2d}}{2m_d}(m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2)\right] \\
&\qquad + s_u c_d m_u \left[\frac{s_{2u}}{2m_u}(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2)\right]\Bigg)^2 \\
&\quad = c_u^2 c_d^2 \left(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - s_d^2(m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2) + s_u^2(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2)\right)^2 \\
&\quad = c_u^2 c_d^2 \left(m_{\tilde{u}_1}^2[c_u^2 + s_u^2] + m_{\tilde{u}_2}^2[s_u^2 - s_u^2] + m_{\tilde{d}_1}^2[-c_d^2 - s_d^2] + m_{\tilde{d}_2}^2[-s_d^2 + s_d^2]\right) \\
&\quad = c_u^2 c_d^2 \left(m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2\right)^2
\end{aligned}
$$

Completely analogous calculations yield:

$$
\begin{aligned}
\Big[- c_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] + c_u c_d m_d(\mu \tan\beta - A_d) + s_u s_d m_u(\mu \cot\beta - A_u)\Big]^2 &= c_u^2 s_d^2 \left(m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2\right)^2 \\
\Big[- s_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] - s_u s_d m_d(\mu \tan\beta - A_d) - c_u c_d m_u(\mu \cot\beta - A_u)\Big]^2 &= s_u^2 c_d^2 \left(m_{\tilde{u}_2}^2 - m_{\tilde{d}_1}^2\right)^2 \\
\Big[+ s_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] - s_u c_d m_d(\mu \tan\beta - A_d) + c_u s_d m_u(\mu \cot\beta - A_u)\Big]^2 &= s_u^2 s_d^2 \left(m_{\tilde{u}_2}^2 - m_{\tilde{d}_2}^2\right)^2
\end{aligned}
$$

Substituting these results in the expression for $\mathbb{Z}$, using eq. B.26 from the first to the second line and eq. C.10 from the second to the third gives:

$$
\begin{aligned}
\mathbb{Z} &= c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{d}_1}) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{d}_2}) \\
&\quad + s_u^2 c_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_1}^2)^2 B_0(m_{\tilde{u}_2}, m_{\tilde{d}_1}) + s_u^2 s_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_2}^2)^2 B_0(m_{\tilde{u}_2}, m_{\tilde{d}_2}) \\
&\quad - m_u^2 (\mu \cot\beta - A_u)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{u}_2}) - m_d^2 (\mu \tan\beta - A_d)^2 B_0(m_{\tilde{d}_1}, m_{\tilde{d}_2}) \\
&= c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{d}_1}) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{d}_2}) \\
&\quad + s_u^2 c_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_1}^2)^2 B_0(m_{\tilde{u}_2}, m_{\tilde{d}_1}) + s_u^2 s_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_2}^2)^2 B_0(m_{\tilde{u}_2}, m_{\tilde{d}_2}) \\
&\quad - m_u^2 \left(\frac{s_{2u}}{2m_u}(m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2)\right)^2 B_0(m_{\tilde{u}_1}, m_{\tilde{u}_2}) - m_d^2 \left(\frac{s_{2d}}{2m_d}(m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2)\right)^2 B_0(m_{\tilde{d}_1}, m_{\tilde{d}_2}) \\
&= c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2)\left[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{d}_1}^2)\right] + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2)\left[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{d}_2}^2)\right] \\
&\quad + s_u^2 c_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_1}^2)\left[A(m_{\tilde{u}_2}^2) - A(m_{\tilde{d}_1}^2)\right] + s_u^2 s_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_2}^2)\left[A(m_{\tilde{u}_2}^2) - A(m_{\tilde{d}_2}^2)\right] \\
&\quad - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2)\left[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{u}_2}^2)\right] - s_d^2 c_d^2 (m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2)\left[A(m_{\tilde{d}_1}^2) - A(m_{\tilde{d}_2}^2)\right]
\end{aligned}
$$

Now add up all contributions:

$$
\begin{aligned}
\mathbb{X} &= \mathbb{Y} + \mathbb{Z} \\
&= -A(m_{\tilde{u}_1}^2)\left[m_u^2 - m_d^2 c_u^2 - m_u^2 s_u^2 + c_u^2 m_W^2 c_{2\beta}\right] - A(m_{\tilde{u}_2}^2)\left[m_u^2 - m_d^2 s_u^2 - m_u^2 c_u^2 + s_u^2 m_W^2 c_{2\beta}\right] \\
&\quad - A(m_{\tilde{d}_1}^2)\left[m_d^2 - m_u^2 c_d^2 - m_d^2 s_d^2 - c_d^2 m_W^2 c_{2\beta}\right] - A(m_{\tilde{d}_2}^2)\left[m_d^2 - m_u^2 s_d^2 - m_d^2 c_d^2 - s_d^2 m_W^2 c_{2\beta}\right] \\
&\quad + c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2)\Big[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{d}_1}^2)\Big] + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2)\Big[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{d}_2}^2)\Big] \\
&\quad + s_u^2 c_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_1}^2)\Big[A(m_{\tilde{u}_2}^2) - A(m_{\tilde{d}_1}^2)\Big] + s_u^2 s_d^2 (m_{\tilde{u}_2}^2 - m_{\tilde{d}_2}^2)\Big[A(m_{\tilde{u}_2}^2) - A(m_{\tilde{d}_2}^2)\Big] \\
&\quad - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2)\Big[A(m_{\tilde{u}_1}^2) - A(m_{\tilde{u}_2}^2)\Big] - s_d^2 c_d^2 (m_{\tilde{d}_1}^2 - m_{\tilde{d}_2}^2)\Big[A(m_{\tilde{d}_1}^2) - A(m_{\tilde{d}_2}^2)\Big]
\end{aligned}
$$

Collect the terms in front of $A(m_{\tilde{u}_1})$ and use eq. B.21 from the third line to the fourth:

$$
\begin{aligned}
&- m_u^2 + m_d^2 c_u^2 + m_u^2 s_u^2 - c_u^2 m_W^2 c_{2\beta} + c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2) - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \\
&= (-m_u^2 + m_d^2 c_u^2 + m_u^2 s_u^2 - c_u^2 m_W^2 c_{2\beta}) + c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2) - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \\
&= -c_u^2 (m_u^2 - m_d^2 + m_W^2 c_{2\beta}) + c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2) - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \\
&= -c_u^2 (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2) + c_u^2 c_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_1}^2) + c_u^2 s_d^2 (m_{\tilde{u}_1}^2 - m_{\tilde{d}_2}^2) - s_u^2 c_u^2 (m_{\tilde{u}_1}^2 - m_{\tilde{u}_2}^2) \\
&= m_{\tilde{u}_1}^2 [-c_u^4 + c_u^2 c_d^2 + c_u^2 s_d^2 - s_u^2 c_u^2] + m_{\tilde{u}_2}^2 [-c_u^2 s_u^2 + s_u^2 c_u^2] + m_{\tilde{d}_1}^2 [c_u^2 c_d^2 - c_u^2 c_d^2] + m_{\tilde{d}_2}^2 [c_u^2 s_d^2 - c_u^2 s_d^2] \\
&= 0
\end{aligned}
$$

Completely analogous calculations yield zero for the coefficients of $A(m_{\tilde{u}_2})$, $A(m_{\tilde{d}_1})$, $A(m_{\tilde{d}_2})$, so as a result, $\mathbb{X}$ is zero.

E.2 Cancellation of Divergences

To prove the cancellation of divergences, collect the coefficient of $2/\bar{\epsilon}$ from all the graphs which contribute to the self-energies in eq. 3.3. Let $\mathbb{X}$ denote the sum of the coefficients. Note that the coefficients from A-boson-loops enter the sum with a plus, those from the $H^{\pm}$- and $W^{\pm}$-loops with a minus. Drop the i to get the self-energy. We want to prove that the sum is zero, so we drop the factor $g^2/(32\pi^2 m_W^2)$ as well. In the following calculations, I will integrate over Δ_{12} without mentioning it explicitly:

$$
\begin{aligned}
\int dx \Delta_{12} &= \int dx \big[- x(1-x)q^2 - x(m_1^2 - m_2^2) + m_1^2 \big] = \left[-\frac{1}{2}x^2 q^2 + \frac{1}{3}x^3 q^2 - \frac{1}{2}x^2 (m_1^2 - m_2^2) + m_1^2 \right]\Bigg|_0^1 \\
&= \left[-\frac{1}{2}q^2 + \frac{1}{3}q^2 - \frac{1}{2}(m_1^2 - m_2^2) + m_1^2 \right] \\
&= -\frac{1}{6}q^2 + \frac{1}{2}(m_1^2 + m_2^2)
\end{aligned}
$$

So let us begin to cancel the divergences.

$$
\begin{aligned}
\mathbb{X} = &+ \Bigg[4(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\left(-\frac{1}{6}q^2 + \frac{1}{2}(m_u^2+m_d^2)\right) - 2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\frac{1}{6}q^2 \\
&\quad + 4m_u^2 m_d^2\Bigg] \quad \text{from fig. F.1} \\
&- 2m_u^2\cot^2\beta\left[3\left(-\frac{1}{6}q^2 + m_u^2\right) - 2m_u^2\right] \quad \text{from fig. F.24} \\
&- 2m_d^2\tan^2\beta\left[3\left(-\frac{1}{6}q^2 + m_d^2\right) - 2m_d^2\right] \quad \text{from fig. F.25} \\
&- \Big[c_u c_d(-m_W^2 s_{2\beta} + m_d^2\tan\beta + m_u^2\cot\beta) + s_u s_d m_u m_d(\tan\beta + \cot\beta) \\
&\quad + c_u s_d m_d(\mu + A_d\tan\beta) + s_u c_d m_u(\mu + A_u\cot\beta)\Big]^2 \quad \text{from fig. F.2} \\
&- \Big[-c_u s_d(-m_W^2 s_{2\beta} + m_d^2\tan\beta + m_u^2\cot\beta) + s_u c_d m_u m_d(\tan\beta + \cot\beta) \\
&\quad + c_u c_d m_d(\mu + A_d\tan\beta) - s_u s_d m_u(\mu + A_u\cot\beta)\Big]^2 \quad \text{from fig. F.3} \\
&- \Big[-s_u c_d(-m_W^2 s_{2\beta} + m_d^2\tan\beta + m_u^2\cot\beta) + c_u s_d m_u m_d(\tan\beta + \cot\beta) \\
&\quad - s_u s_d m_d(\mu + A_d\tan\beta) + c_u c_d m_u(\mu + A_u\cot\beta)\Big]^2 \quad \text{from fig. F.4} \\
&- \Big[s_u s_d(-m_W^2 s_{2\beta} + m_d^2\tan\beta + m_u^2\cot\beta) + c_u c_d m_u m_d(\tan\beta + \cot\beta) \\
&\quad - s_u c_d m_d(\mu + A_d\tan\beta) - c_u s_d m_u(\mu + A_u\cot\beta)\Big]^2 \quad \text{from fig. F.5} \\
&+ m_u^2(\mu + A_u\cot\beta)^2 \quad \text{from figs. F.26, F.27} \\
&+ m_d^2(\mu + A_d\tan\beta)^2 \quad \text{from figs. F.28, F.29} \\
&+ \Bigg[-c_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 + \frac{1}{2}\frac{c_u^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 - (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2}m_W^2 m_{\tilde{u}_1}^2 - m_d^2\tan^2\beta c_u^2 m_{\tilde{u}_1}^2 \\
&\quad - m_u^2\cot^2\beta s_u^2 m_{\tilde{u}_1}^2\Bigg] \quad \text{from fig. F.6} \\
&+ \Bigg[-s_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 + \frac{1}{2}\frac{s_u^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 + (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2}m_W^2 m_{\tilde{u}_2}^2 - m_d^2\tan^2\beta s_u^2 m_{\tilde{u}_2}^2 \\
&\quad - m_u^2\cot^2\beta c_u^2 m_{\tilde{u}_2}^2\Bigg] \quad \text{from fig. F.7} \\
&+ \Bigg[c_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 - \frac{1}{2}\frac{c_d^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 - (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2}m_W^2 m_{\tilde{d}_1}^2 - m_u^2\cot^2\beta c_d^2 m_{\tilde{d}_1}^2 \\
&\quad - m_d^2\tan^2\beta s_d^2 m_{\tilde{d}_1}^2\Bigg] \quad \text{from fig. F.8} \\
&+ \Bigg[s_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_2}^2 - \frac{1}{2}\frac{s_d^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{d}_s}^2 + (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2}m_W^2 m_{\tilde{d}_2}^2 - m_u^2\cot^2\beta s_d^2 m_{\tilde{d}_2}^2 \\
&\quad - m_d^2\tan^2\beta c_d^2 m_{\tilde{d}_2}^2\Bigg] \quad \text{from fig. F.9} \\
&- \left[\frac{1}{2}\frac{c_u^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 - (c_u^2 - s_u^2)e_u m_W^2 c_{2\beta}\frac{s_w^2}{c_w^2}m_{\tilde{u}_1}^2 - m_u^2\cot^2\beta m_{\tilde{u}_1}^2\right] \quad \text{from fig. F.30}
\end{aligned}
$$

$$
\begin{aligned}
&- \left[\frac{1}{2} \frac{s_u^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 + (c_u^2 - s_u^2) e_u m_W^2 c_{2\beta} \frac{s_w^2}{c_w^2} m_{\tilde{u}_2}^2 - m_u^2 \cot^2\beta m_{\tilde{u}_2}^2 \right] \quad \text{from fig. F.31} \\
&+ \left[\frac{1}{2} \frac{c_d^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 + (c_d^2 - s_d^2) e_d m_W^2 c_{2\beta} \frac{s_w^2}{c_w^2} m_{\tilde{d}_1}^2 + m_d^2 \tan^2\beta m_{\tilde{d}_1}^2 \right] \quad \text{from fig. F.32} \\
&+ \left[\frac{1}{2} \frac{s_d^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{d}_2}^2 - (c_d^2 - s_d^2) e_d m_W^2 c_{2\beta} \frac{s_w^2}{c_w^2} m_{\tilde{d}_2}^2 + m_d^2 \tan^2\beta m_{\tilde{d}_2}^2 \right] \quad \text{from fig. F.33} \\
&- 2m_W^2 \left[-\frac{1}{6} q^2 - \frac{1}{6} q^2 + \frac{1}{2}(m_u^2 + m_d^2) \right] \quad \text{from fig. F.41} \\
&- m_W^2 c_u^2 c_d^2 \left[2\left(-\frac{1}{6} q^2 + \frac{1}{2}(m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2) \right) \right] \quad \text{from fig. F.42} \\
&- m_W^2 c_u^2 s_d^2 \left[2\left(-\frac{1}{6} q^2 + \frac{1}{2}(m_{\tilde{u}_1}^2 + m_{\tilde{d}_2}^2) \right) \right] \quad \text{from fig. F.43} \\
&- m_W^2 s_u^2 c_d^2 \left[2\left(-\frac{1}{6} q^2 + \frac{1}{2}(m_{\tilde{u}_2}^2 + m_{\tilde{d}_1}^2) \right) \right] \quad \text{from fig. F.44} \\
&- m_W^2 s_u^2 s_d^2 \left[2\left(-\frac{1}{6} q^2 + \frac{1}{2}(m_{\tilde{u}_2}^2 + m_{\tilde{d}_2}^2) \right) \right] \quad \text{from fig. F.45} \\
&+ m_W^2 c_u^2 m_{\tilde{u}_1}^2 + m_W^2 s_u^2 m_{\tilde{u}_2}^2 + m_W^2 c_d^2 m_{\tilde{d}_1}^2 + m_W^2 s_d^2 m_{\tilde{d}_2}^2 \quad \text{from figs. F.46, F.47, F.48, F.49}
\end{aligned}
$$

Parts of the above expression for $\mathbb{X}$ can be simplified.

