
Low Energy Spectra in Mirage Mediation

VALÉRI LÖWEN

Diplomarbeit in Physik
angefertigt im
Physikalischen Institut

vorgelegt der
Mathematisch–Naturwissenschaftlichen Fakultät
der
Rheinischen Friedrich–Wilhelms–Universität
Bonn

im November 2006

I hereby confirm that I have authored this diploma thesis independently and without use of others than the indicated resources.
All passages which are literally or in general matter taken out of publications or other resources are marked as such.

Examiner : Prof. Dr. Hans Peter Nilles
Second Examiner : Prof. Dr. Herbert Dreiner

Acknowledgments

I would like to express my gratitude to all those who have supported me, directly and indirectly, throughout my studies. First and foremost, I would like to thank Prof. Dr. Nilles for giving me the opportunity to work on such innovative topic and to breath the air of science. I would also like to thank Michael Ratz for his guidance in my research. Special thanks go to Oleg Lebedev for his helpful discussions. Furthermore, I am deeply grateful to the members of Prof. Dr. Nilles' group for their suggestions and encouragement. I wish to express my sincere thanks to Prof. Dr. Dreiner for his readiness to accept the task as a co-examiner.

Very special thanks go to my parents. Without their love and support, this thesis would not have seen the light of day.

All these feelings that give away from judgement, that confuse the hell out of us, that make second guess ourselves... We need them to help us fill in the missing pieces because we almost never have all the facts.

(A. Nissim, L. Perlmutter)

Contents

1	Introduction	1
2	Accession to Supersymmetry	3
2.1	Glimpse of the Standard Model	3
2.2	Way out	4
2.3	SUSY	5
2.3.1	Construction of supermultiplets	5
2.3.2	Superfield formalism	6
2.3.3	Lagrangians	7
2.4	The MSSM	9
2.4.1	R-Parity	10
2.5	SUGRA	12
3	Breakdown of Supersymmetry	15
3.1	Concatenation	15
3.2	Soft Terms in the MSSM	20
3.2.1	Soft Breaking Universality	20
3.2.2	Electroweak Symmetry Breaking	21
3.2.3	Mixing	23
3.3	Spontaneously broken SUGRA	24
4	Mediations	27
4.1	Gravity Mediation	27
4.1.1	mSUGRA	30
4.1.2	Properties of mSUGRA	30
4.2	Anomaly Mediation	31
4.2.1	Properties of Anomaly Mediation	33
5	Mirage™ Scale Formation	35
5.1	SUSY violating de Sitter Vacua	35
5.1.1	Joining String Theory	35
5.1.2	KKLT Model	36
5.1.3	Little Hierarchy	38
5.1.4	Mirage Unification	40
5.1.5	de Sitter Vacua from Spontaneously Broken SUGRA	42
5.2	Soft Terms in Mirage Mediation	43

5.3	Low Energy Phenomenology	45
5.3.1	Case $\Delta\xi = 0$ (Ordinary Mirage Mediation)	47
5.3.2	Case $\Delta\xi \neq 0$ (Extended Mirage Mediation)	53
5.4	Heterotic Mirage Mediation	57
5.4.1	Phenomenological Aspects	59
5.5	Conclusion	62
A	Notations and Conventions	63
A.1	Two-Component Spinors	63
A.2	Superfield Language	64
A.3	Dimensions	65
B	Renormalization Group	67
B.1	RGE for the MSSM	67
B.2	Anomalous Dimension	70
	Glossary	73
	Bibliography	75
	Index	79

Chapter 1

Introduction

There might be different paths one can follow in order to improve the present status in particle physics. The direction suggested by the Standard Model (SM) [1, 2] seems to lead to interesting physical insights. On the one hand, one has theoretical milestones followed by successful experimental confirmation and, on the other hand, the SM gives rise to exceed the frontier and lay down new foundations.

One of the reasons to stay forward-bound is the so-called *hierarchy problem* which deals with the difficulty of stabilizing the large gap between the electroweak scale M_{EW} and the (reduced) Planck scale $M_P = (8\pi G)^{-1/2} \simeq 2.6 \times 10^{18}$ GeV. This problem is connected to the masses of the scalar particles which, in the framework of the SM, suffer from quadratic divergences. Such issue arises from a general property of the scalar fields in a gauge theory, namely the tendency of scalars to get their mass close to the largest available energy scale in the theory. Even though it is possible to keep the Higgs mass finite by performing unnatural fine-tuning, this procedure is unstable in perturbation theory [3].

In order to protect the masses of the scalar particles against radiative corrections, one can make use of the powerful concept known in physics as symmetry. Since there is no symmetry in the SM that protects the masses of the scalars, one needs an extension. The glory of being one possible solution to the hierarchy problem is enjoyed by the so-called *supersymmetry* (SUSY) [4, 5]. This symmetry introduces to every SM particle a *superparticle* which differs by spin. The particle content of the *Minimal Supersymmetric Standard Model* (MSSM) [6, 7] ensures that dangerous quadratic divergent contributions to the masses of scalar particles vanish. However, the experimental evidence for superpartners (being as light as their SM counterparts) is inexistent and, thus, SUSY must be broken at low energies. The understanding of SUSY breaking is a difficult enterprise. From the theoretical point of view, it is natural to consider a spontaneous breakdown, but this is problematic from the phenomenological perspective [7]. In the framework of the MSSM the breakdown can only occur *explicitly* but *softly* in order to keep the protection of the scalar masses. This trouble, caused by theoretical and phenomenological demands, can be solved if one introduces two separate sectors [7]. In the so-called “hidden” sector SUSY is broken spontaneously by new fields which are gauge singlets under the SM gauge group. This spontaneous breakdown appears parameterized as softly broken MSSM in the “observable” sector. In this manner the problem of describing the SUSY breakdown is traced to the issue of *mediating* the SUSY breakdown to the observable sector.

In the progress of SUSY several mediation types have been developed. Two of them have become widely accepted. In the so-called *gravity mediation* [8] the two sectors communicate through gravitational strength interactions whereas in the *anomaly mediation* [9] the connection between the sectors is established at loop level via a Weyl anomaly. These two mediation scenarios require only few parameters but, unfortunately, both are at odds with phenomenology. In gravity mediation the light gravitino (superpartner of the graviton) can spoil baryogenesis in the early universe [10, 11] whereas in anomaly mediation sleptons are tachyonic. Thus, an improved scenario would be desirable.

Providing the possibility for the unification and quantum description of all fundamental forces, string theory [12] has become a leading light in particle physics. Consistency requires string theory to live in 10 spacetime dimensions. Accordance with observation can be achieved e. g. if one compactifies the extra-dimensions on internal manifolds. In the effective low energy formulation, the compactification is parameterized by *moduli* [12]. These are massless scalar fields which transform as gauge singlets under the SM gauge group and, due to its presence, the hidden sector is naturally built-in in string theory. One essential task in string theory is to stabilize the moduli since its values, describing the shape and size of the internal manifolds, are connected to physical couplings. Moduli can be stabilized perturbatively (e. g. fluxes [13]) as well as non-perturbatively (e. g. gaugino condensation [14]). Recently, it was shown [15–17] that in schemes where moduli are stabilized in both ways, soft parameters receive *comparable* contributions from both gravity and anomaly mediation. In this scenario, called *mirage mediation*, the pattern of soft masses is very distinct [18–23]. One characteristic feature of the spectrum is a moderately large hierarchy among the soft, the gravitino and the moduli masses. Furthermore, the low energy phenomenology is described just by two parameters. Mirage mediation also provides possible solutions to the cosmological gravitino/moduli problems [24, 25]. It partially solves the SUSY CP and flavor problems [26] and it can reduce the fine-tuning of the weak scale [27, 28]. The presence of anomaly mediation, however, provides a scheme with tachyonic sleptons and squarks, and therefore further modifications are needed. In this work one possible solution to this problem is presented. It is based on a set-up, where mirage mediation with additional pattern arises.