Auxiliary Calculation

The sum of the contributions from figs. F.2, F.3, F.4, F.5 can be simplified. Define

$$
A \equiv -m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta, \qquad B \equiv m_u m_d (\tan\beta + \cot\beta),
$$
$$
C \equiv m_d(\mu + A_d \tan\beta), \qquad D \equiv m_u(\mu + A_u \cot\beta),
$$

and write the sum of these contributions in terms of A, B, C, and D:

$$
\begin{aligned}
&(c_u c_d A + s_u s_d B + c_u s_d C + s_u s_d D)^2 + (-c_u s_d A + s_u c_d B + c_u c_d C - s_u s_d D)^2 \\
&\quad + (-s_u c_d A + c_u s_d B - s_u s_d C + c_u c_d D)^2 + (s_u s_d A + c_u c_d B - s_u c_d C - c_u s_d D)^2 \\
&\quad = c_u^2 c_d^2 A^2 + c_u^2 s_d^2 C^2 + s_u^2 c_d^2 D^2 + c_u^2 s_d^2 A^2 + s_u^2 c_d^2 B^2 + c_u^2 c_d^2 C^2 + s_u^2 s_d^2 D^2 + s_u^2 c_d^2 A^2 + c_u^2 s_d^2 B^2 \\
&\quad + s_u^2 s_d^2 C^2 + c_u^2 c_d^2 D^2 + s_u^2 s_d^2 A^2 + c_u^2 c_d^2 B^2 + s_u^2 c_d^2 C^2 + c_u^2 s_d^2 D^2 + s_u^2 s_d^2 B^2 \\
&\quad = A^2 (c_u^2 c_d^2 + c_u^2 s_d^2 + s_u^2 c_d^2 + s_u^2 s_d^2) + B^2 (s_u^2 c_d^2 + c_u^2 s_d^2 + c_u^2 c_d^2 + s_u^2 s_d^2) \\
&\quad + C^2 (c_u^2 s_d^2 + c_u^2 c_d^2 + s_u^2 s_d^2 + s_u^2 c_d^2) + D^2 (s_u^2 c_d^2 + s_u^2 s_d^2 + c_u^2 c_d^2 + c_u^2 s_d^2) \\
&\quad = A^2 + B^2 + C^2 + D^2 \\
&\quad = (-m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta)^2 + m_u^2 m_d^2 (\tan\beta + \cot\beta)^2 + m_d^2 (\mu + A_d \tan\beta)^2 + m_u^2 (\mu + A_u \cot\beta)^2
\end{aligned}
\tag{E.2}
$$

Auxiliary Calculation

The sum of the contributions from figs. F.42, F.43, F.44, F.45, F.46, F.47, F.48, F.49 be simplified.

$$
\begin{aligned}
&-m_W^2 \left[c_u^2 c_d^2 \left(-\frac{1}{3}q^2 + m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2\right) + c_u^2 s_d^2 \left(-\frac{1}{3}q^2 + m_{\tilde{u}_1}^2 + m_{\tilde{d}_2}^2\right) + s_u^2 c_d^2 \left(-\frac{1}{3}q^2 + m_{\tilde{u}_2}^2 + m_{\tilde{d}_1}^2\right)\right. \\
&\quad \left. + s_u^2 s_d^2 \left(-\frac{1}{3}q^2 + m_{\tilde{u}_2}^2 + m_{\tilde{d}_2}^2\right) - c_u^2 m_{\tilde{u}_1}^2 - s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2\right] \\
&= - m_W^2 \Big[-\frac{1}{3}q^2 \left(c_u^2 c_d^2 + c_u^2 s_d^2 + s_u^2 c_d^2 + s_u^2 s_d^2\right) + m_{\tilde{u}_1}^2 \left(c_u^2 c_d^2 + c_u^2 s_d^2 - c_u^2\right) + m_{\tilde{u}_2}^2 \left(s_u^2 c_d^2 + s_u^2 s_d^2 - s_u^2\right) \\
&\quad + m_{\tilde{d}_1}^2 \left(c_u^2 c_d^2 + s_u^2 c_d^2 - c_d^2\right) + m_{\tilde{d}_2}^2 \left(c_u^2 s_d^2 + s_u^2 s_d^2 - s_d^2\right) \Big] \\
&= m_W^2 \frac{1}{3} q^2 = \frac{m_W^4}{3}
\end{aligned}
\tag{E.3}
$$

Now proceed with the cancellation of divergences. Remember that the self-energies are to be calculated at $q^2 = M^2$, where M is the mass of the respective particle. With these substitutions, and eqs. E.2, E.3, we have:

$$
\begin{aligned}
X = &+ \Big[- m_{H^\pm}^2 (m_d^2 \tan^2\beta + m_u^2 \cot^2\beta) + 2(m_u^2 + m_d^2)(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta) + 4 m_u^2 m_d^2 \Big] \\
&- \Big[- m_A^2 (m_d^2 \tan^2\beta + m_u^2 \cot^2\beta) + 2 m_u^4 \cot^2\beta + 2 m_d^4 \tan^2\beta \Big] \\
&- \Big[(-m_W^2 s_{2\beta} + m_d^2 \tan^2\beta + m_u^2 \cot^2\beta)^2 + m_u^2 m_d^2 (\tan\beta + \cot\beta)^2 + m_d^2 (\mu + A_d \tan\beta)^2 \\
&\quad + m_u^2 (\mu + A_u \cot\beta)^2 \Big] \\
&+ \Big[m_u^2 (\mu + A_u \cot\beta)^2 + m_d^2 (\mu + A_d \tan\beta)^2 \Big] \\
&+ \Big[- c_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 + \frac{1}{2}\frac{c_u^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 - (c_u^2 - s_u^2) e_u c_{2\beta} \frac{s_w^2}{c_w^2} m_W^2 m_{\tilde{u}_1}^2 - m_d^2 \tan^2\beta\, c_u^2 m_{\tilde{u}_1}^2 \\
&\quad - m_u^2 \cot^2\beta\, s_u^2 m_{\tilde{u}_1}^2 \Big] \\
&+ \Big[- s_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 + \frac{1}{2}\frac{s_u^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 + (c_u^2 - s_u^2) e_u c_{2\beta} \frac{s_w^2}{c_w^2} m_W^2 m_{\tilde{u}_2}^2 - m_d^2 \tan^2\beta\, s_u^2 m_{\tilde{u}_2}^2 \\
&\quad - m_u^2 \cot^2\beta\, c_u^2 m_{\tilde{u}_2}^2 \Big] \\
&+ \Big[c_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 - \frac{1}{2}\frac{c_d^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 - (c_d^2 - s_d^2) e_d c_{2\beta} \frac{s_w^2}{c_w^2} m_W^2 m_{\tilde{d}_1}^2 - m_u^2 \cot^2\beta\, c_d^2 m_{\tilde{d}_1}^2 \\
&\quad - m_d^2 \tan^2\beta\, s_d^2 m_{\tilde{d}_1}^2 \Big] \\
&+ \Big[s_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_2}^2 - \frac{1}{2}\frac{s_d^2}{c_w^2} m_W^2 c_{2\beta} m_{\tilde{d}_s}^2 + (c_d^2 - s_d^2) e_d c_{2\beta} \frac{s_w^2}{c_w^2} m_W^2 m_{\tilde{d}_2}^2 - m_u^2 \cot^2\beta\, s_d^2 m_{\tilde{d}_2}^2 \\
&\quad - m_d^2 \tan^2\beta\, c_d^2 m_{\tilde{d}_2}^2 \Big]
\end{aligned}
$$

$$\begin{aligned}
&-\Bigg[\frac{1}{2}\frac{c_u^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 - (c_u^2 - s_u^2)e_u m_W^2 c_{2\beta}\frac{s_w^2}{c_w^2}m_{\tilde{u}_1}^2 - m_u^2\cot^2\beta m_{\tilde{u}_1}^2 + \frac{1}{2}\frac{s_u^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{u}_2}^2\\
&\quad + (c_u^2 - s_u^2)e_u m_W^2 c_{2\beta}\frac{s_w^2}{c_w^2}m_{\tilde{u}_2}^2 - m_u^2\cot^2\beta m_{\tilde{u}_2}^2\Bigg]\\
&+\Bigg[\frac{1}{2}\frac{c_d^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{d}_1}^2 + (c_d^2 - s_d^2)e_d m_W^2 c_{2\beta}\frac{s_w^2}{c_w^2}m_{\tilde{d}_1}^2 + m_d^2\tan^2\beta m_{\tilde{d}_1}^2 + \frac{1}{2}\frac{s_d^2}{c_w^2}m_W^2 c_{2\beta} m_{\tilde{d}_2}^2\\
&\quad - (c_d^2 - s_d^2)e_d m_W^2 c_{2\beta}\frac{s_w^2}{c_w^2}m_{\tilde{d}_2}^2 + m_d^2\tan^2\beta m_{\tilde{d}_2}^2\Bigg]\\
&-\Bigg[-\frac{2}{3}m_W^4 - \frac{1}{3}m_W^4 + m_W^2(m_u^2 + m_d^2)\Bigg]
\end{aligned}$$

Now substitute the tree-level relation $m_{H^\pm}^2 = m_A^2 + m_W^2$ (see eq. 3.3) and perform some trivial cancellations:

$$\begin{aligned}
\mathbb{X} = &+\Big[-m_W^2(m_d^2\tan^2\beta + m_u^2\cot^2\beta) + 2m_u^2 m_d^2\tan^2\beta + 2m_u^2 m_d^2\cot^2\beta + 4m_u^2 m_d^2\Big]\\
&-\Big[(-m_W^2 s_{2\beta} + m_d^2\tan\beta + m_u^2\cot\beta)^2 + m_u^2 m_d^2(\tan\beta + \cot\beta)^2\Big]\\
&+\Big[-c_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_1}^2 - m_d^2\tan^2\beta c_u^2 m_{\tilde{u}_1}^2 - m_u^2\cot^2\beta s_u^2 m_{\tilde{u}_1}^2 + m_u^2\cot^2\beta m_{\tilde{u}_1}^2\\
&\quad - s_u^2 m_W^2 c_{2\beta} m_{\tilde{u}_2}^2 - m_d^2\tan^2\beta s_u^2 m_{\tilde{u}_2}^2 - m_u^2\cot^2\beta c_u^2 m_{\tilde{u}_2}^2 + m_u^2\cot^2\beta m_{\tilde{u}_2}^2 + c_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_1}^2\\
&\quad - m_u^2\cot^2\beta c_d^2 m_{\tilde{d}_1}^2 - m_d^2\tan^2\beta s_d^2 m_{\tilde{d}_1}^2 + m_d^2\tan^2\beta m_{\tilde{d}_1}^2 + s_d^2 m_W^2 c_{2\beta} m_{\tilde{d}_2}^2 - m_u^2\cot^2\beta s_d^2 m_{\tilde{d}_2}^2\\
&\quad - m_d^2\tan^2\beta c_d^2 m_{\tilde{d}_2}^2 + m_d^2\tan^2\beta m_{\tilde{d}_2}^2\Big]\\
&-\left[-m_W^4 + m_W^2(m_u^2 + m_d^2)\right]
\end{aligned}$$

Substitute $m_W^2 c_{2\beta} = c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - c_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2$ (see eq. B.21). Some trivial cancellations are performed to simplify the result.

$$\begin{aligned}
\mathbb{X} = &- m_W^2(m_d^2\tan^2\beta + m_u^2\cot^2\beta) + m_u^2 m_d^2\tan^2\beta + m_u^2 m_d^2\cot^2\beta - m_W^4 s_{2\beta} - m_d^4\tan^2\beta - m_u^4\cot^2\beta\\
&+ 2m_W^2 m_d^2 s_{2\beta}\tan\beta + 2m_W^2 m_u^2 s_{2\beta}\cot\beta - c_u^2 m_{\tilde{u}_1}^2(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2)\\
&- m_d^2\tan^2\beta c_u^2 m_{\tilde{u}_1}^2 - m_u^2\cot^2\beta s_u^2 m_{\tilde{u}_1}^2 + m_u^2\cot^2\beta m_{\tilde{u}_1}^2 - s_u^2 m_{\tilde{u}_2}^2(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2\\
&- m_u^2 + m_d^2) - m_d^2\tan^2\beta s_u^2 m_{\tilde{u}_2}^2 - m_u^2\cot^2\beta c_u^2 m_{\tilde{u}_2}^2 + m_u^2\cot^2\beta m_{\tilde{u}_2}^2 + c_d^2 m_{\tilde{d}_1}^2(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2\\
&- s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2\tan^2\beta s_d^2 m_{\tilde{d}_1}^2 - m_u^2\cot^2\beta c_d^2 m_{\tilde{d}_1}^2 + m_d^2\tan^2\beta m_{\tilde{d}_1}^2 + s_d^2 m_{\tilde{d}_2}^2(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2\\
&- c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2\tan^2\beta c_d^2 m_{\tilde{d}_2}^2 - m_u^2\cot^2\beta s_d^2 m_{\tilde{d}_2}^2 + m_d^2\tan^2\beta m_{\tilde{d}_2}^2 + m_W^4\\
&- m_W^2(m_u^2 + m_d^2)
\end{aligned}$$

Next, we need expressions for m_W^2 and $m_W^2 s_{2\beta}$. Simply express them by $m_W^2 c_{2\beta}$ and then use eq. B.21:

$$\begin{aligned}
m_W^2 &= \frac{1}{c_{2\beta}}m_W^2 c_{2\beta} = \frac{1}{c_{2\beta}}\Big[c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - c_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2\Big],\\
m_W^2 s_{2\beta} &= m_W^2 c_{2\beta}\tan 2\beta = \tan 2\beta\Big[c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - c_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2\Big].
\end{aligned}$$

Substituting these expressions in $\mathbb{X}$ gives:

$$
\begin{aligned}
\mathbb{X} = &-\frac{1}{c_{2\beta}}(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2)(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta) + m_u^2 m_d^2 \cot^2\beta \\
&- \tan^2 2\beta (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2)^2 - m_d^4 \tan^2\beta - m_u^4 \cot^2\beta \\
&+ 2\tan 2\beta (m_d^2 \tan\beta + m_u^2 \cot\beta)(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) \\
&- c_u^2 m_{\tilde{u}_1}^2 (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2 \tan^2\beta c_u^2 m_{\tilde{u}_1}^2 - m_u^2 \cot^2\beta s_u^2 m_{\tilde{u}_1}^2 + m_u^2 \cot^2\beta m_{\tilde{u}_1}^2 \\
&- s_u^2 m_{\tilde{u}_2}^2 (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2 \tan^2\beta s_u^2 m_{\tilde{u}_2}^2 - m_u^2 \cot^2\beta c_u^2 m_{\tilde{u}_2}^2 + m_u^2 \cot^2\beta m_{\tilde{u}_2}^2 \\
&+ c_d^2 m_{\tilde{d}_1}^2 (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2 \tan^2\beta s_d^2 m_{\tilde{d}_1}^2 - m_u^2 \cot^2\beta c_d^2 m_{\tilde{d}_1}^2 + m_d^2 \tan^2\beta m_{\tilde{d}_1}^2 \\
&+ s_d^2 m_{\tilde{d}_2}^2 (c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2) - m_d^2 \tan^2\beta c_d^2 m_{\tilde{d}_2}^2 - m_u^2 \cot^2\beta s_d^2 m_{\tilde{d}_2}^2 + m_d^2 \tan^2\beta m_{\tilde{d}_2}^2 \\
&+ \frac{1}{c_{2\beta}^2}(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2)^2 \\
&- \frac{1}{c_{2\beta}}(c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 - c_d^2 m_{\tilde{d}_1}^2 - s_d^2 m_{\tilde{d}_2}^2 - m_u^2 + m_d^2)(m_u^2 + m_d^2)
\end{aligned}
$$

Next, we organize $\mathbb{X}$ in powers of the masses, and show in the following auxiliary calculations, that every coefficient is zero.

$$
\begin{aligned}
\mathbb{X} = &+ m_u^4 \left[\frac{1}{c_{2\beta}}\cot^2\beta - \tan^2 2\beta - \cot^2\beta - 2\tan 2\beta \cot\beta + \frac{1}{c_{2\beta}^2} + \frac{1}{c_{2\beta}}\right] \quad \text{(see eq. E.4)} \\
&+ m_d^4 \left[-\frac{1}{c_{2\beta}}\tan^2\beta - \tan^2 2\beta - \tan^2\beta + 2\tan 2\beta \tan\beta + \frac{1}{c_{2\beta}^2} - \frac{1}{c_{2\beta}}\right] \quad \text{(see eq. E.5)} \\
&+ m_u^2 m_d^2 \left[\frac{1}{c_{2\beta}}\tan^2\beta - \frac{1}{c_{2\beta}}\cot^2\beta + \tan^2\beta + \cot^2\beta + 2\tan^2 2\beta - 2\tan 2\beta \tan\beta + 2\tan 2\beta \cot\beta \right. \\
&\qquad \left. - \frac{2}{c_{2\beta}^2} + \frac{1}{c_{2\beta}} - \frac{1}{c_{2\beta}}\right] \quad \text{(see eq. E.6)} \\
&+ m_u^2 m_{\tilde{u}_1}^2 \left[-\frac{1}{c_{2\beta}}\cot^2\beta c_u^2 + 2\tan^2 2\beta c_u^2 + 2\tan 2\beta \cot\beta c_u^2 + c_u^2 - \cot^2\beta s_u^2 + \cot^2\beta \right. \\
&\qquad \left. - \frac{2}{c_{2\beta}^2}c_u^2 - \frac{1}{c_{2\beta}}c_u^2\right] \quad \text{(see eq. E.7)} \\
&+ m_u^2 m_{\tilde{u}_2}^2 \left[-\frac{1}{c_{2\beta}}\cot^2\beta s_u^2 + 2\tan^2 2\beta s_u^2 + 2\tan 2\beta \cot\beta s_u^2 + s_u^2 - \cot^2\beta c_u^2 + \cot^2\beta \right. \\
&\qquad \left. - \frac{2}{c_{2\beta}^2}s_u^2 - \frac{1}{c_{2\beta}}s_u^2\right] \quad \text{(see eq. E.7)} \\
&+ m_u^2 m_{\tilde{d}_1}^2 \left[\frac{1}{c_{2\beta}}\cot^2\beta c_d^2 - 2\tan^2 2\beta c_d^2 - 2\tan 2\beta \cot\beta c_d^2 - c_d^2 - \cot^2\beta c_d^2 \right. \\
&\qquad \left. + \frac{2}{c_{2\beta}^2}c_d^2 + \frac{1}{c_{2\beta}}c_d^2\right] \quad \text{(see eq. E.7)}
\end{aligned}
$$

$$
\begin{aligned}
&+ m_u^2 m_{\tilde{d}_2}^2 \left[\frac{1}{c_{2\beta}} \cot^2\beta s_d^2 - 2\tan^2 2\beta s_d^2 - 2\tan 2\beta \cot\beta s_d^2 - s_d^2 - \cot^2\beta s_d^2 \right.\\
&\qquad \left. + \frac{2}{c_{2\beta}^2} s_d^2 + \frac{1}{c_{2\beta}} s_d^2 \right] \quad \text{(see eq. E.7)}\\
&+ m_d^2 m_{\tilde{u}_1}^2 \left[-\frac{1}{c_{2\beta}} \tan^2\beta c_u^2 - 2\tan^2 2\beta c_u^2 + 2\tan 2\beta \tan\beta c_u^2 - c_u^2 - \tan^2\beta c_u^2 \right.\\
&\qquad \left. + \frac{2}{c_{2\beta}^2} c_u^2 - \frac{1}{c_{2\beta}} c_u^2 \right] \quad \text{(see eq. E.8)}\\
&+ m_d^2 m_{\tilde{u}_2}^2 \left[-\frac{1}{c_{2\beta}} \tan^2\beta s_u^2 - 2\tan^2 2\beta s_u^2 + 2\tan 2\beta \tan\beta s_u^2 - s_u^2 - \tan^2\beta s_u^2 \right.\\
&\qquad \left. + \frac{2}{c_{2\beta}^2} s_u^2 - \frac{1}{c_{2\beta}} s_u^2 \right] \quad \text{(see eq. E.8)}\\
&+ m_d^2 m_{\tilde{d}_1}^2 \left[\frac{1}{c_{2\beta}} \tan^2\beta c_d^2 + 2\tan^2 2\beta c_d^2 - 2\tan 2\beta \tan\beta c_d^2 + c_d^2 - \tan^2\beta s_d^2 + \tan^2\beta \right.\\
&\qquad \left. - \frac{2}{c_{2\beta}^2} c_d^2 + \frac{1}{c_{2\beta}} c_d^2 \right] \quad \text{(see eq. E.8)}\\
&+ m_d^2 m_{\tilde{d}_2}^2 \left[\frac{1}{c_{2\beta}} \tan^2\beta s_d^2 + 2\tan^2 2\beta s_d^2 - 2\tan 2\beta \tan\beta s_d^2 + s_d^2 - \tan^2\beta c_d^2 + \tan^2\beta \right.\\
&\qquad \left. - \frac{2}{c_{2\beta}^2} s_d^2 + \frac{1}{c_{2\beta}} s_d^2 \right] \quad \text{(see eq. E.8)}\\
&+ m_{\tilde{u}_1}^4 \left[-\tan^2 2\beta c_u^4 - c_u^4 + \frac{1}{c_{2\beta}^2} c_u^4 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{u}_2}^4 \left[-\tan^2 2\beta s_u^4 - s_u^4 + \frac{1}{c_{2\beta}^2} s_u^4 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{d}_1}^4 \left[-\tan^2 2\beta c_d^4 - c_d^4 + \frac{1}{c_{2\beta}^2} c_d^4 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{d}_2}^4 \left[-\tan^2 2\beta s_d^4 - s_d^4 + \frac{1}{c_{2\beta}^2} s_d^4 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{u}_1}^2 m_{\tilde{u}_2}^2 \left[-2\tan^2 2\beta c_u^2 s_u^2 - c_u^2 s_u^2 - s_u^2 c_u^2 + \frac{2}{c_{2\beta}^2} c_u^2 s_u^2 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{u}_1}^2 m_{\tilde{d}_1}^2 \left[2\tan^2 2\beta c_u^2 c_d^2 + c_u^2 c_d^2 + c_u^2 c_d^2 - \frac{2}{c_{2\beta}^2} c_u^2 c_d^2 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{u}_1}^2 m_{\tilde{d}_2}^2 \left[2\tan^2 2\beta c_u^2 s_d^2 + c_u^2 s_d^2 + c_u^2 s_d^2 - \frac{2}{c_{2\beta}^2} c_u^2 s_d^2 \right] \quad \text{(see eq. E.9)}\\
&+ m_{\tilde{u}_2}^2 m_{\tilde{d}_1}^2 \left[2\tan^2 2\beta s_u^2 c_d^2 + s_u^2 c_d^2 + s_u^2 c_d^2 - \frac{2}{c_{2\beta}^2} s_u^2 c_d^2 \right] \quad \text{(see eq. E.9)}
\end{aligned}
$$

$$+ m^2_{\tilde{u}_2} m^2_{\tilde{d}_2} \left[2\tan^2 2\beta s_u^2 s_d^2 + s_u^2 s_d^2 + s_u^2 s_d^2 - \frac{2}{c^2_{2\beta}} s_u^2 s_d^2 \right] \quad \text{(see eq. E.9)}$$
$$+ m^2_{\tilde{d}_1} m^2_{\tilde{d}_2} \left[-2\tan^2 2\beta c_d^2 s_d^2 - c_d^2 s_d^2 - c_d^2 s_d^2 + \frac{2}{c^2_{2\beta}} c_d^2 s_d^2 \right] \quad \text{(see eq. E.9)}$$

As stated before, our strategy is to show that every coefficient in the above sum is zero. To do so, we need some trigonometric formulae, which are listed below for convenience:

$$\sin 2\beta = \frac{2\tan\beta}{1+2\tan^2\beta}, \qquad \cos 2\beta = \frac{1-\tan^2\beta}{1+\tan^2\beta}, \qquad \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}, \qquad \cot\beta = \frac{1}{\tan\beta}$$

Define $t \equiv \tan\beta$. Then these formulae take on the form:

$$\sin 2\beta = \frac{2t}{1+2t^2}, \qquad \cos 2\beta = \frac{1-t^2}{1+t^2}, \qquad \tan 2\beta = \frac{2t}{1-t^2}, \qquad \cot\beta = \frac{1}{t}$$

Now using the above formulae, express all the trigonometric functions by t, and then it is clear that the resulting polynomial in t is identically zero in each case. The following auxiliary calculations deal with each coefficient separately.

Auxiliary Calculation

$$\begin{aligned}
&\frac{1}{c_{2\beta}} \cot^2\beta - \tan^2 2\beta - \cot^2\beta - 2\tan 2\beta \cot\beta + \frac{1}{c^2_{2\beta}} + \frac{1}{c_{2\beta}} \\
&= \frac{1+t^2}{1-t^2}\frac{1}{t^2} - \left(\frac{2t}{1-t^2}\right)^2 - \frac{1}{t^2} - 2\frac{2t}{1-t^2}\frac{1}{t} + \left(\frac{1+t^2}{1-t^2}\right)^2 + \frac{1+t^2}{1-t^2} \\
&= \frac{1}{t^2(1-t^2)^2}\left\{(1+t^2)(1-t^2) - 4t^4 - (1-t^2)^2 - 4t^2(1-t^2) + (1+t^2)t^2 + (1+t^2)(1-t^2)t^2\right\} \\
&= \frac{1}{t^2(1-t^2)^2}\left\{1 - t^4 - 4t^4 - 1 + 2t^2 - t^4 - 4t^2 + 4t^4 + t^2 + 2t^4 + t^6 + t^2 - t^6\right\} \\
&= \frac{1}{t^2(1-t^2)^2}\left\{(1-1) + t^2(2-4+1+1) + t^4(-1-4-1+4+2) + t^6(1-1)\right\} \\
&= 0
\end{aligned} \tag{E.4}$$

Auxiliary Calculation

$$\begin{aligned}
&-\frac{1}{c_{2\beta}}\tan^2\beta-\tan^2 2\beta-\tan^2\beta+2\tan 2\beta\tan\beta+\frac{1}{c_{2\beta}^2}-\frac{1}{c_{2\beta}}\\
&=-\frac{1+t^2}{1-t^2}t^2-\left(\frac{2t}{1-t^2}\right)^2-t^2+2\frac{2t}{1-t^2}t+\left(\frac{1+t^2}{1-t^2}-\frac{1+t^2}{1-t^2}\right)^2\\
&=\frac{1}{(1-t^2)^2}\left\{-t^2(1+t^2)(1-t^2)-4t^2-t^2(1-t^2)^2+4t^2(1-t^2)+(1+t^2)^2-(1+t^2)(1-t^2)\right\}\\
&=\frac{1}{(1-t^2)^2}\left\{-t^2+t^6-4t^2-t^2+2t^4-t^6+4t^2-4t^4+1+2t^2+t^4-1+t^4\right\}\\
&=\frac{1}{(1-t^2)^2}\left\{(1-1)+t^2(-1-4-1+4+2)+t^4(2-4+1+1)+t^6(1-1)\right\}\\
&=0
\end{aligned}\tag{E.5}$$

Auxiliary Calculation

$$\begin{aligned}
&\frac{1}{c_{2\beta}}\tan^2\beta-\frac{1}{c_{2\beta}}\cot^2\beta+\tan^2\beta+\cot^2\beta+2\tan^2 2\beta-2\tan 2\beta\tan\beta+2\tan 2\beta\cot\beta-\frac{2}{c_{2\beta}^2}+\frac{1}{c_{2\beta}}-\frac{1}{c_{2\beta}}\\
&=-\left[-\frac{1}{c_{2\beta}}\tan^2\beta-\tan^2 2\beta-\tan^2\beta+2\tan 2\beta\tan\beta+\frac{1}{c_{2\beta}^2}-\frac{1}{c_{2\beta}}\right]\\
&\quad-\left[-\frac{1}{c_{2\beta}}\cot^2\beta-\tan^2 2\beta-\cot^2\beta-2\tan 2\beta\cot\beta+\frac{1}{c_{2\beta}^2}+\frac{1}{c_{2\beta}}\right]\\
&=0-0\\
&=0
\end{aligned}\tag{E.6}$$