Overview

The outline of this work is as follows. Chapter 2 briefly describes the SM and the construction of the MSSM with exact SUSY. In chapter 3, the softly broken MSSM is illustrated. In chapter 4, both mediation scenarios, gravity and anomaly mediation, are explained. Chapter 5 is the main part of this work. It describes the motivation for mirage mediation, and using an alternative approach, derives the soft breaking terms. These contain additional contributions which will be used in order to remove tachyons. At first “ordinary” mirage mediation is studied. Subsequently, a version of mirage mediation without tachyons will be analysed.

Chapter 2

Accession to Supersymmetry

In this chapter we will briefly refer to the Standard Model of particle physics and introduce supersymmetry as a possible solution to the hierarchy problem.

2.1 Glimpse of the Standard Model

The Standard Model (SM) with its local $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge invariance describes the strong, weak and electromagnetic interactions up to a scale of approximately 100 GeV [1, 2]. The SM consists of fermions and bosons. The fermionic sector contains 3 generations of left- and right-handed leptons and quarks.

$$L_i = \begin{pmatrix} \nu_{e_i} \\ e_i \end{pmatrix}_L, \quad e_{i_R}, \quad Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad u_{i_R}, \quad d_{i_R}, \quad \begin{matrix} e_i = \{e^-, \mu^-, \tau^-\} \\ u_i = \{u, c, t\} \\ d_i = \{d, s, b\} \end{matrix}.$$

The left-chiral fermion fields transform as doublets and the right-chiral as singlets under $SU(2)_L$. The quark fields transform as color triplets whereas the leptons are color singlets.

The gauge bosons of the SM gauge group are 8 gluons g_μ^a , 3 bosons W_μ and the B boson, respectively. The gauge fields transform as the adjoint representation of the gauge group. The gluons are always massless. The $SU(2)_L$ and $U(1)_Y$ gauge bosons are massless in the limit of exact electroweak (EW) symmetry.

At the electroweak scale $M_{EW} \sim 100$ GeV, the local $SU(2)_L \otimes U(1)_Y$ gets spontaneously broken to $U(1)_{EM}$. The spontaneous breakdown is initiated through the $SU(2)_L$ Higgs doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

A non-zero vacuum expectation value (VEV) of ϕ^0 , arising from minimization of the Higgs potential, spontaneously breaks the local $SU(2)_L \otimes U(1)_Y$ symmetry. This breakdown generates 3 massless Goldstone bosons which are “eaten” by the originally massless gauge bosons. While the photon γ remains massless, the weak bosons W^\pm and Z acquire mass through the VEV of the Higgs field. This VEV also generates fermion masses through Yukawa interactions.

Though baryon B and lepton L numbers are automatic symmetries in the SM, they can be violated at quantum level. In what follows we consider $m_\nu \equiv 0$.



Fig. 2.1 :: Quantum corrections to the Higgs (mass)² from a fermion loop (a) and a scalar loop (b).

2.2 Way out

Scalar particles entail a certain “ominousity” within the SM. Quantum corrections to the mass of a scalar particle suffer from quadratic divergences.

Consider for example 1-loop contributions to the Higgs mass, which come from every particle that couples to the Higgs field (see fig. 2.1 a). The most significant contribution comes from heavy Dirac fermions f (in the SM $f = top$) [6, 7]

$$\delta^f m_H^2 = \frac{|\lambda_f|^2}{16\pi^2} \left(-2\Lambda_{UV}^2 + 6m_f^2 \log \frac{\Lambda_{UV}}{m_f} + \dots \right), \quad (2.1)$$

where λ_f is a dimensionless coupling constant, m_f the mass of the fermion and Λ_{UV} is an ultraviolet momentum cut-off regulating the loop integral. As evident from eq. (2.1), the (mass)² correction badly diverges as $\Lambda_{UV} \rightarrow \infty$.

One can evade the problem of quadratic divergences by considering the 1-loop contribution to the Higgs mass coming from a (heavy) complex scalar particle S (fig. 2.1 b) which couples to the Higgs through $\Delta\mathcal{L} = -\lambda_S |H|^2 |S|^2$. Then [6, 7]

$$\delta^S m_H^2 = \frac{\lambda_S}{16\pi^2} \left(\Lambda_{UV}^2 - 3m_S^2 \log \frac{\Lambda_{UV}}{m_S} + \dots \right). \quad (2.2)$$

Assigning two scalars to one fermion and providing $|\lambda_f|^2 = \lambda_S$, the total quantum corrections to m_H^2 , namely $\delta_H = \delta^f + 2\delta^S$, would be free of quadratic divergences at 1-loop order. In this case, $\delta_H \sim \Delta m \cdot \log(\Lambda_{UV})$ with $\Delta m = m_S - m_f$. In addition, if claim $\Delta m = 0$ then at 1-loop level δ_H would neatly vanish. To ensure that quadratic divergences are absent to all orders in perturbation theory, our simple framework leads rather to a fine-tuning affair than to a “natural” protection of the Higgs mass to remain in the 100 GeV region. Nevertheless, the idea of connecting fermions and bosons was used to structure a strategy which has the power of a symmetry [4], known as *supersymmetry* (SUSY).

Indeed, it is the only “graded” Lie algebra of symmetries of the S -Matrix consistent with relativistic quantum field theory [29]. This symmetry lays down conditions for cancellation of quadratic divergences in perturbation theory. The reason for this is founded in the *non-renormalization theorem* [30,31] which implies that masses of scalar particles are not renormalized to any order in perturbation theory [5].

We see that the Higgs mass is very sensitive to new physics masked by Λ_{UV} . The new scalar particles which we have introduced cannot be arbitrarily heavy.

In fact, the SM requires the new scalars to be in the TeV region, otherwise the couplings in the Higgs sector would reach unnatural size [3]. We can also deduce it from our 1-loop toy-statement: the total correction $\delta_H \sim \Delta m$ should not be significantly larger than the mass of the Higgs.

2.3 SUSY

A generator of supersymmetry transformations Q turns a fermionic state into a bosonic one and vice versa:

$$Q|\text{fermion}\rangle = |\text{boson}\rangle, \quad Q|\text{boson}\rangle = |\text{fermion}\rangle.$$

The complex generator Q therefore is a fermionic operator featuring spin-1/2 angular momentum. Theories with $N > 1$ distinct copies of Q and \bar{Q} are called extended supersymmetries. Such theories have nice mathematical allurements but no phenomenological relevance [6]. In this work we will solely concentrate on $N = 1$ supersymmetry. Using the usual two-component notation, the SUSY algebra is given by [7, 32]

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}^\mu P_\mu, \quad (2.3a)$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \quad (2.3b)$$

$$[P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot{\alpha}}] = 0, \quad (2.3c)$$

where σ^μ are Pauli matrices and $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$ are the component indices. Conventions and notations are presented in the appendix A.