Auxiliary Calculation

$$\begin{aligned}
&-\frac{1}{c_{2\beta}}\cot^2\beta c_u^2+2\tan^2 2\beta c_u^2+2\tan^2 2\beta\cot\beta c_u^2+c_u^2+(1-s_u^2)\cot^2\beta-\frac{2}{c_{2\beta}^2}c_u^2-\frac{1}{c_{2\beta}}c_u^2\\
&=-\frac{1+t^2}{1-t^2}\frac{1}{t^2}+2\left(\frac{2t}{1-t^2}\right)^2+2\frac{2t}{1-t^2}\frac{1}{t}+1+\frac{1}{t^2}-2\left(\frac{1+t^2}{1-t^2}\right)^2-\frac{1+t^2}{1-t^2}\\
&=\frac{1}{t^2(1-t^2)^2}\left\{-(1+t^2)(1-t^2)+8t^4+4t^2(1-t^2)+t^2(1-t^2)^2+(1-t^2)^2-2t^2(1+t^2)^2-(1+t^2)(1-t^2)t^2\right\}\\
&=\frac{1}{t^2(1-t^2)^2}\left\{-1+t^4+8t^4+4t^2-4t^4+t^2-2t^4+t^6+1-2t^2+t^4-2t^2-4t^4-2t^6-t^2+t^6\right\}\\
&=\frac{1}{t^2(1-t^2)^2}\left\{(-1+1)+t^2(4+1-2-2-1)+t^4(1+8-4-2+1-4)+t^6(1-2+1)\right\}\\
&=0
\end{aligned}\tag{E.7}$$

Auxiliary Calculation

$$
\begin{aligned}
&-\frac{1}{c_{2\beta}}\tan^2\beta - 2\tan^2 2\beta + 2\tan 2\beta\tan\beta - 1 - \tan^2\beta + \frac{2}{c_{2\beta}^2} - \frac{1}{c_{2\beta}}\\
&= -\frac{1+t^2}{1-t^2}t^2 - 2\left(\frac{2t}{1-t^2}\right)^2 + 2\frac{2t}{1-t^2}t - 1 - t^2 + 2\left(\frac{1+t^2}{1-t^2}\right)^2 - \frac{1+t^2}{1-t^2}\\
&= \frac{1}{(1-t^2)^2}\{-(1+t^2)(1-t^2)t^2 - 8t^2 + 4t^2(1-t^2) - (1-t^2)^2 - t^2(1-t^2)^2\\
&+ 2(1+t^2)^2 - (1+t^2)(1-t^2)\}\\
&= \frac{1}{(1-t^2)^2}\left\{-t^2 + t^6 - 8t^2 + 4t^2 - 4t^4 - 1 + 2t^2 - t^4 - t^2 + 2t^4 - t^6 + 2 + 4t^2 + 2t^4 - 1 + t^4\right\}\\
&= \frac{1}{(1-t^2)^2}\left\{(-1+2-1) + t^2(-1-8+4+2-1+4) + t^4(-4-1+2+2+1) + t^6(1-1)\right\}\\
&= 0
\end{aligned}
\tag{E.8}
$$

Auxiliary Calculation

$$
\begin{aligned}
&-\tan^2 2\beta - 1 + \frac{1}{c_{2\beta}^2} = -\left(\frac{2t}{1-t^2}\right)^2 - 1 + \left(\frac{1+t^2}{1-t^2}\right)^2\\
&= \frac{1}{(1-t^2)^2}\left\{-4t^2 - (1-t^2)^2 + (1+t^2)^2\right\} = \frac{1}{(1-t^2)^2}\left\{-4t^2 - 1 + 2t^2 - t^4 + 1 + 2t^2 + t^4\right\}\\
&= 0
\end{aligned}
\tag{E.9}
$$

Conclusion

Every coefficient of vanishes, so the sum is identically zero, i.e. all divergences sum up to zero.

E.3 The Finite Part

E.3.1 Definitions and Notation

To compare my results with ref. [8], I will write $\widetilde{\Delta}$ in the notation of that paper. I will express $\widetilde{\Delta}$ as

$$
\widetilde{\Delta} = \frac{3g^2}{32\pi^2 m_W^2}\left(S_W^q + S_W^{\tilde{q}} + S_{A,H^\pm}^q + S_{A,H^\pm}^{\tilde{q}}\right). \tag{E.10}
$$

The notation is self-explanatory. I have factored out $g^2/(32\pi^2 m_W^2)$, and the factor of 3 in the numerator is for color.
To express the result more concisely, we adopt the following definitions:

$$
V \equiv m_d^2\tan^2\beta - m_u^2\cot^2\beta + m_W^2 c_{2\beta}, \quad V_{\ell\ell} \equiv m_d^2\tan\beta + m_u^2\cot\beta + m_W^2 s_{2\beta}, \tag{E.11}
$$

$$
V_{rr} \equiv m_u m_d(\tan\beta + \cot\beta), \quad V_{\ell r} \equiv m_d(\mu + A_d\tan\beta), \quad V_{r\ell} \equiv m_u(\mu + A_u\cot\beta). \tag{E.12}
$$

Note that these definitions slightly differ from ref. [8], because in my conventions, μ has the opposite sign. The loop integrals are rewritten in terms of four functions as given in eqs. C.23-C.26. The relations given in eqs. C.30-C.33 are useful to note for the following calculations.

E.3.2 W^- and Quarks

Let us calculate the self-energy of the W-boson originating from quarks running in the loops. Consider the graph of fig. F.41. Take the coefficient of $g^{\mu\nu}$ (the transverse part of the self-energy), drop the i, factor out $g^2/(32\pi^2 m_W^2)$, and note that it enters the mass sum rule with a minus (see eq. 3.3).

$$\begin{aligned}
S_W^q &= (-1)\cdot 2m_W^2 \int dx \Big\{ x(1-x)q^2 \log\Delta_{ud} - \Delta_{ud}\log\Delta_{ud} \Big\} \\
&= 2m_W^2 \int dx \Big\{ -m_W^2 x(1-x)\log\Delta_{ud} + \Big(-x(1-x)m_W^2 - x(m_u^2 - m_d^2) + m_u^2 \Big)\log\Delta_{ud} \Big\} \\
&= 2m_W^2 \int dx \Big\{ -2m_W^2 x(1-x)\log\Delta_{ud} - \Big(x(m_u^2 - m_d^2) - m_u^2 \Big)\log\Delta_{ud} \Big\} \\
&= m_W^2 \int dx \Big\{ -4m_W^2 x(1-x)\log\Delta_{ud} - 2\Big((-1+x)m_u^2 - x m_d^2 \Big)\log\Delta_{ud} \Big\} \\
&= m_W^2 \int dx \Big\{ -4G(m_W^2, m_u^2, m_d^2) + 2\Big((1-x)m_u^2 + x m_d^2 \Big)\log\Delta_{ud} \Big\} \\
&= m_W^2 \Big\{ 2H(m_W^2, m_u^2, m_d^2) - 4G(m_W^2, m_u^2, m_d^2) \Big\}
\end{aligned}$$

E.3.3 W^- and Squarks

Let us calculate the self-energy of the W-boson originating from squarks running in the loops. Consider the graphs of figs. F.46, F.47, F.48, F.49, F.42, F.43, F.44, F.45. Take the coefficient of $g^{\mu\nu}$ (the transverse part of the self-energy), drop the i, factor out $g^2/(32\pi^2 m_W^2)$, and note that it enters the mass sum rule with a minus (see eq. 3.3).

Auxiliary Calculation

$$\begin{aligned}
&\int dx \Big\{ 2c_u^2 c_d^2 \Delta_{\tilde{u}_1\tilde{d}_1} + 2c_u^2 s_d^2 \Delta_{\tilde{u}_1\tilde{d}_2} + 2s_u^2 c_d^2 \Delta_{\tilde{u}_2\tilde{d}_1} + 2s_u^2 s_d^2 \Delta_{\tilde{u}_2\tilde{d}_2} \Big\} \\
&\quad = 2c_u^2 c_d^2 \left[-\frac{1}{6}k^2 + \frac{1}{2}(m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2) \right] + 2c_u^2 s_d^2 \left[-\frac{1}{6}k^2 + \frac{1}{2}(m_{\tilde{u}_1}^2 + m_{\tilde{d}_2}^2) \right] + 2s_u^2 c_d^2 \left[-\frac{1}{6}k^2 + \frac{1}{2}(m_{\tilde{u}_2}^2 + m_{\tilde{d}_1}^2) \right] \\
&\qquad + 2s_u^2 s_d^2 \left[-\frac{1}{6}k^2 + \frac{1}{2}(m_{\tilde{u}_2}^2 + m_{\tilde{d}_2}^2) \right] \\
&\quad = -\frac{1}{3}k^2 \Big(c_u^2 c_d^2 + c_u^2 s_d^2 + s_u^2 c_d^2 + s_u^2 s_d^2 \Big) + c_u^2 m_{\tilde{u}_1}^2 + s_u^2 m_{\tilde{u}_2}^2 + c_d^2 m_{\tilde{d}_1}^2 + s_d^2 m_{\tilde{d}_2}^2
\end{aligned} \tag{E.13}$$

$$\begin{aligned}
S_W^{\tilde{q}} = (-1)\cdot m_W^2 \Big\{ &-c_u^2 m_{\tilde{u}_1}^2 \Big(-\log m_{\tilde{u}_1}^2 + 1 \Big) - s_u^2 m_{\tilde{u}_2}^2 \Big(-\log m_{\tilde{u}_2}^2 + 1 \Big) - c_d^2 m_{\tilde{d}_1}^2 \Big(-\log m_{\tilde{d}_1}^2 + 1 \Big) \\
&- s_d^2 m_{\tilde{d}_2}^2 \Big(-\log m_{\tilde{d}_2}^2 + 1 \Big) + 2c_u^2 c_d^2 \int dx \left(-\Delta_{\tilde{u}_1\tilde{d}_1}\log\Delta_{\tilde{u}_1\tilde{d}_1} + \Delta_{\tilde{u}_1\tilde{d}_1} \right) \\
&+ 2c_u^2 s_d^2 \int dx \left(-\Delta_{\tilde{u}_1\tilde{d}_2}\log\Delta_{\tilde{u}_1\tilde{d}_2} + \Delta_{\tilde{u}_1\tilde{d}_2} \right) + 2s_u^2 c_d^2 \int dx \left(-\Delta_{\tilde{u}_2\tilde{d}_1}\log\Delta_{\tilde{u}_2\tilde{d}_1} + \Delta_{\tilde{u}_2\tilde{d}_1} \right) \\
&+ 2s_u^2 s_d^2 \int dx \left(-\Delta_{\tilde{u}_2\tilde{d}_2}\log\Delta_{\tilde{u}_2\tilde{d}_2} + \Delta_{\tilde{u}_2\tilde{d}_2} \right) \Big\}
\end{aligned}$$

$$
\begin{aligned}
&= (-1)\cdot m_W^2\Big\{c_u^2 m_{\tilde u_1}^2 F(0,m_{\tilde u_1}^2,m_{\tilde u_1}^2) + s_u^2 m_{\tilde u_2}^2 F(0,m_{\tilde u_2}^2,m_{\tilde u_2}^2) + c_d^2 m_{\tilde d_1}^2 F(0,m_{\tilde d_1}^2,m_{\tilde d_2}^2) + s_d^2 m_{\tilde d_2}^2 F(0,m_{\tilde d_2}^2,m_{\tilde d_2}^2) \\
&\quad - c_u^2 m_{\tilde u_1}^2 - s_u^2 m_{\tilde u_2}^2 - c_d^2 m_{\tilde d_1}^2 - s_d^2 m_{\tilde d_2}^2 + c_u^2 c_d^2 \int dx \left(-2\Delta_{\tilde u_1 \tilde d_1}\log\Delta_{\tilde u_1 \tilde d_1} - \frac{1}{3}k^2\right) \\
&\quad + c_u^2 s_d^2 \int \left(-2\Delta_{\tilde u_1 \tilde d_2}\log\Delta_{\tilde u_1 \tilde d_2} - \frac{1}{3}k^2\right) + s_u^2 c_d^2 \int dx \left(-2\Delta_{\tilde u_2 \tilde d_1}\log\Delta_{\tilde u_2 \tilde d_1} - \frac{1}{3}k^2\right) \\
&\quad + s_u^2 s_d^2 \int dx \left(-2\Delta_{\tilde u_2 \tilde d_2}\log\Delta_{\tilde u_2 \tilde d_2} - \frac{1}{3}k^2\right) + c_u^2 m_{\tilde u_1}^2 + s_u^2 m_{\tilde u_2}^2 + c_d^2 m_{\tilde d_1}^2 + s_d^2 m_{\tilde d_2}^2\Big\} \\
&= m_W^2\Big\{ - c_u^2 m_{\tilde u_1}^2 F(0,m_{\tilde u_1}^2,m_{\tilde u_1}^2) - s_u^2 m_{\tilde u_2}^2 F(0,m_{\tilde u_2}^2,m_{\tilde u_2}^2) - c_d^2 m_{\tilde d_1}^2 F(0,m_{\tilde d_1}^2,m_{\tilde d_2}^2) - s_d^2 m_{\tilde d_2}^2 F(0,m_{\tilde d_2}^2,m_{\tilde d_2}^2) \\
&\quad + c_u^2 c_d^2 J(k^2,m_{\tilde u_1}^2,m_{\tilde d_1}^2) + c_u^2 s_d^2 J(k^2,m_{\tilde u_1}^2,m_{\tilde d_2}^2) + s_u^2 c_d^2 J(k^2,m_{\tilde u_2}^2,m_{\tilde d_1}^2) + s_u^2 s_d^2 J(k^2,m_{\tilde u_2}^2,m_{\tilde d_2}^2)\Big\}
\end{aligned}
$$

From the first line to the second, we used the result of the auxiliary calculation, eq. E.13.

E.3.4 A, $H^\pm$ and Quarks

Let us calculate the self-energy of the A and $H^\pm$ originating from quarks running in the loops. Consider the graphs of figs. F.1, F.24, F.25. Drop the i, factor out $g^2/(32\pi^2 m_W^2)$, and note that contributions from A enter the mass sum rule with a plus, those from $H^\pm$ with a minus (see eq. 3.3).

$$
\begin{aligned}
S^q_{A,H^\pm} &= \int dx \Big\{2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\Delta_{ud}(1-2\log\Delta_{ud}) \\
&\qquad + \left[2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)x(1-x)q^2 - 4m_u^2 m_d^2\right]\log\Delta_{ud}\Big\} \\
&\qquad - m_u^2\cot^2\beta\int dx\Big\{-\log\Delta_{uu}\big(3\Delta_{uu} - 2m_u^2\big) + \Delta_{uu}\Big\} \\
&\qquad - m_d^2\tan^2\beta\int dx\Big\{-\log\Delta_{dd}\big(3\Delta_{dd} - 2m_d^2\big) + \Delta_{dd}\Big\} \\
&= 2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\int dx\Delta_{ud} - 4(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\int dx\Delta_{ud}\log\Delta_{ud} \\
&\quad + 2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\int dx x(1-x)q^2\log\Delta_{ud} - 4m_u^2 m_d^2\int dx\log\Delta_{ud} \\
&\quad + 6m_u^2\cot^2\beta\int dx\Delta_{uu}\log\Delta_{uu} - 4m_u^4\cot^2\beta\int dx\log\Delta_{uu} - 2m_u^2\cot^2\beta\int dx\Delta_{uu} \\
&\quad + 6m_d^2\tan^2\beta\int dx\Delta_{dd}\log\Delta_{dd} - 4m_d^4\tan^2\beta\int dx\log\Delta_{dd} - 2m_d^2\tan^2\beta\int dx\Delta_{dd}
\end{aligned}
$$

$$
\begin{aligned}
&= 2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\left[-\frac{1}{2}m_H^2 + \frac{1}{3}m_H^2 - \frac{1}{2}(m_u^2 - m_d^2) + m_u^2\right] \\
&\quad - 4(m_d^2\tan^2\beta + m_u^2\cot^2\beta)\big[-G(m_u,m_d,m_H) + H(m_u,m_d,m_H)\big] \\
&\quad + 2(m_d^2\tan^2\beta + m_u^2\cot^2\beta)G(m_u,m_d,m_H) - 4m_u^2m_d^2F(m_u,m_d,m_H) \\
&\quad + 6m_u^2\cot^2\beta\big[-G(m_u,m_u,m_A) + m_u^2F(m_u,m_u,m_A)\big] - 4m_u^4\cot^2\beta F(m_u,m_u,m_A) \\
&\quad + 6m_d^2\tan^2\beta\big[-G(m_d,m_d,m_A) + m_d^2F(m_d,m_d,m_A)\big] - 4m_d^4\tan^2\beta F(m_d,m_d,m_A) \\
&\quad - 2m_u^2\cot^2\beta\left[-\frac{1}{2}m_A^2 + \frac{1}{3}m_A^2 + m_u^2\right] - 2m_d^2\tan^2\beta\left[-\frac{1}{2}m_A^2 + \frac{1}{3}m_A^2 + m_u^2\right] \\
&= 2m_u^4\cot^2\beta F(m_u,m_u,m_A) - 2m_u^4\cot^2\beta - 6m_u^2\cot^2\beta G(m_u,m_u,m_A) + \frac{1}{3}m_u^2\cot^2\beta m_A^2 \\
&\quad + 2m_d^4\tan^2\beta F(m_d,m_d,m_A) - 2m_d^4\tan^2\beta - 6m_d^2\tan^2\beta G(m_d,m_d,m_A) + \frac{1}{3}m_d^2\tan^2\beta m_A^2 \\
&\quad - 4m_u^2m_d^2F(m_u,m_d,m_H) + (m_d^2\tan^2\beta + m_u^2\cot^2\beta)\left[2\left(-\frac{1}{6}m_H^2 + \frac{1}{2}(m_u^2 + m_d^2)\right)\right. \\
&\quad \left. + 4\big[G(m_u,m_d,m_H) - H(m_u,m_d,m_H)\big] + 2G(m_u,m_d,m_H)\right] \\
&= 2m_u^2\cot^2\beta\left\{m_u^2\big[F(m_u,m_u,m_A) - 1\big] - 3G(m_u,m_u,m_A + \frac{1}{6}m_A^2)\right\} \\
&\quad + 2m_d^2\tan^2\beta\left\{m_d^2\big[F(m_d,m_d,m_A) - 1\big] - 3G(m_d,m_d,m_A + \frac{1}{6}m_A^2)\right\} - 4m_u^2m_d^2F(m_u,m_d,m_H) \\
&\quad + (m_d^2\tan^2\beta + m_u^2\cot^2\beta)\left[m_u^2 + m_d^2 - \frac{1}{3}m_H^2 + 6G(m_u,m_d,m_H) - 4H(m_u,m_d,m_H)\right]
\end{aligned}
$$

E.3.5 A, $H^\pm$ and Squarks

Let us calculate the self-energy of the A and $H^\pm$ originating from squarks running in the loops. Consider the graphs of figs. F.2, F.3, F.4, F.5, F.26, F.27, F.28, F.29, F.6, F.7, F.8, F.9, F.30, F.31, F.32, F.33. Drop the i, factor out $g^2/(32\pi^2 m_W^2)$, and note that contributions from A enter the mass sum rule with a plus, those from $H^\pm$ with a minus (see eq. 3.3).

$$
\begin{aligned}
S^{\tilde q}_{A,H^\pm} &= -(c_uc_dV_{\ell\ell} + s_us_dV_{rr} + c_us_dV_{\ell r} + s_uc_dV_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_1\tilde d_1}\big) \\
&\quad - (c_us_dV_{\ell\ell} + s_uc_dV_{rr} + c_uc_dV_{\ell r} - s_us_dV_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_1\tilde d_2}\big) \\
&\quad - (s_uc_dV_{\ell\ell} + c_us_dV_{rr} - s_us_dV_{\ell r} + c_uc_dV_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_2\tilde d_1}\big) \\
&\quad - (s_us_dV_{\ell\ell} + c_uc_dV_{rr} - s_uc_dV_{\ell r} - c_us_dV_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_2\tilde d_2}\big) \\
&\quad + \frac{1}{2}(V_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_1\tilde u_2}\big) + \frac{1}{2}(V_{r\ell})^2\int dx\big(-\log\Delta_{\tilde u_2\tilde u_1}\big) \\
&\quad + \frac{1}{2}(V_{\ell r})^2\int dx\big(-\log\Delta_{\tilde d_1\tilde d_2}\big) + \frac{1}{2}(V_{\ell r})^2\int dx\big(-\log\Delta_{\tilde d_2\tilde d_1}\big)
\end{aligned}
$$

$$
\begin{aligned}
&+ \big[- c_u^2 m_W^2 c_{2\beta} - m_d^2 \tan^2\beta c_u^2 - m_u^2 \cot^2\beta s_u^2 + m_u^2 \cot^2\beta \big] \big(- \log m_{\tilde u_1}^2 + 1 \big) m_{\tilde u_1}^2 \\
&+ \big[- s_u^2 m_W^2 c_{2\beta} - m_d^2 \tan^2\beta s_u^2 - m_u^2 \cot^2\beta c_u^2 + m_u^2 \cot^2\beta \big] \big(- \log m_{\tilde u_2}^2 + 1 \big) m_{\tilde u_2}^2 \\
&+ \big[c_d^2 m_W^2 c_{2\beta} - m_u^2 \cot^2\beta c_d^2 - m_d^2 \tan^2\beta s_d^2 + m_d^2 \tan^2\beta \big] \big(- \log m_{\tilde d_1}^2 + 1 \big) m_{\tilde d_1}^2 \\
&+ \big[s_d^2 m_W^2 c_{2\beta} - m_u^2 \cot^2\beta s_d^2 - m_d^2 \tan^2\beta c_d^2 + m_d^2 \tan^2\beta \big] \big(- \log m_{\tilde d_2}^2 + 1 \big) m_{\tilde d_2}^2 \\
= &\ (c_u c_d V_{\ell\ell} + s_u s_d V_{rr} + c_u s_d V_{\ell r} + s_u c_d V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde d_1}, m_H) \\
&+ (c_u s_d V_{\ell\ell} + s_u c_d V_{rr} + c_u c_d V_{\ell r} - s_u s_d V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde d_2}, m_H) \\
&+ (s_u c_d V_{\ell\ell} + c_u s_d V_{rr} - s_u s_d V_{\ell r} + c_u c_d V_{r\ell})^2 F(m_{\tilde u_2}, m_{\tilde d_1}, m_H) \\
&+ (s_u s_d V_{\ell\ell} + c_u c_d V_{rr} - s_u c_d V_{\ell r} - c_u s_d V_{r\ell})^2 F(m_{\tilde u_2}, m_{\tilde d_2}, m_H) \\
&- (V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde u_2}, m_A) - (V_{\ell r})^2 F(m_{\tilde d_1}, m_{\tilde d_2}, m_A) \\
&+ c_u^2 m_{\tilde u_1}^2 \big[- m_W^2 c_{2\beta} - m_d^2 \tan^2\beta + m_u^2 \cot^2\beta \big] \big(- \log m_{\tilde u_1}^2 + 1 \big) \\
&+ s_u^2 m_{\tilde u_2}^2 \big[- m_W^2 c_{2\beta} - m_d^2 \tan^2\beta + m_u^2 \cot^2\beta \big] \big(- \log m_{\tilde u_2}^2 + 1 \big) \\
&+ c_d^2 m_{\tilde d_1}^2 \big[m_W^2 c_{2\beta} - m_u^2 \cot^2\beta + m_d^2 \tan^2\beta \big] \big(- \log m_{\tilde d_1}^2 + 1 \big) \\
&+ s_d^2 m_{\tilde d_2}^2 \big[m_W^2 c_{2\beta} - m_u^2 \cot^2\beta + m_d^2 \tan^2\beta \big] \big(- \log m_{\tilde d_2}^2 + 1 \big) \\
= &\ (c_u c_d V_{\ell\ell} + s_u s_d V_{rr} + c_u s_d V_{\ell r} + s_u c_d V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde d_1}, m_H) \\
&+ (c_u s_d V_{\ell\ell} + s_u c_d V_{rr} + c_u c_d V_{\ell r} - s_u s_d V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde d_2}, m_H) \\
&+ (s_u c_d V_{\ell\ell} + c_u s_d V_{rr} - s_u s_d V_{\ell r} + c_u c_d V_{r\ell})^2 F(m_{\tilde u_2}, m_{\tilde d_1}, m_H) \\
&+ (s_u s_d V_{\ell\ell} + c_u c_d V_{rr} - s_u c_d V_{\ell r} - c_u s_d V_{r\ell})^2 F(m_{\tilde u_2}, m_{\tilde d_2}, m_H) \\
&- (V_{r\ell})^2 F(m_{\tilde u_1}, m_{\tilde u_2}, m_A) - (V_{\ell r})^2 F(m_{\tilde d_1}, m_{\tilde d_2}, m_A) \\
&+ c_u^2 m_{\tilde u_1}^2 V \big[F(m_{\tilde u_1}, m_{\tilde u_1}, 0) - 1 \big] + s_u^2 m_{\tilde u_2}^2 V \big[F(m_{\tilde u_2}, m_{\tilde u_2}, 0) - 1 \big] \\
&- c_d^2 m_{\tilde d_1}^2 V \big[F(m_{\tilde d_1}, m_{\tilde d_1}, 0) - 1 \big] - s_d^2 m_{\tilde d_2}^2 V \big[F(m_{\tilde d_2}, m_{\tilde d_2}, 0) - 1 \big]
\end{aligned}
$$

E.3.6 Conclusion

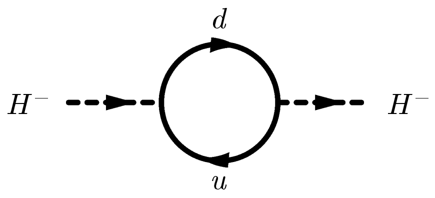
Our result is in perfect agreement with that of ref. [8]. The difference in the sign of μ, the coefficient of $H_1 H_2$ in the superpotential, is due to different conventions.

Appendix F

Feynman Graphs

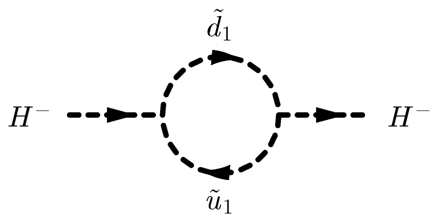
In calculating the self-energy graphs listed in this appendix, we used the Feynman rules from refs. [25, 26] and crosschecked them with ref. [27]. For each diagram, the final result was checked with the literature [22, 23, 43].