The single particle states fall into irreducible representations of the SUSY algebra called *supermultiplets*. The bosons and fermions within a supermultiplet are known as *superpartners* of each other. In an exact supersymmetric world the superpartners are degenerate in mass. Generators of SUSY transformations commute with those of gauge transformations. Therefore the superpartners of a supermultiplet are in the same representation of the gauge group, i.e. they have the same gauge charges. Each supermultiplet contains the same number of fermionic and bosonic degrees of freedom (DOF): $n_F = n_B$.

2.3.1 Construction of supermultiplets

The simplest possibility for a supermultiplet, which is consistent with the requirements in section 2.3, contains a Weyl fermion ($n_F = 2$) and a complex scalar ($n_B = 2$) called *sfermion* (short for scalar fermion). For the SUSY algebra to close off-shell, one has to introduce the so called *auxiliary fields* which do not propagate. Off-shell, the Weyl fermion has 4 DOF but the complex scalar still 2. So one complex auxiliary field with $n_B^a = 2$ is needed. This formation of a 2-component Weyl fermion and a complex scalar (with corresponding auxiliary field) is called *chiral supermultiplet*.

The next to the simplest supermultiplet would be a massless spin-1 gauge boson ($n_B = 2$) accompanied by a massless spin-1/2 Weyl fermion ($n_F = 2$) called gauge fermion or *gaugino*. In the off-shell situation, the gauge boson acquires an additional DOF (longitudinal polarization) while the Weyl fermion has four DOF. Thus one needs to introduce one real auxiliary field ($n_B^g = 1$).

Since gauge bosons transform as the adjoint representation of the gauge group, this is also obligatory for the gaugino and the auxiliary field. Moreover, for gauginos this implies that their left- and right-handed parts transform equivalently. Such a combination of a gauge boson and a gaugino is called *gauge* or *vector supermultiplet*. A further irreducible representation would be a combination of a spin-2 boson and a spin-3/2 gaugino.

2.3.2 Superfield formalism

To describe the physics of a supersymmetric theory it is advantageous to make use of the *superfield* formalism [7, 32].

Consider anticommuting Grassman parameters $\theta^\alpha, \bar{\theta}^{\dot{\beta}}$ ($\alpha, \dot{\beta} = 1, 2$) with

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0.$$

Superfields are functions that live on the $(x, \theta, \bar{\theta})$ space, called *superspace*. It is useful to introduce left- and right-chiral representations of the superalgebra. In the left-chiral representation a left-handed chiral superfield is given by [7]

$$\begin{aligned} \Phi_L(x, \theta) &= \varphi(x) + \sqrt{2}\theta^\alpha \xi_\alpha(x) + \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta F(x) \\ &= \varphi(x) + \sqrt{2}\theta\xi(x) + \theta^2 F(x), \end{aligned} \quad (2.4)$$

where φ is a complex scalar field, ξ is a left-handed (2-component) Weyl fermion and F the complex auxiliary field¹. These fields are often called component fields as they appear as components in the superspace language. These component fields behave under a SUSY transformation as

$$\begin{aligned} \delta\varphi &= \sqrt{2}\varepsilon\xi, \\ \delta\xi &= \sqrt{2}\varepsilon F - i\sqrt{2}\sigma_\mu\bar{\varepsilon}\partial^\mu\varphi, \\ \delta F &= i\sqrt{2}\partial^\mu\xi\sigma_\mu\bar{\varepsilon}, \end{aligned} \quad (2.5)$$

where ε is an infinitesimal anticommuting Weyl fermion parametrizing the SUSY transformation. The lowest components transform into higher ones; the highest component (F) transforms into a total derivative.

The vector superfields Γ are characterized by the reality condition $\Gamma^\dagger = \Gamma$. In the Wess-Zumino gauge, a vector superfield can be expressed as [7]

$$\Gamma(x, \theta, \bar{\theta}) = \theta\sigma_\mu\bar{\theta}A^\mu(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x), \quad (2.6)$$

¹Right-handed superfields can be obtained through conjugation and depend on $(x, \bar{\theta})$.

with A^μ being a spin-1 field, λ a Weyl spinor and D a real auxiliary field. Transformation properties of the component fields of a vector superfield are

$$\begin{aligned}\delta A_\mu &= \lambda \sigma_\mu \bar{\varepsilon} + \varepsilon \sigma_\mu \bar{\lambda}, \\ \delta \lambda &= \varepsilon D + \frac{i}{2} \varepsilon \partial^\mu A_\mu - \sigma^{\mu\nu} \varepsilon \partial_\mu A_\nu, \\ \delta D &= i \partial_\mu \lambda \sigma^\mu \varepsilon + i \partial_\mu \bar{\lambda} \bar{\sigma}^\mu \varepsilon,\end{aligned}\tag{2.7}$$

with $\sigma^{\mu\nu} = \frac{1}{4} (\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$. Here again the lowest components transform into higher ones and the highest component (D) transforms into a total derivative.

2.3.3 Lagrangians

The description of supersymmetric interactions requires appropriate Lagrangians. As we have seen in the previous section, the F - and D -terms² of the superfields transform into total derivatives, so they can be used for the construction of supersymmetric Lagrangians. A general renormalizable supersymmetric Lorentz invariant Lagrangian reads [7]

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} &= \frac{1}{4} \left[\Xi^2 + \Xi^\dagger{}^2 \right]_F + \left[\Phi^{\dagger j} (e^{2g\Gamma^a T^a})^i_j \Phi_i \right]_D + \left[W(\Phi_i) + \text{h.c.} \right]_F, \\ W(\Phi_i) &= \frac{1}{2} \mathcal{M}^{ij} \Phi_i \Phi_j + \frac{1}{6} \mathcal{Y}^{ijk} \Phi_i \Phi_j \Phi_k.\end{aligned}$$

Here F and D means that only the highest component contributes. Φ and Γ are chiral and vector superfields, respectively. Ξ is a spinorial field strength, T^a are generators of the gauge group, g is a coupling constant and ijk are type indices. W is called *superpotential*³, a very fundamental object in supersymmetric theories. \mathcal{M} and \mathcal{Y} are symmetric in their indices. See appendix A.2 for details.