F.1 Graphs Contributing to the Self-energy of $H^\pm$



$$
\begin{aligned}
= \frac{-ig^2}{32\pi^2 m_W^2} \int dx\, \frac{2}{\bar{\epsilon}} & \left[4(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta)\Delta_{ud} - 2(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta)x(1-x)q^2 + 4m_u^2 m_d^2\right] \\
& + 2(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta)\Delta_{ud}(1 - 2\log\Delta_{ud}) \\
& + \left[2(m_d^2 \tan^2\beta + m_u^2 \cot^2\beta)x(1-x)q^2 - 4m_u^2 m_d^2\right] \log\Delta_{ud}
\end{aligned}
$$

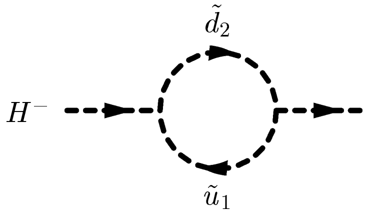
Figure F.1: Graph contributing to the self-energy of $H^\pm$



$$
= \frac{ig^2}{32\pi^2 m_W^2} [V(H^- \tilde{u}_1 \tilde{d}_1)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{d}_1}\right)
$$

$$
\begin{aligned}
V(H^- \tilde{u}_1 \tilde{d}_1) = {} & c_u c_d \left[-m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta\right] \\
& + s_u s_d m_u m_d(\tan\beta + \cot\beta) + c_u s_d m_d(\mu + A_d \tan\beta) + s_u c_d m_u(\mu + A_u \cot\beta)
\end{aligned}
$$

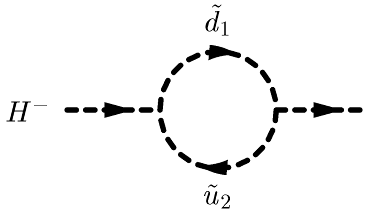
Figure F.2: Graph contributing to the self-energy of $H^\pm$



$$= \frac{ig^2}{32\pi^2 m_W^2}[V(H^-\tilde{u}_1\tilde{d}_2)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1\tilde{d}_2}\right)$$

$$\begin{aligned} V(H^-\tilde{u}_1\tilde{d}_2) &= -c_u s_d \left[-m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta\right] \\ &+ s_u c_d m_u m_d(\tan\beta + \cot\beta) + c_u c_d m_d(\mu + A_d \tan\beta) - s_u s_d m_u(\mu + A_u \cot\beta) \end{aligned}$$

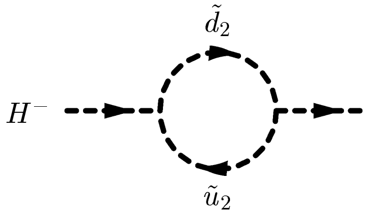
Figure F.3: Graph contributing to the self-energy of $H^\pm$



$$= \frac{ig^2}{32\pi^2 m_W^2}[V(H^-\tilde{u}_2\tilde{d}_1)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{d}_1}\right)$$

$$\begin{aligned} V(H^-\tilde{u}_2\tilde{d}_1) &= -s_u c_d \left[-m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta\right] \\ &+ c_u s_d m_u m_d(\tan\beta + \cot\beta) - s_u s_d m_d(\mu + A_d \tan\beta) + c_u c_d m_u(\mu + A_u \cot\beta) \end{aligned}$$

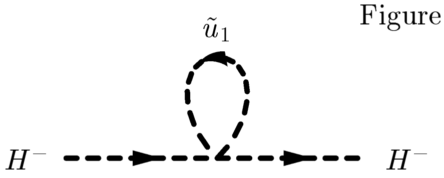
Figure F.4: Graph contributing to the self-energy of $H^\pm$



$$= \frac{ig^2}{32\pi^2 m_W^2}[V(H^-\tilde{u}_2\tilde{d}_2)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{d}_2}\right)$$

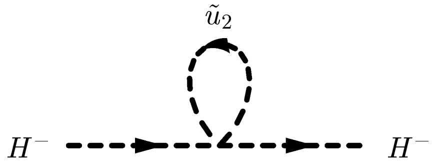
$$\begin{aligned} V(H^-\tilde{u}_2\tilde{d}_2) &= s_u s_d \left[-m_W^2 s_{2\beta} + m_d^2 \tan\beta + m_u^2 \cot\beta\right] \\ &+ c_u c_d m_u m_d(\tan\beta + \cot\beta) - s_u c_d m_d(\mu + A_d \tan\beta) - c_u s_d m_u(\mu + A_u \cot\beta) \end{aligned}$$

Figure F.5: Graph contributing to the self-energy of $H^\pm$



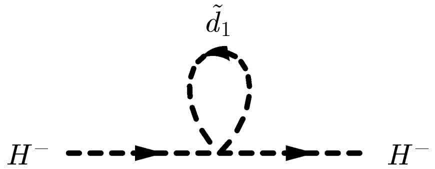
$$= -\frac{ig^2}{32\pi^2}\left[-c_u^2 c_{2\beta} + \frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\beta} - (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2\tan^2\beta}{m_W^2}c_u^2 - \frac{m_u^2\cot^2\beta}{m_W^2}s_u^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right)m_{\tilde{u}_1}^2$$

Figure F.6: Graph contributing to the self-energy of $H^\pm$



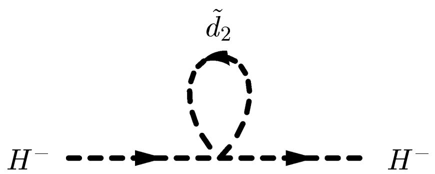
$$= -\frac{ig^2}{32\pi^2}\left[-s_u^2 c_{2\beta} + \frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\beta} + (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2\tan^2\beta}{m_W^2}s_u^2 - \frac{m_u^2\cot^2\beta}{m_W^2}c_u^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right)m_{\tilde{u}_2}^2$$

Figure F.7: Graph contributing to the self-energy of $H^\pm$



$$= -\frac{ig^2}{32\pi^2}\left[c_d^2 c_{2\beta} - \frac{1}{2}\frac{c_d^2}{c_w^2}c_{2\beta} - (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2\cot^2\beta}{m_W^2}c_d^2 - \frac{m_d^2\tan^2\beta}{m_W^2}s_d^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1\right)m_{\tilde{d}_1}^2$$

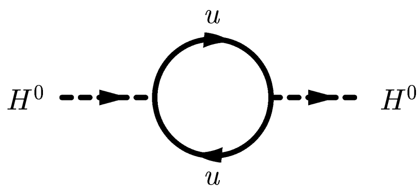
Figure F.8: Graph contributing to the self-energy of $H^\pm$



$$= -\frac{ig^2}{32\pi^2}\left[s_d^2 c_{2\beta} - \frac{1}{2}\frac{s_d^2}{c_w^2}c_{2\beta} + (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2\cot^2\beta}{m_W^2}s_d^2 - \frac{m_d^2\tan^2\beta}{m_W^2}c_d^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1\right)m_{\tilde{d}_2}^2$$

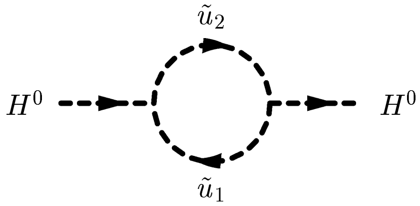
Figure F.9: Graph contributing to the self-energy of $H^\pm$

F.2 Graphs Contributing to the Self-energy of H^0



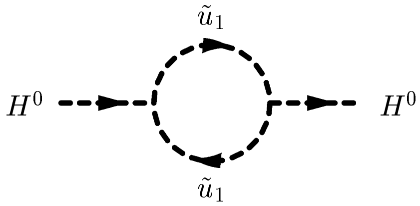
$$= -i\frac{g^2 m_u^2 \sin^2\alpha}{16\pi^2 m_W^2 \sin^2\beta}\int dx\,\left[\left(\frac{2}{\bar{\epsilon}} - \log\Delta_{uu}\right)3\Delta_{uu} + \Delta_{uu}\right]$$

Figure F.10: Graph contributing to the self-energy of H^0



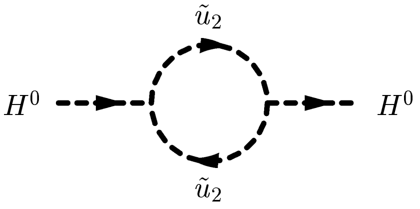
$$= -\frac{i}{16\pi^2}\left(\frac{igm_Z}{c_w}s_u c_u\left(\frac{1}{2} - 2e_u s_w^2\right)\cos(\alpha+\beta) + \frac{igm_u}{2m_W s_\beta}(c_u^2 - s_u^2)(\mu c_\alpha - A_u s_\alpha)\right)^2 \int dx\,\left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_2\tilde{u}_1}\right)$$

Figure F.11: Graph contributing to the self-energy of H^0



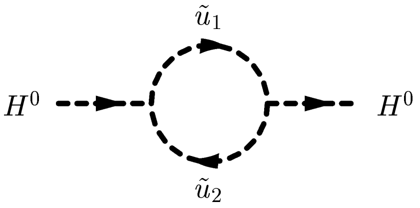
$$\begin{aligned} = -\frac{i}{16\pi^2}\Bigg(&-\frac{igm_Z}{c_w}\cos(\alpha+\beta)\left[\frac{1}{2}c_u^2 - (c_u^2 - s_u^2)e_u s_w^2\right] \\ &- \frac{igm_u^2}{m_W s_\beta}s_\alpha + \frac{igm_u}{m_W s_\beta}c_u s_u(\mu c_\alpha - A_u s_\alpha)\Bigg)^2 \int dx\,\left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{u}_1}\right) \end{aligned}$$

Figure F.12: Graph contributing to the self-energy of H^0



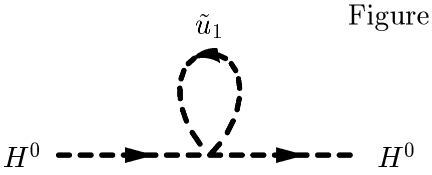
$$= -\frac{i}{16\pi^2}\Bigg(-\frac{igm_Z}{c_w}\cos(\alpha+\beta)\left[\frac{1}{2}s_u^2 + (c_u^2 - s_u^2)e_u s_w^2\right] \\ - \frac{igm_u^2}{m_W s_\beta}s_\alpha - \frac{igm_u}{m_W s_\beta}c_u s_u(\mu c_\alpha - A_u s_\alpha)\Bigg)^2 \int dx \, \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_2\tilde{u}_2}\right)$$

Figure F.13: Graph contributing to the self-energy of H^0



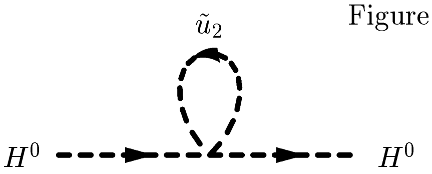
$$= -\frac{i}{16\pi^2}\left(\frac{igm_Z}{c_w}s_u c_u\left(\frac{1}{2} - 2e_u s_w^2\right)\cos(\alpha+\beta) + \frac{igm_u}{2m_W s_\beta}(c_u^2 - s_u^2)(\mu c_\alpha - A_u s_\alpha)\right)^2 \int dx \, \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{u}_2}\right)$$

Figure F.14: Graph contributing to the self-energy of H^0



$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\alpha} - (c_u^2 - s_u^2)e_u c_{2\alpha}\frac{s_w^2}{c_w^2} + \frac{m_u^2}{m_W^2}\frac{s_\alpha^2}{s_\beta^2}\right]\int dx \, \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right) m_{\tilde{u}_1}^2$$

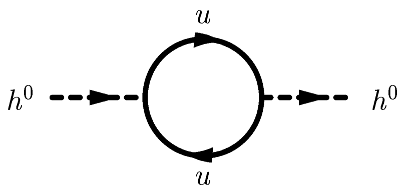
Figure F.15: Graph contributing to the self-energy of H^0



$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\alpha} + (c_u^2 - s_u^2)e_u c_{2\alpha}\frac{s_w^2}{c_w^2} + \frac{m_u^2}{m_W^2}\frac{s_\alpha^2}{s_\beta^2}\right]\int dx \, \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right) m_{\tilde{u}_2}^2$$

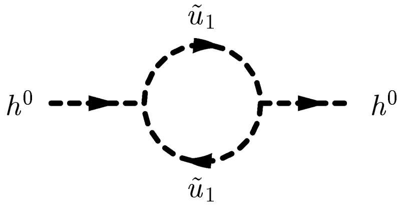
Figure F.16: Graph contributing to the self-energy of H^0

F.3 Graphs Contributing to the Self-energy of h^0



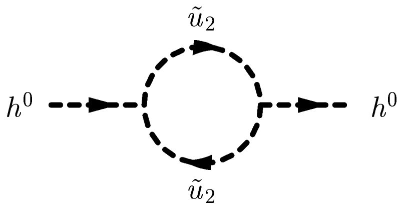
$$= -i \frac{g^2 m_u^2 \cos^2\alpha}{16\pi^2 m_W^2 \sin^2\beta} \int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{uu} \right) 3\Delta_{uu} + \Delta_{uu} \right]$$

Figure F.17: Graph contributing to the self-energy of h^0



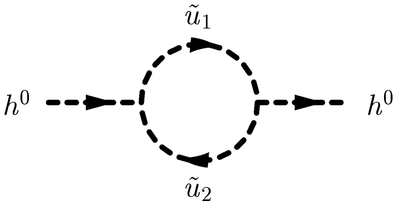
$$= -\frac{i}{16\pi^2} \left(\frac{igm_Z}{c_w} \sin(\alpha+\beta) \left[\frac{1}{2} c_u^2 - (c_u^2 - s_u^2) e_u s_w^2 \right] - \frac{igm_u^2}{m_W s_\beta} c_\alpha - \frac{igm_u}{m_W s_\beta} c_u s_u (\mu s_\alpha + A_u c_\alpha) \right)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1 \tilde{u}_1} \right)$$

Figure F.18: Graph contributing to the self-energy of h^0



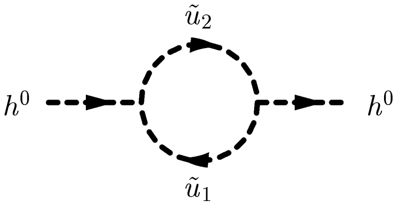
$$= -\frac{i}{16\pi^2} \left(\frac{igm_Z}{c_w} \sin(\alpha+\beta) \left[\frac{1}{2} s_u^2 + (c_u^2 - s_u^2) e_u s_w^2 \right] - \frac{igm_u^2}{m_W s_\beta} c_\alpha + \frac{igm_u}{m_W s_\beta} c_u s_u (\mu s_\alpha + A_u c_\alpha) \right)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2 \tilde{u}_2} \right)$$

Figure F.19: Graph contributing to the self-energy of h^0



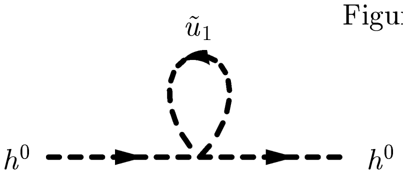
$$= -\frac{i}{16\pi^2}\left(-\frac{igm_Z}{c_w}s_u c_u\left(\frac{1}{2}-2e_u s_w^2\right)\sin(\alpha+\beta)-\frac{igm_u}{2m_W s_\beta}(c_u^2-s_u^2)(\mu s_\alpha + A_u c_\alpha)\right)^2\int dx\,\left(\frac{2}{\bar{\epsilon}}-\log\Delta_{\tilde{u}_1\tilde{u}_2}\right)$$

Figure F.20: Graph contributing to the self-energy of h^0



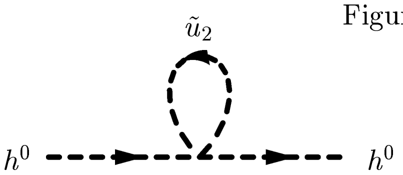
$$= -\frac{i}{16\pi^2}\left(-\frac{igm_Z}{c_w}s_u c_u\left(\frac{1}{2}-2e_u s_w^2\right)\sin(\alpha+\beta)-\frac{igm_u}{2m_W s_\beta}(c_u^2-s_u^2)(\mu s_\alpha + A_u c_\alpha)\right)^2\int dx\,\left(\frac{2}{\bar{\epsilon}}-\log\Delta_{\tilde{u}_2\tilde{u}_1}\right)$$

Figure F.21: Graph contributing to the self-energy of h^0



$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\alpha}-(c_u^2-s_u^2)e_u c_{2\alpha}\frac{s_w^2}{c_w^2}-\frac{m_u^2}{m_W^2}\frac{c_\alpha^2}{s_\beta^2}\right]\int dx\,\left(\frac{2}{\bar{\epsilon}}-\log m_{\tilde{u}_1}^2+1\right)m_{\tilde{u}_1}^2$$

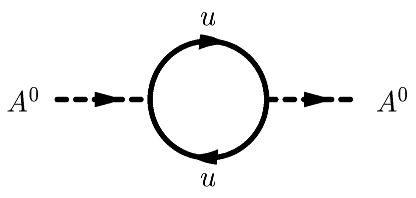
Figure F.22: Graph contributing to the self-energy of h^0



$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\alpha}+(c_u^2-s_u^2)e_u c_{2\alpha}\frac{s_w^2}{c_w^2}-\frac{m_u^2}{m_W^2}\frac{c_\alpha^2}{s_\beta^2}\right]\int dx\,\left(\frac{2}{\bar{\epsilon}}-\log m_{\tilde{u}_2}^2+1\right)m_{\tilde{u}_2}^2$$

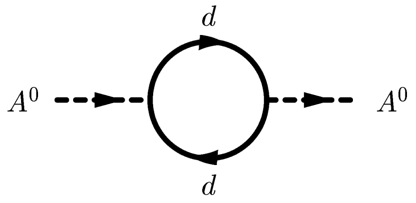
Figure F.23: Graph contributing to the self-energy of h^0

F.4 Graphs Contributing to the Self-energy of A^0



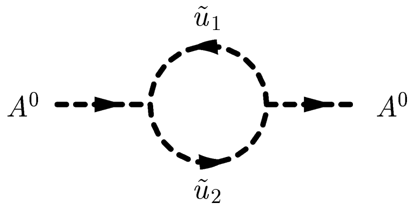
$$= -i\frac{g^2 m_u^2 \cot^2\beta}{16\pi^2 m_W^2}\int dx\,\left[\left(\frac{2}{\bar\epsilon} - \log\Delta_{uu}\right)(3\Delta_{uu} - 2m_u^2) + \Delta_{uu}\right]$$

Figure F.24: Graph contributing to the self-energy of A^0



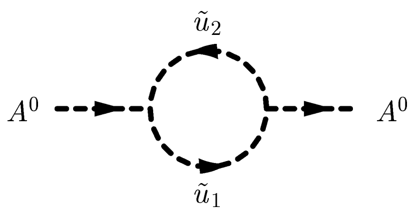
$$= -i\frac{g^2 m_d^2 \tan^2\beta}{16\pi^2 m_W^2}\int dx\,\left[\left(\frac{2}{\bar\epsilon} - \log\Delta_{dd}\right)(3\Delta_{dd} - 2m_d^2) + \Delta_{dd}\right]$$

Figure F.25: Graph contributing to the self-energy of A^0



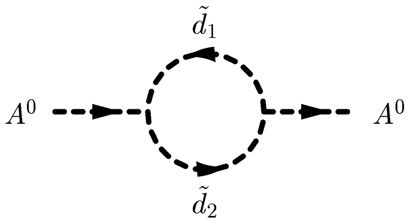
$$= i\frac{g^2 m_u^2}{64\pi^2 m_W^2}(\mu + A_u\cot\beta)^2\int dx\,\left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_1\tilde u_2}\right)$$

Figure F.26: Graph contributing to the self-energy of A^0



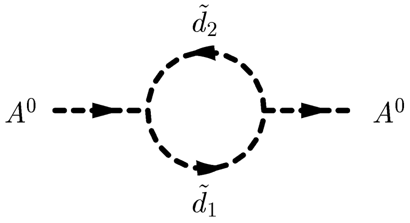
$$= i\frac{g^2 m_u^2}{64\pi^2 m_W^2}(\mu + A_u\cot\beta)^2\int dx\,\left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde u_2\tilde u_1}\right)$$

Figure F.27: Graph contributing to the self-energy of A^0



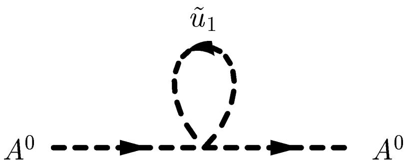
$$= i\frac{g^2 m_d^2}{64\pi^2 m_W^2}(\mu + A_d \tan\beta)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{d}_1 \tilde{d}_2}\right)$$

Figure F.28: Graph contributing to the self-energy of A^0



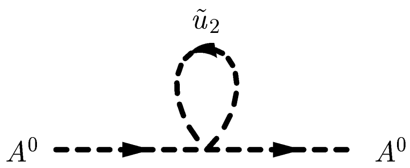
$$= i\frac{g^2 m_d^2}{64\pi^2 m_W^2}(\mu + A_d \tan\beta)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{d}_2 \tilde{d}_1}\right)$$

Figure F.29: Graph contributing to the self-energy of A^0



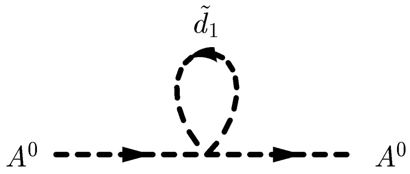
$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\beta} - (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2}{m_W^2}\cot^2\beta\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right) m_{\tilde{u}_1}^2$$

Figure F.30: Graph contributing to the self-energy of A^0



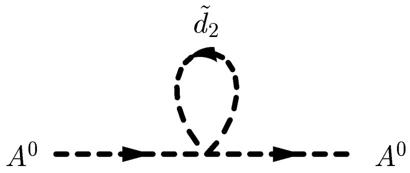
$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\beta} + (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2}{m_W^2}\cot^2\beta\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right)m_{\tilde{u}_2}^2$$

Figure F.31: Graph contributing to the self-energy of A^0



$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_d^2}{c_w^2}c_{2\beta} + (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} + \frac{m_d^2}{m_W^2}\tan^2\beta\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1\right)m_{\tilde{d}_1}^2$$

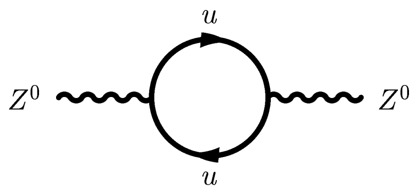
Figure F.32: Graph contributing to the self-energy of A^0



$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_d^2}{c_w^2}c_{2\beta} - (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} + \frac{m_d^2}{m_W^2}\tan^2\beta\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1\right)m_{\tilde{d}_2}^2$$

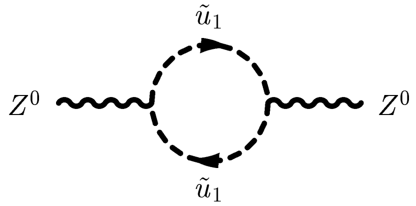
Figure F.33: Graph contributing to the self-energy of A^0

F.5 Graphs Contributing to the Self-energy of Z^0



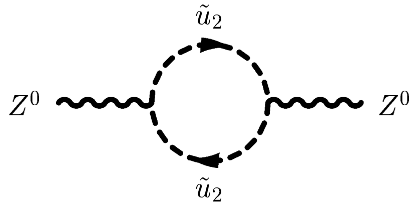
$$= \frac{ig^2}{16\pi^2 c_w^2} \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{uu}\right) \left(2\left[\frac{1}{2} - \frac{4}{3}s_w^2 + \frac{16}{9}s_w^4\right] x(1-x)(q^\mu q^\nu - g^{\mu\nu}q^2) + \frac{1}{2}m_u^2 g^{\mu\nu}\right)$$

Figure F.34: Graph contributing to the self-energy of Z^0



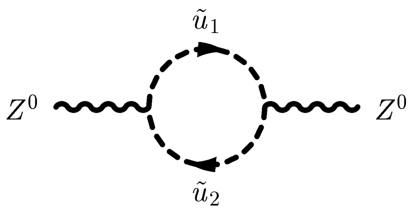
$$= \frac{ig^2}{64\pi^2 c_w^2} \left[- c_u^2 + 2e_u s_w^2\right]^2 \int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1\tilde{u}_1}\right) \left(2\Delta_{\tilde{u}_1\tilde{u}_1} g^{\mu\nu} + (1-2x)^2 q^\mu q^\nu\right) + 2\Delta_{\tilde{u}_1\tilde{u}_1} g^{\mu\nu}\right]$$

Figure F.35: Graph contributing to the self-energy of Z^0



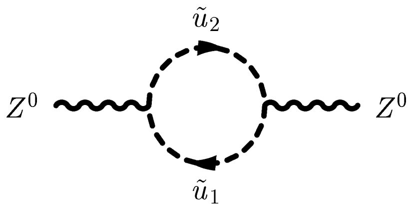
$$= \frac{ig^2}{64\pi^2 c_w^2} \left[- s_u^2 + 2e_u s_w^2\right]^2 \int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{u}_2}\right) \left(2\Delta_{\tilde{u}_2\tilde{u}_2} g^{\mu\nu} + (1-2x)^2 q^\mu q^\nu\right) + 2\Delta_{\tilde{u}_2\tilde{u}_2} g^{\mu\nu}\right]$$

Figure F.36: Graph contributing to the self-energy of Z^0



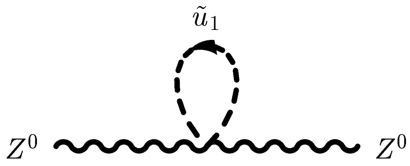
$$= \frac{ig^2}{64\pi^2 c_w^2} c_u^2 s_u^2 \int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_1 \tilde{u}_2} \right) \left(2\Delta_{\tilde{u}_1 \tilde{u}_2} g^{\mu\nu} + (1-2x)^2 q^\mu q^\nu \right) + 2\Delta_{\tilde{u}_1 \tilde{u}_2} g^{\mu\nu} \right]$$

Figure F.37: Graph contributing to the self-energy of Z^0



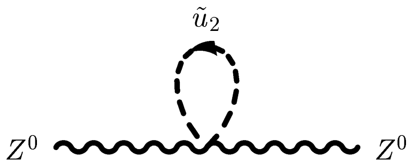
$$= \frac{ig^2}{64\pi^2 c_w^2} c_u^2 s_u^2 \int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2 \tilde{u}_1} \right) \left(2\Delta_{\tilde{u}_2 \tilde{u}_1} g^{\mu\nu} + (1-2x)^2 q^\mu q^\nu \right) + 2\Delta_{\tilde{u}_2 \tilde{u}_1} g^{\mu\nu} \right]$$

Figure F.38: Graph contributing to the self-energy of Z^0



$$= -\frac{ig^2}{8\pi^2 c_w^2} \left(\frac{1}{4} c_u^2 - \frac{2}{3} s_w^2 c_u^2 + \frac{4}{9} s_w^4 \right) g^{\mu\nu} \int dx \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1 \right) m_{\tilde{u}_1}^2$$

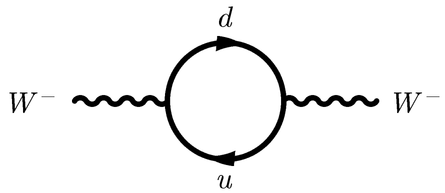
Figure F.39: Graph contributing to the self-energy of Z^0



$$= -\frac{ig^2}{8\pi^2 c_w^2} \left(\frac{1}{4} s_u^2 - \frac{2}{3} s_w^2 s_u^2 + \frac{4}{9} s_w^4 \right) g^{\mu\nu} \int dx \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1 \right) m_{\tilde{u}_2}^2$$

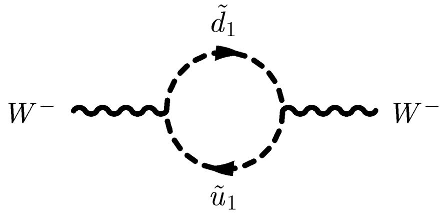
Figure F.40: Graph contributing to the self-energy of Z^0

F.6 Graphs Contributing to the Self-energy of $W^\pm$



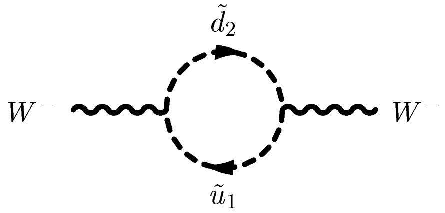
$$= \frac{ig^2}{16\pi^2}\int dx \left(\frac{2}{\bar\epsilon} - \log\Delta_{ud}\right)\left\{x(1-x)(2q^\mu q^\nu - g^{\mu\nu}q^2) + g^{\mu\nu}\Delta_{ud}\right\}$$

Figure F.41: Graph contributing to the self-energy of $W^\pm$



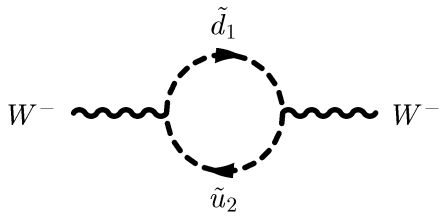
$$= \frac{ig^2}{32\pi^2}\, c_u^2 c_d^2 \int dx\, \left[\left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_1\tilde{d}_1} + 1\right) 2\Delta_{\tilde{u}_1\tilde{d}_1}\, g^{\mu\nu} + \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_1\tilde{d}_1}\right)(1-2x)^2 q^\mu q^\nu\right]$$

Figure F.42: Graph contributing to the self-energy of $W^\pm$



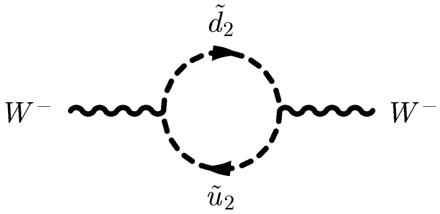
$$= \frac{ig^2}{32\pi^2}\, c_u^2 s_d^2 \int dx\, \left[\left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_1\tilde{d}_2} + 1\right) 2\Delta_{\tilde{u}_1\tilde{d}_2}\, g^{\mu\nu} + \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_1\tilde{d}_2}\right)(1-2x)^2 q^\mu q^\nu\right]$$

Figure F.43: Graph contributing to the self-energy of $W^\pm$



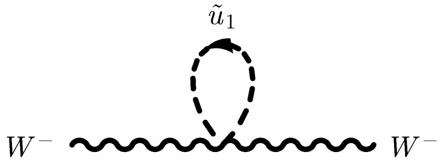
$$= \frac{ig^2}{32\pi^2}\, s_u^2 c_d^2 \int dx\, \left[\left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_2\tilde{d}_1} + 1\right) 2\Delta_{\tilde{u}_2\tilde{d}_1}\, g^{\mu\nu} + \left(\frac{2}{\bar\epsilon} - \log\Delta_{\tilde{u}_2\tilde{d}_1}\right)(1-2x)^2 q^\mu q^\nu\right]$$