Up to total derivatives, the Lagrangian for chiral multiplets extracts to

$$\begin{aligned}\mathcal{L}_{\text{CHIRAL}} &= \partial^\mu \bar{\varphi}^i \partial_\mu \varphi_i + i \xi_i \sigma^\mu [\partial_\mu] \bar{\xi}^i - V(\varphi_i, \bar{\varphi}^i) \\ &\quad - \frac{1}{2} \left(\mathcal{M}^{ij} \xi_i \xi_j - \mathcal{M}_{ij}^* \bar{\xi}^i \bar{\xi}^j - \mathcal{Y}^{ijk} \xi_i \xi_j \varphi_k - \mathcal{Y}_{ijk}^* \bar{\xi}^i \bar{\xi}^j \bar{\varphi}^k \right),\end{aligned}\tag{2.8}$$

where $A[\partial_\mu]B = \frac{1}{2} A \partial_\mu B - \frac{1}{2} (\partial_\mu A) B$. The contribution from chiral superfields to the scalar potential V is given by

$$\begin{aligned}V(\varphi_i, \bar{\varphi}^i) &= \frac{\partial \bar{W}}{\partial \bar{\varphi}^i} \frac{\partial W}{\partial \varphi_i} = \bar{W}_i W^i = F_i \bar{F}^i \\ &= \mathcal{M}_{ik}^* \mathcal{M}^{kj} \bar{\varphi}^i \varphi_j + \frac{1}{2} \mathcal{M}^{in} \mathcal{Y}_{jkn}^* \varphi_i \bar{\varphi}^j \bar{\varphi}^k \\ &\quad + \frac{1}{2} \mathcal{M}_{in}^* \mathcal{Y}^{jkn} \bar{\varphi}^i \varphi_j \varphi_k + \frac{1}{4} \mathcal{Y}^{ijn} \mathcal{Y}_{klm}^* \varphi_i \varphi_j \bar{\varphi}^k \bar{\varphi}^l.\end{aligned}\tag{2.9}$$

²In general, F -terms are coefficients of θ^2 and D -terms are coefficients of $\theta^2 \bar{\theta}^2$. F -terms are the highest components of chiral superfields and D -terms are those of the vector superfields.

Vector superfields can be constructed from chiral superfields via $\Phi^\dagger \Phi$.

³The superpotential is a holomorphic function of the fields: $W = W(\Phi)$ & $W^\dagger = W^\dagger(\Phi^\dagger)$.



Fig. 2.2 :: SUSY chiral and gauge interactions: (a) scalar–fermion from eq. (2.8), (b) (scalar)⁴ eq. (2.9), (c) gauge boson–gaugino from eq. (2.10), (d) gaugino–fermion–scalar from eq. (2.11).

Written in component fields, the Lagrangian for gauge supermultiplets reads

$$\mathcal{L}_{\text{GAUGE}} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} - i\bar{\lambda}^a \bar{\sigma}^\mu \mathcal{D}_\mu \lambda^a + \frac{1}{2}D^a D^a, \quad (2.10)$$

where a runs over the adjoint representation of the gauge group and we use $\mathcal{D}_\mu \lambda^a = \partial_\mu \lambda^a - gf^{abc}A_\mu^b \lambda^c$ and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - f^{abc}A_\mu^b A_\nu^c$.

The full Lagrangian for supersymmetric chiral gauge interactions in the component field language contains eq. (2.8) and eq. (2.10) as well as some additional terms. All ordinary derivatives ∂_μ have to be replaced by gauge covariant derivatives $\mathcal{D}_\mu = \partial_\mu + igA_\mu^a T^a$ in order to maintain gauge invariance⁴:

$$\mathcal{L} = \mathcal{L}_{\text{CHIRAL}} + \mathcal{L}_{\text{GAUGE}} - \sqrt{2} \sum_{\alpha, a} \left[\bar{\lambda}^a \bar{\xi}^i (T^{\alpha a})_i^j \varphi_j + \xi_i (T^{\alpha a})_j^i \bar{\varphi}^j \lambda^a + \frac{g_\alpha^2}{\sqrt{2}} D^{\alpha a} D^{\alpha a} \right], \quad (2.11)$$

where $D^{\alpha a} = \bar{\varphi}^i (T^{\alpha a})_i^j \varphi_j$ and we have allowed for the gauge group to be a direct product of group factors

$$\mathcal{G} = \bigotimes_{\alpha} \mathcal{G}_{\alpha},$$

with respective gauge coupling constants g_α ⁵. The scalar potential generalizes to

$$V(\varphi_i, \bar{\varphi}^i) = \bar{F}^i F_i + \frac{1}{2} \sum_{\alpha, a} g_\alpha^2 D^{\alpha a} D^{\alpha a}. \quad (2.12)$$

Note that in global SUSY the scalar potential is always non-negative, $V \geq 0$, since it is a sum of squares of absolute values.

In a renormalizable supersymmetric field theory, all couplings and masses are determined by gauge invariance and by the superpotential W . In fig. 2.2 some of the new interactions resulting from eq. (2.11) are depicted. These results can be used to construct supersymmetric models, especially supersymmetric extensions of the Standard Model which are subject to phenomenological perceptions.

⁴The generators of the gauge group T^a satisfy $[T^a, T^b] = if^{abc}T^c$, with f^{abc} being the structure constants of the gauge group.

⁵In the limit of SM the gauge couplings are $g_1 = \sqrt{5/3}g'$, $g_2 = g$, and $g_3 = g_s$.

Superfields	Fermions	Bosons	$SU(3)_C \times SU(2)_L \times U(1)_Y$	P_M
MATTER SECTOR				
Q_i	$\begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$	$\begin{pmatrix} \tilde{u}_{iL} \\ \tilde{d}_{iL} \end{pmatrix}$	$(3, 2, +1/6)$	-1
\bar{u}_i	u_{iR}^\dagger	\tilde{u}_{iR}^*	$(\bar{3}, 1, -2/3)$	-1
\bar{d}_i	d_{iR}^\dagger	\tilde{d}_{iR}^*	$(\bar{3}, 1, +1/3)$	-1
L_i	$\begin{pmatrix} \nu_{e_i} \\ e_{iL} \end{pmatrix}$	$\begin{pmatrix} \tilde{\nu}_{e_i} \\ \tilde{e}_{iL} \end{pmatrix}$	$(1, 2, -1/2)$	-1
\bar{e}_i	e_{iR}^\dagger	\tilde{e}_{iR}^*	$(1, 1, +1)$	-1
HIGGS SECTOR				
$H_u = H_2$	$\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$(1, 2, +1/2)$	+1
$H_d = H_1$	$\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$(1, 2, -1/2)$	+1
GAUGE SECTOR				
B	\tilde{B}^0	B^0	$(1, 1, 0)$	+1
W	$\tilde{W}^0 \tilde{W}^\pm$	$W^0 W^\pm$	$(1, 3, 0)$	+1
g	\tilde{g}	g	$(8, 1, 0)$	+1

Tab. 2.1 :: Particle/sparticle content of the MSSM including SM gauge quantum numbers and matter parity.

2.4 The MSSM

We are ready now to bring the SM in contact with SUSY to obtain the *Minimal Supersymmetric Standard Model* (MSSM) [6, 7]. From our discussion in section 2.3.2, we know that only chiral superfields can contain chiral fermions. Thus the SM fermions must reside in chiral superfields (supermultiplets). The scalar superpartners of leptons and quarks are called *sleptons* and *squarks*, respectively. The left- and right-handed pieces have their own scalar partners⁶.

The Higgs boson must be within a chiral supermultiplet; otherwise inconsistencies would arise⁷. Their superpartners are called *higgsinos*. A distinctive feature of the MSSM is that it contains two Higgs supermultiplets. There are several reasons why this is unavoidable [7]. As mentioned in section 2.3.3, the superpotential W is a holomorphic function, it cannot contain conjugates. To make the up-type, the down-type quarks and the leptons massive, two Higgs multiplets with different hypercharges are needed. Another reason is that one Higgs multiplet would cause gauge anomalies which cannot be cancelled.