Figure F.44: Graph contributing to the self-energy of $W^\pm$



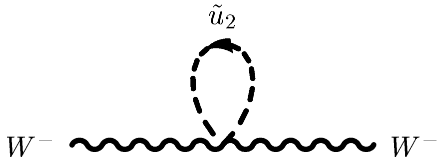
$$= \frac{ig^2}{32\pi^2}\, s_u^2 s_d^2 \int dx \, \left[\left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2 \tilde{d}_2} + 1 \right) 2\Delta_{\tilde{u}_2 \tilde{d}_2}\, g^{\mu\nu} + \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2 \tilde{d}_2} \right) (1-2x)^2 q^\mu q^\nu \right]$$

Figure F.45: Graph contributing to the self-energy of $W^\pm$



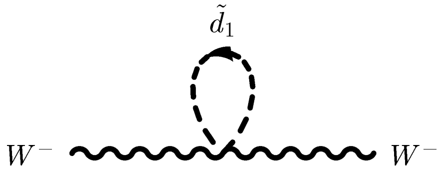
$$= -\frac{ig^2 c_u^2}{32\pi^2} g^{\mu\nu} \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1 \right) m_{\tilde{u}_1}^2$$

Figure F.46: Graph contributing to the self-energy of $W^\pm$



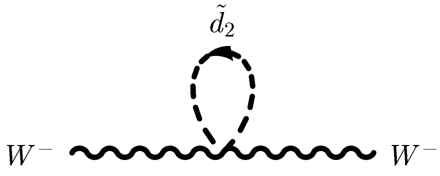
$$= -\frac{ig^2 s_u^2}{32\pi^2} g^{\mu\nu} \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1 \right) m_{\tilde{u}_2}^2$$

Figure F.47: Graph contributing to the self-energy of $W^\pm$



$$= -\frac{ig^2 c_d^2}{32\pi^2} g^{\mu\nu} \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1 \right) m_{\tilde{d}_1}^2$$

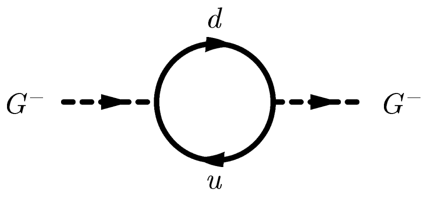
Figure F.48: Graph contributing to the self-energy of $W^\pm$



$$= -\frac{ig^2 s_d^2}{32\pi^2} g^{\mu\nu} \left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1 \right) m_{\tilde{d}_2}^2$$

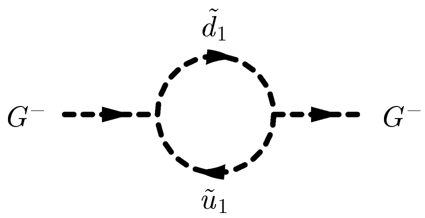
Figure F.49: Graph contributing to the self-energy of $W^\pm$

F.7 Graphs Contributing to the Self-energy of $G^{\pm}$



$$
= \frac{-ig^2}{32\pi^2 m_W^2} \int dx \, \frac{2}{\bar{\epsilon}} \left[4(m_u^2 + m_d^2)\Delta_{ud} - 2(m_u^2 + m_d^2)x(1-x)q^2 - 4m_u^2 m_d^2\right] + 2(m_u^2 + m_d^2)\Delta_{ud}(1 - 2\log\Delta_{ud}) \\
+ \left[2(m_u^2 + m_d^2)x(1-x)q^2 + 4m_u^2 m_d^2\right] \log\Delta_{ud}
$$

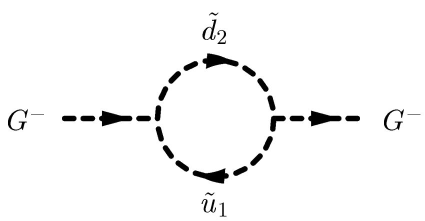
Figure F.50: Graph contributing to the self-energy of $G^{\pm}$



$$
= \frac{ig^2}{32\pi^2 m_W^2} [V(G^- \tilde{u}_1 \tilde{d}_1)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{d}_1}\right)
$$

$$
V(G^- \tilde{u}_1 \tilde{d}_1) = c_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] + c_u s_d m_d(\mu\tan\beta - A_d) - s_u c_d m_u(\mu\cot\beta - A_u)
$$

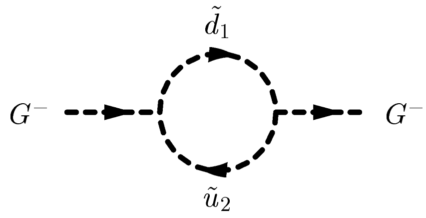
Figure F.51: Graph contributing to the self-energy of $G^{\pm}$



$$
= \frac{ig^2}{32\pi^2 m_W^2} [V(G^- \tilde{u}_1 \tilde{d}_2)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{d}_2}\right)
$$

$$
V(G^- \tilde{u}_1 \tilde{d}_2) = -c_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] + c_u c_d m_d(\mu\tan\beta - A_d) + s_u s_d m_u(\mu\cot\beta - A_u)
$$

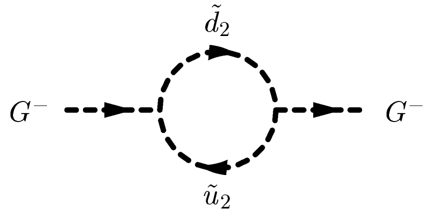
Figure F.52: Graph contributing to the self-energy of $G^{\pm}$



$$= \frac{ig^2}{32\pi^2 m_W^2}[V(G^-\tilde{u}_2\tilde{d}_1)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{d}_1}\right)$$

$$V(G^-\tilde{u}_2\tilde{d}_1) = -s_u c_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] - s_u s_d m_d(\mu \tan\beta - A_d) - c_u c_d m_u(\mu \cot\beta - A_u)$$

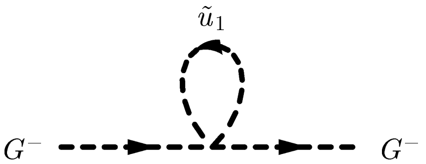
Figure F.53: Graph contributing to the self-energy of $G^{\pm}$



$$= \frac{ig^2}{32\pi^2 m_W^2}[V(G^-\tilde{u}_2\tilde{d}_2)]^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2\tilde{d}_2}\right)$$

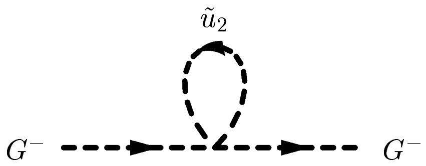
$$V(G^-\tilde{u}_2\tilde{d}_2) = s_u s_d \left[m_W^2 c_{2\beta} + m_u^2 - m_d^2\right] - s_u c_d m_d(\mu \tan\beta - A_d) + c_u s_d m_u(\mu \cot\beta - A_u)$$

Figure F.54: Graph contributing to the self-energy of $G^{\pm}$



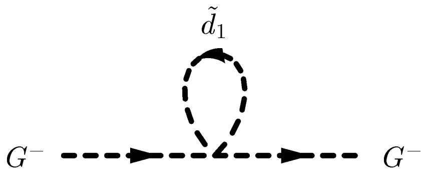
$$= -\frac{ig^2}{32\pi^2}\left[c_u^2 c_{2\beta} - \frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\beta} + (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2}{m_W^2}c_u^2 - \frac{m_u^2}{m_W^2}s_u^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right) m_{\tilde{u}_1}^2$$

Figure F.55: Graph contributing to the self-energy of $G^{\pm}$



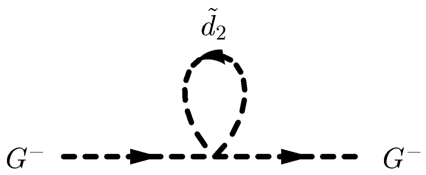
$$= -\frac{ig^2}{32\pi^2}\left[s_u^2 c_{2\beta} - \frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\beta} - (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2}{m_W^2}s_u^2 - \frac{m_u^2}{m_W^2}c_u^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right)m_{\tilde{u}_2}^2$$

Figure F.56: Graph contributing to the self-energy of $G^\pm$



$$= -\frac{ig^2}{32\pi^2}\left[-c_d^2 c_{2\beta} + \frac{1}{2}\frac{c_d^2}{c_w^2}c_{2\beta} + (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2}{m_W^2}c_d^2 - \frac{m_d^2}{m_W^2}s_d^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1\right)m_{\tilde{d}_1}^2$$

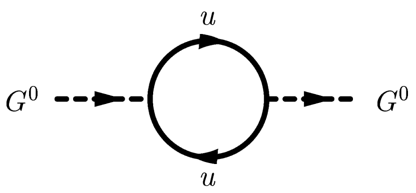
Figure F.57: Graph contributing to the self-energy of $G^\pm$



$$= -\frac{ig^2}{32\pi^2}\left[-s_d^2 c_{2\beta} + \frac{1}{2}\frac{s_d^2}{c_w^2}c_{2\beta} - (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_u^2}{m_W^2}s_d^2 - \frac{m_d^2}{m_W^2}c_d^2\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1\right)m_{\tilde{d}_2}^2$$

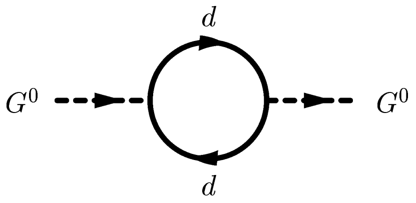
Figure F.58: Graph contributing to the self-energy of $G^\pm$

F.8 Graphs Contributing to the Self-energy of G^0



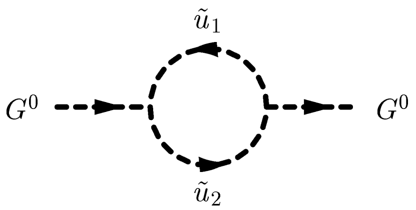
$$= -i\frac{g^2 m_u^2}{16\pi^2 m_W^2}\int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log\Delta_{uu}\right)(3\Delta_{uu} - 2m_u^2) + \Delta_{uu}\right]$$

Figure F.59: Graph contributing to the self-energy of G^0



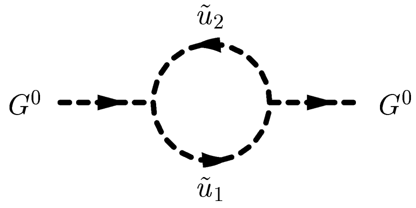
$$= -i\frac{g^2 m_d^2}{16\pi^2 m_W^2}\int dx \left[\left(\frac{2}{\bar{\epsilon}} - \log\Delta_{dd}\right)(3\Delta_{dd} - 2m_d^2) + \Delta_{dd}\right]$$

Figure F.60: Graph contributing to the self-energy of G^0



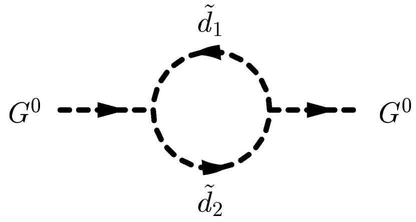
$$= i\frac{g^2 m_u^2}{64\pi^2 m_W^2}(\mu\cot\beta - A_u)^2\int dx \left(\frac{2}{\bar{\epsilon}} - \log\Delta_{\tilde{u}_1\tilde{u}_2}\right)$$

Figure F.61: Graph contributing to the self-energy of G^0



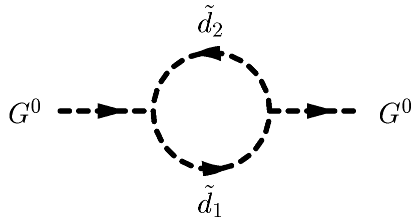
$$= i\frac{g^2 m_u^2}{64\pi^2 m_W^2}(\mu \cot\beta - A_u)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{u}_2 \tilde{u}_1}\right)$$

Figure F.62: Graph contributing to the self-energy of G^0



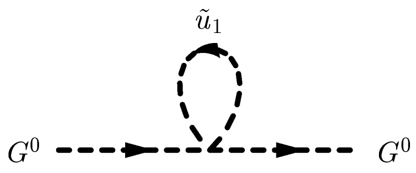
$$= i\frac{g^2 m_d^2}{64\pi^2 m_W^2}(\mu \tan\beta - A_d)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{d}_1 \tilde{d}_2}\right)$$

Figure F.63: Graph contributing to the self-energy of G^0



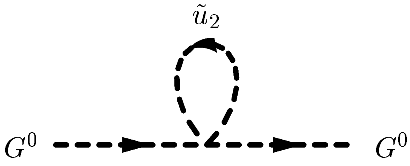
$$= i\frac{g^2 m_d^2}{64\pi^2 m_W^2}(\mu \tan\beta - A_d)^2 \int dx \left(\frac{2}{\bar{\epsilon}} - \log \Delta_{\tilde{d}_2 \tilde{d}_1}\right)$$

Figure F.64: Graph contributing to the self-energy of G^0



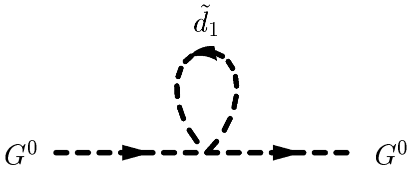
$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_u^2}{c_w^2}c_{2\beta} - (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} + \frac{m_u^2}{m_W^2}\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_1}^2 + 1\right) m_{\tilde{u}_1}^2$$

Figure F.65: Graph contributing to the self-energy of G^0



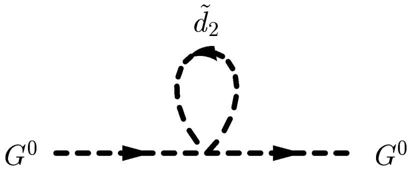
$$= \frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_u^2}{c_w^2}c_{2\beta} + (c_u^2 - s_u^2)e_u c_{2\beta}\frac{s_w^2}{c_w^2} + \frac{m_u^2}{m_W^2}\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{u}_2}^2 + 1\right) m_{\tilde{u}_2}^2$$

Figure F.66: Graph contributing to the self-energy of G^0



$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{c_d^2}{c_w^2}c_{2\beta} + (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2}{m_W^2}\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_1}^2 + 1\right) m_{\tilde{d}_1}^2$$

Figure F.67: Graph contributing to the self-energy of G^0



$$= -\frac{ig^2}{32\pi^2}\left[\frac{1}{2}\frac{s_d^2}{c_w^2}c_{2\beta} - (c_d^2 - s_d^2)e_d c_{2\beta}\frac{s_w^2}{c_w^2} - \frac{m_d^2}{m_W^2}\right]\left(\frac{2}{\bar{\epsilon}} - \log m_{\tilde{d}_2}^2 + 1\right) m_{\tilde{d}_2}^2$$

Figure F.68: Graph contributing to the self-energy of G^0

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