The gauge bosons of the SM must be put into gauge superfields. The superpartners of the gluons are the *gluinos*, those of the W and B bosons the *winos* and *bino*, respectively. All superfields are labeled the same way as the ordinary SM particles. The superpartners, also known as *sparticles* are denoted by a tilde.

⁶The handedness of the superfields corresponds to the chirality of the fermions. To *distinguish* which scalar belongs to which fermion, scalars symbolically carry a handedness subscript.

⁷By looking at the gauge quantum numbers one could try to combine the left-handed leptons L_i with H_d . Since they differ in lepton numbers this would lead to lepton number violation.

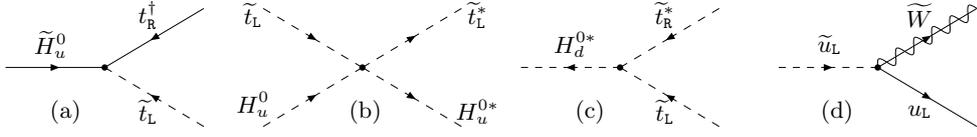


Fig. 2.3 :: Some of the MSSM couplings. (a) top-(s)quark Yukawa interaction, (b) (scalar)⁴, (c) (scalar)³, (d) gaugino interaction.

The bar indicates a left-handed CP conjugate of right-handed fields. Tab. 2.1 summarizes the field content of the MSSM. The superpotential for the MSSM reads

$$W_{\text{MSSM}} = \bar{u} \mathcal{Y}_u Q H_u - \bar{d} \mathcal{Y}_d Q H_d - \bar{e} \mathcal{Y}_e L H_d + \mu H_u H_d, \quad (2.13)$$

where all fields are chiral superfields and the couplings $\mathcal{Y}_{u,d,e}$ are 3×3 Yukawa matrices in family space. Since the third family particles are the heaviest only the (3, 3) componets of the matrices will have significant contributions. This leads to the approximation

$$W_{\text{MSSM}} \simeq y_t (\bar{t} t H_u^0 - \bar{t} b H_u^+) - y_b (\bar{b} t H_d^- - \bar{b} b H_d^0) - y_\tau (\bar{\tau} \nu_\tau H_d^- - \bar{\tau} \tau H_d^0) + \mu (H_u^+ H_d^- - H_u^0 H_d^0). \quad (2.14)$$

The μ -term in eq. (2.13) is the supersymmetrized version of the SM Higgs boson mass term. It supports higgsino mass terms as well as Higgs (mass)² terms.

$$\mathcal{L}_{\text{MSSM}} \supset -\mu \left(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0 + \text{h.c.} \right) - |\mu|^2 \left(|H_u^0|^2 + \dots \right) \quad (2.15)$$

In general one can treat the μ -term as an independent parameter of the MSSM.

Masses of the fermions and the mixing angles are obtained from Yukawa matrices after EW symmetry gets spontaneously broken by the VEVs of the neutral Higgs fields. Alongside the usual Yukawa interactions, new Yukawa interactions become possible. In addition to known Higgs–fermion couplings, sfermion–higgsino–fermion interactions appear [6, 7].

Furthermore, there are also diverse (scalar)³, (scalar)⁴ and (scalar)²(Higgs)² interactions. The couplings of the gauge bosons to sparticles are determined by gauge invariance of the kinetic terms in the Lagrangian. The gauginos do couple to (s)quark, (s)lepton and Higgs(ino) pairs [6, 7].

2.4.1 R–Parity

The superpotential eq. (2.13) could also contain further holomorphic gauge invariant renormalizable terms like [6, 7]

$$W_{\text{EXT}} = \varsigma_1^{ijk} L_i L_j \bar{e}_k + \varsigma_2^{ijk} L_i Q_j \bar{d}_k + \varsigma_3^i L_i H_u + \varsigma_4^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k. \quad (2.16)$$

The trouble coming from these terms is that they violate total lepton and baryon numbers. The first three terms in eq. (2.16) violate the total lepton number by

one unit and the last term violates the baryon number by one unit. Since B and L violating processes have not been confirmed experimentally (absence of proton decay) such terms must be either prohibited or at least suppressed⁸.

An elegant way to draw a curtain over eq. (2.16) is to impose a symmetry, called *matter parity* P_M . It is a discrete symmetry defined by

$$P_M = (-1)^{3(B-L)}, \quad (2.17)$$

where B and L is the baryon and lepton number, respectively. Matter parity is a multiplicative quantum number defined for any superfield. Chiral superfields have odd matter parity while the Higgs- and vector superfields have even P_M . Tab. 2.1 contains P_M for the field content of the MSSM.

For the new symmetry to come into effect, we allow only such interaction terms for which the product of individual P_M is $+1$. By this procedure, terms in eq. (2.16) are forbidden. Important to note is that matter parity provides a candidate for an exactly conserved symmetry while B and L numbers cannot be⁹.

From the phenomenological point of view, however, it is useful to define for each particle and sparticle the so called *R-parity* by

$$P_R = (-1)^{3(B-L)+2s}, \quad (2.18)$$

with s being the spin of a (s)particle. The transformation property is given by

$$\begin{aligned} (\text{particle}) &\longmapsto +(\text{particle}), \\ (\text{sparticle}) &\longmapsto -(\text{sparticle}). \end{aligned}$$

Particles always have positive R -parity whereas sparticles have negative one. R -parity of a multiple (s)particle state is the product of individual R -parities. Members of a superfield have the same P_M but different P_R .

The conservation of these parities has several phenomenological consequences. The assumption of conserved P_R (or equivalently P_M) leads to the conservation of total B and L numbers¹⁰. Phenomenologically, this means:

- any interaction vertex contains an even number of sparticles
- at colliders sparticles can only appear in pairs
- the *lightest supersymmetric particle* (LSP) is stable
- sparticles heavier than the LSP decay into an odd number of LSPs
- if the LSP is neutral, it can be a cold dark matter candidate.

The MSSM is *defined* to conserve R -parity.

⁸In the SM, renormalizable B and L violating terms are not possible.

⁹ B and L numbers can be violated in the SM by non-perturbative electroweak effects. With P_M assumed to be conserved, no B and L violating terms appear in the MSSM.

¹⁰This is only true at the renormalizable level. In the non-renormalizable case the conserved quantity is $(-1)^{3(B-L)}$.

2.5 SUGRA

So far, we have considered global SUSY: $\partial_\mu \varepsilon = 0$. We now switch on the spacetime dependence of the infinitesimal transformation parameter $\varepsilon = \varepsilon(x)$. Implementing 4-component notation the SUSY algebra can be recasted as [5]

$$[\varepsilon Q, \bar{Q} \bar{\varepsilon}] = 2 \varepsilon \gamma^\mu \bar{\varepsilon} P_\mu. \quad (2.19)$$

The product of two local SUSY transformations leads to spacetime translations which change from point to point—a general coordinate transformation. In other words, the invariance under local SUSY transformations implies an invariance under local coordinate transformations.

In analogy to ordinary symmetries, global SUSY can be made local by following the Noether procedure [5]. Starting with the global case and replacing $\varepsilon \rightarrow \varepsilon(x)$ is not enough to make the total Lagrangian density locally supersymmetric. In order to restore invariance, a new spinorial gauge field ψ_μ^α has to be introduced. It transforms as¹¹

$$\delta \psi_\mu^\alpha = \frac{2}{\varkappa} \partial_\mu \varepsilon^\alpha + \dots,$$

with \varkappa being a coupling constant of dimension $(\text{mass})^{-1}$. Since ψ_μ^α carries a spinor and a vector index, it must represent a spin-3/2 fermion.

The introduction of the new spinorial field causes an imbalance in the multiplet structure as ψ_μ^α has no bosonic partner. The appropriate superpartner can be found by adding the Noether coupling \mathcal{L}_N to the total Lagrangian (Noether procedure). Its variation gives [5]

$$\delta \mathcal{L}_N \simeq \varkappa \bar{\psi}_\nu \gamma_\mu T^{\mu\nu} \varepsilon,$$

with $T^{\mu\nu}$ being the energy-momentum tensor of scalar fields, and γ_μ denotes the γ -matrices. These terms can only be canceled if one introduces a tensor field $g_{\mu\nu}$ which transforms as

$$\delta g_{\mu\nu} \simeq \varkappa \bar{\psi}_\mu \gamma_\nu \varepsilon$$

and contributes to the total Lagrangian through

$$\mathcal{L}_g = -g_{\mu\nu} T^{\mu\nu}.$$

This is actually the coupling of the graviton to the energy-momentum tensor.

Promoting global supersymmetry to local symmetry automatically includes gravity. Therefore, local supersymmetry is often dubbed *supergravity* (SUGRA). The new fermionic field ψ_μ^α is nothing else than the superpartner of the spin-2 graviton. It is called *gravitino*. Graviton $g_{\mu\nu}$ and gravitino ψ_μ^α together with their auxiliary field build up the SUGRA multiplet. The appearance of the dimensionful coupling constant \varkappa signals that SUGRA is a non-renormalizable theory. Since the graviton has interactions of gravitational strength, $\varkappa = M_{\text{P}}^{-1}$. In the effective low

¹¹The factor of 2 is needed because the transformation parameter $\varepsilon(x)$ now is a 4-spinor.

energy Lagrangian such non-renormalizable terms appear suppressed by negative powers of M_{P} . The minimal formulation of SUGRA contains one SUGRA multiplet and couplings of chiral and vector superfields to the former. The phenomenologically relevant part in the Lagrangian [5, 33] describing such interactions is parametrized by two quantities: the function

$$G(\varphi_i, \bar{\varphi}^i) = \frac{K(\varphi_i, \bar{\varphi}^i)}{M_{\text{P}}^2} + \log \frac{|W(\varphi_i)|^2}{M_{\text{P}}^6} \quad (2.20)$$

and the analytic *gauge kinetic function* \mathbb{f}_{ab} , with a and b being gauge group indices. Both functions depend on scalar components of chiral multiplets of type i . The function G is real and transforms as a gauge singlet. K is likewise a real, gauge-invariant function and is called *Kähler potential*. $W(\varphi_i)$ is the analytic superpotential defined in section 2.3.3. The Kähler metric

$$K_j^i = \frac{\partial^2 K}{\partial \varphi_i \partial \bar{\varphi}^j} \quad (2.21)$$

describes the form of the kinetic energy terms of the scalar components of the chiral superfields. \mathbb{f}_{ab} does the same for gauge superfields. The gauge couplings are related to the the gauge kinetic function via

$$\frac{1}{g_a^2} = \Re \mathbb{f}_a, \quad (2.22)$$

with $\mathbb{f}_a = \delta_a^b \mathbb{f}_{ab}$ (sum over b). The scalar potential at tree level is given by [7]

$$V(\varphi_i, \bar{\varphi}^i) = M_{\text{P}}^4 e^G [G^i G_{\bar{j}} (K^{-1})_{\bar{i}}^{\bar{j}} - 3] + \sum_{\alpha} \frac{g^2}{2} \mathbb{f}_{ab}^{-1} D^{\alpha a} D^{\alpha b}, \quad (2.23)$$

where $D^{\alpha a} = G^i (T^{\alpha a})_i^j \varphi_j$, $G^i = \partial G / \partial \varphi_i$ and $(K^{-1})_{\bar{i}}^{\bar{j}}$ is the inverse Kähler metric. Compared to the scalar potential of global SUSY eq. (2.9), in the local case the scalar potential can be negative.

Chapter 3

Breakdown of Supersymmetry

If supersymmetry were an exact symmetry particles and sparticles would be degenerate in mass. Since no sparticles have been discovered, supersymmetry must be broken (at low energies). The problem of addressing the mechanism of supersymmetry breakdown is a deep one. In general, there are two algorithms how a symmetry can be broken: *spontaneously* through the VEV of a non-singlet field (Nambu-Goldstone mode) or *explicitly* through a small non-invariant part in the Lagrangian (Weinberg-Wigner mode). The aim of this chapter is to show how these two modes are encapsulated in the scheme of supersymmetry/supergravity breakdown.

3.1 Concatenation

This section is a short survey starting from exact supersymmetry, passing by theoretical restrictions, and arriving at a possible description of SUSY breakdown. Let us begin by looking at the SUSY algebra eq. (2.3). Its direct consequence is that the generator of SUSY transformations Q (supercharge) commutes with the Hamiltonian. By the use of eq. (2.3c) and $\sigma_{\alpha\dot{\beta}}^{\mu} \sigma_{\nu}^{\alpha\beta} = 2g_{\nu}^{\mu}$ one obtains

$$H = P^0 = \frac{1}{4} \left((Q_1 + \bar{Q}_1)^2 + (Q_2 + \bar{Q}_2)^2 \right). \quad (3.1)$$

This relationship states that H is positive semidefinite and, of course, well-defined. A supersymmetric vacuum state is defined by

$$Q_{\alpha} |0\rangle = 0, \quad \bar{Q}_{\dot{\alpha}} |0\rangle = 0,$$

which means that in the case of preserved SUSY the supercharges annihilate the vacuum state. *Supersymmetry is spontaneously broken if and only if at least one of the supercharges does not annihilate the vacuum.* This affects the conjunction between the vacuum energy and the spontaneous violation of SUSY. Eq. (3.1) guarantees the positivity of the vacuum energy

$$E_{\text{vac}} \equiv \langle 0 | H | 0 \rangle \geq 0. \quad (3.2)$$

Thus, if global supersymmetry is preserved, the vacuum energy is exactly zero and the vacuum state resides at the absolute minimum of the potential (fig. 3.1). Otherwise, if $E_{\text{vac}} \neq 0$ SUSY is spontaneously broken.

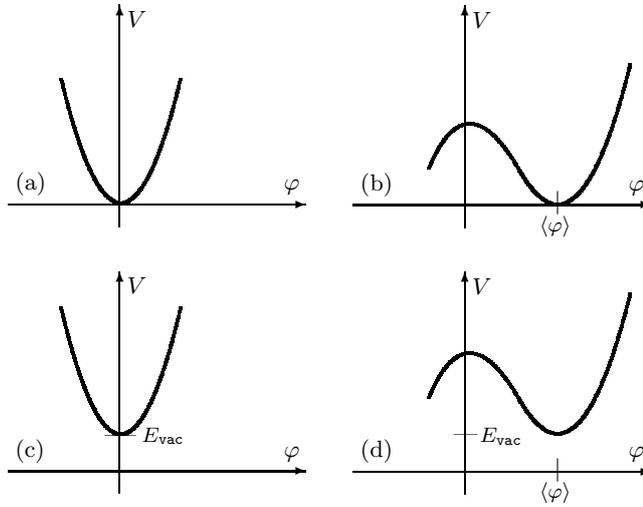


Fig. 3.1 :: Scalar potential V in a theory including supersymmetry and some internal gauge theory.
 (a) SUSY and internal symmetry preserved,
 (b) SUSY preserved but internal symmetry broken,
 (c) broken SUSY due to $E_{\text{vac}} \neq 0$ but internal symmetry valid,
 (d) SUSY and internal symmetry broken by $E_{\text{vac}} \neq 0$ and $\langle\varphi\rangle \neq 0$, respectively.

Unlike ordinary symmetries, the existence of an invariant state does not necessarily imply that this is the ground state of the theory [5]. In exact global supersymmetry groundstates are always at $E_{\text{vac}} = 0$.

In the case of spontaneous supersymmetry breaking (SSB), we obviously have $Q_\alpha |0\rangle = |\xi_\alpha\rangle \neq 0$, where $|\xi_\alpha\rangle$ is some fermionic state. Using supercurrent formalism [5] one can rewrite

$$\langle\xi_\alpha|J_\beta^\mu|0\rangle = f\sigma_{\alpha\beta}^\mu.$$

The supercurrent J_β^μ creates a fermion whose coupling $f^2 \simeq E_{\text{vac}}$ measures the breakdown of SUSY. This is the analogon to the spontaneous breakdown of an ordinary symmetry where massless Goldstone bosons emerge. In SUSY the Goldstone particle is a massless Weyl fermion called *goldstino*¹. Depending on what supermultiplet is involved in SSB, the fermion ξ_α is either a chiral fermion or a gaugino. The SSB is associated with a non-zero VEV of an operator that changes under a SUSY transformation [7]. To find out what fields are involved in SSB, we look at the transformation properties of chiral fermions eq. (2.5) and gauginos eq. (2.7). Since $\partial_\mu\varphi$ and $\partial_\mu A_\nu$ are not supposed to acquire VEVs², only the auxiliary fields can characterize the spontaneous breakdown of SUSY:

$$\langle 0|F|0\rangle \equiv \Lambda_F^2, \quad \langle 0|D|0\rangle \equiv \Lambda_D^2.$$

So we conclude that SUSY is spontaneously broken if and only if the auxiliary fields F and/or D acquire non-zero VEVs. Λ_F^2 and Λ_D^2 have dimension (mass)² and represent the scale of SUSY breaking. SSB with non-zero $\langle F \rangle$ is called *F-type breaking*, whereas such with non-zero $\langle D \rangle$ is known as *D-type breaking*. For realistic SSB F -type breaking is always necessary whereas D -terms can additionally contribute. In what follows we will concentrate on F -term contributions. As

¹Note that the goldstino is not the superpartner of the Goldstone boson.

²VEVs of spinorial, vector or tensor quantities ruin Lorentz invariance.

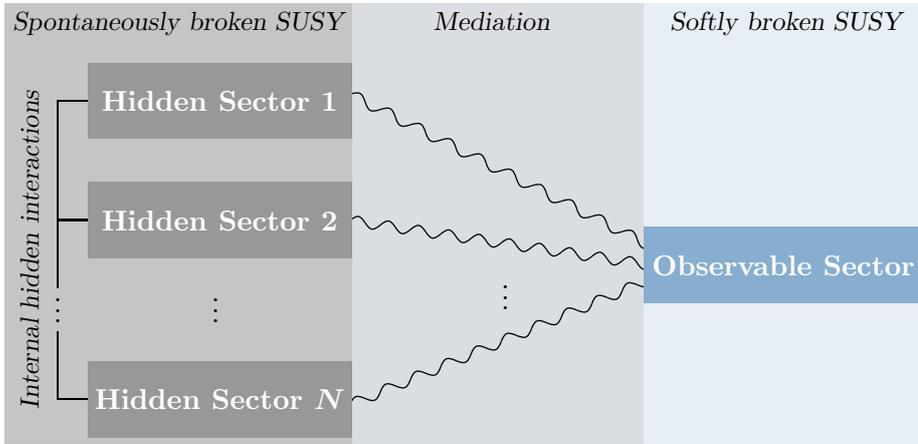


Fig. 3.2 :: Mediation of spontaneous SUSY breakdown from hidden to observable sector.

already discussed in the previous chapter, all members of a superfield carry the same quantum numbers (except for the spin). Now, if an auxiliary field of some observable field is equipped with a non-zero VEV to break SUSY, it will also break miscellaneous internal symmetries like color, electromagnetism, etc. Also the massless goldstinos would have SM quantum numbers. Thus, so as to avoid a disaster, SUSY can be spontaneously broken only by fields which are singlets under the SM gauge group. Since such fields cannot be part of the ordinary matter one places them into the so-called *hidden sector*, whereas the ordinary matter inhabits the *observable sector*. Fig. 3.2 depicts this schematically .

These two sectors are postulated to have no direct couplings. But in order to *mediate* spontaneously broken SUSY from the hidden to the observable sector, these two sectors must share some, albeit very weak, interactions. Such weak interactions can be arranged gravitationally through operators suppressed by inverse powers of M_{P} or at loop level suppressed by loop factors $(16\pi^2)^{-1}$. In general, several hidden sectors can be involved in mediating SSB (fig. 3.2). The hidden sectors can have certain non-SM gauge couplings among themselves.

In the MSSM, the breakdown of SUSY must be explicit, but cannot be arbitrary. In section 2.2 we saw that unbroken SUSY provides the cancellation of quadratic divergences in scalar (masses)² to all orders in perturbation theory. If we break SUSY, we also challenge the cancellation. This poses a constraint on the nature of explicit breaking terms. To maintain convergence to all orders in perturbation theory in presence of explicit broken SUSY, it was shown [34] that the breaking terms must contain couplings of positive mass dimension. Such breaking terms are called *soft*. The (explicit) soft breaking can be effectively illustrated as

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SOFT}},$$

where $\mathcal{L}_{\text{SUSY}}$ denotes the SUSY preserving part eq. (2.11) and $\mathcal{L}_{\text{SOFT}}$ contains soft

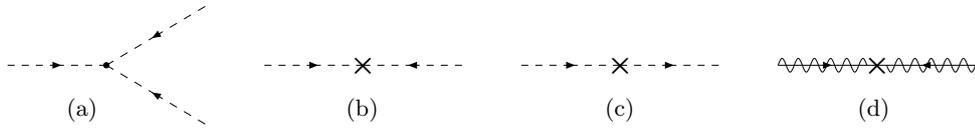


Fig. 3.3 :: Soft SUSY-breaking interactions: (a) (scalar)³ couplings, (b) (scalar)² couplings, (c) non-analytic (scalar)² couplings, (d) gaugino mass insertion.

breaking terms. The most general soft breaking terms at renormalizable level are

$$\begin{aligned} \mathcal{L}_{\text{SOFT}} = & -\frac{1}{2} (M_a \lambda^a \lambda^a + \text{h.c.}) - (m^2)^i_j \varphi_i \bar{\varphi}^j \\ & - \left(\frac{1}{6} \mathcal{A}^{ijk} \varphi_i \varphi_j \varphi_k + \frac{1}{2} \mathcal{B}^{ij} \varphi_i \varphi_j + \mathcal{C}^i \varphi_i + \text{h.c.} \right), \end{aligned} \quad (3.3)$$

with φ_i being the scalar components of some chiral superfield and λ^a are the 2-component gauginos. The soft breaking terms include Majorana gaugino mass terms, scalar (mass)² and scalar (mass)³ terms³. \mathcal{A}^{ijk} and \mathcal{B}^{ij} are symmetric in their indices, m^2 is a hermitean matrix. The \mathcal{C} -terms are non-zero only if the theory contains gauge invariant scalar fields. Observe that \mathcal{A} - and \mathcal{B} -terms are similar to the \mathcal{Y} - and \mathcal{M}^2 -terms in the superpotential. The fact, that $\mathcal{L}_{\text{SOFT}}$ contains only sparticles highlights the explicit violation of supersymmetry. The scale of the soft breaking parameters, M_{SOFT} , characterizes the mass splittings in the supermultiplets.

Now, let us try to figure out how the spontaneous breakdown of SUSY might be arranged in the hidden sector. The softly broken MSSM with its $\mathcal{O}(100)$ free parameters does not solve this problem on its own. Therefore, one needs a simpler underlying theory, containing spontaneously broken SUSY, which explains the softly broken MSSM in a natural way. One fact we certainly can expect is that the scale associated with this new theory is (significantly) larger than M_{EW} .

Signals towards this assumption come from the scale dependence of the SM gauge couplings. The renormalization group (RG) equations of the three gauge couplings [6, 7], up to 1-loop order, are given by

$$\frac{d g_a}{d \log \varrho} = \frac{1}{16\pi^2} b_a g_a^3, \quad (3.4)$$

where $a = 1, 2, 3$, $\varrho = Q/Q_0$ with Q denoting the renormalization scale and Q_0 is some input scale. The β -function coefficients b_a define the slopes of the curves. In the SM, where $b_a^{\text{SM}} = (41/10, -19/6, -7)$, the three gauge couplings tend to unify around $Q \sim 10^{15}$ GeV (see fig. 3.4). By contrast, the MSSM with its $b_a^{\text{MSSM}} = (33/5, 1, -3)$ ⁴ provides just the right slopes for a unification of the gauge

³Although non-analytic scalar (mass)³ terms $\varphi\varphi\bar{\varphi}$ are renormalizable, they are ruled out due to the encompassing of quadratic divergences at loop level.

⁴The b_a are enlarged through additional loop contributions coming from the superpartners [7].

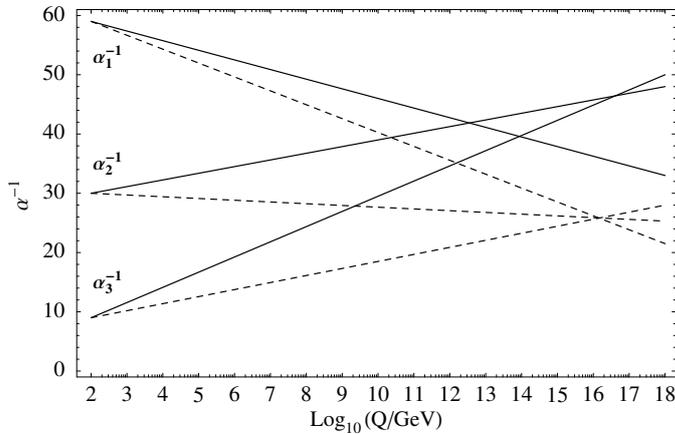


Fig. 3.4 :: RG evolution of the three SM gauge couplings g_a . For clarity $\alpha^{-1} = 4\pi/g_a^2$ is shown. Solid lines represent the behavior in the SM. Dashed lines correspond to the MSSM. 2-loop corrections included.

couplings at $Q = M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV. This result might be a strong hint that at the scale M_{GUT} some *Grand Unified Theory* (GUT) could play an important role. Nevertheless, the conclusion is that the SM gauge couplings may be unified within the MSSM. The MSSM on his part is an effective theory and thus has a limited range of validity which at least terminates at some (high) input scale M_{IN} . On the other hand, if the gauge singlet fields, which break supersymmetry spontaneously in the hidden sector, are supposed to be active at some very high scale M_X with $M_X > M_{\text{IN}} \gg M_{\text{EW}}$ then at M_{IN} these heavy DOF are integrated out and the residual theory is described by $\mathcal{L}_{\text{SOFT}}$.

This cognition alters our hitherto situation. We are now faced with two different scales: M_{SOFT} , specifying the superpartner masses and M_{IN} , setting the frontier where the MSSM fields perceive the SUSY breaking coming from the hidden sector. The unification signature within the MSSM insinuates that the GUT scale is most likely the input scale. In this work we will use $M_{\text{IN}} = M_{\text{GUT}}$.

Finally let us throw a glance at or rather beyond M_{GUT} . There, gravity becomes dominant and everything beyond M_{GUT} has to incorporate gravity. Having said that we know that SUGRA automatically embeds gravity. Thus we arrive at the key point: as has been richly explored in the literature e. g. [5–7], the most phenomenologically acceptable scenario is that of a spontaneously broken SUGRA in the hidden sector, which leads to an effective low energy theory in the observable sector exhibiting explicit but softly broken SUSY.

From these considerations, the theory of SUSY breaking manifests itself in *boundary conditions* given at the GUT scale M_{GUT} . The shape of these boundary conditions depends on the mechanism which is responsible for mediating the spontaneous breakdown of SUGRA. Using the RG utilities one can evolve the soft terms from M_{GUT} down to, say, M_{EW} .

3.2 Soft Terms in the MSSM

To determine the shape of soft breaking terms in the MSSM, we resume our general discussion from the previous section. Since the MSSM does not contain any gauge singlet scalar fields, the \mathcal{C} terms in eq. (3.3) are prohibited. All other terms keep valid. Thus, the possible soft breaking terms compatible with R -parity conservation are

$$\begin{aligned}
\mathcal{L}_{\text{SOFT}} = & -\frac{1}{2} \left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \text{h.c.} \right) \\
& - \left(\widetilde{u}_R^* \mathcal{A}_u \widetilde{Q} H_u + \widetilde{d}_R^* \mathcal{A}_d \widetilde{Q} H_d + \widetilde{e}_R^* \mathcal{A}_e \widetilde{L} H_d + \text{h.c.} \right) \\
& - \left(\widetilde{Q}^* \mathcal{M}_Q^2 \widetilde{Q} + \widetilde{u}_R^* \mathcal{M}_u^2 \widetilde{u}_R + \widetilde{d}_R^* \mathcal{M}_d^2 \widetilde{d}_R + \widetilde{L}^* \mathcal{M}_L^2 \widetilde{L} + \widetilde{e}_R^* \mathcal{M}_e^2 \widetilde{e}_R \right) \\
& - \left(m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^* H_d \right) \\
& - \left(B\mu H_u H_d + \text{h.c.} \right), \tag{3.5}
\end{aligned}$$

where \widetilde{Q} , \widetilde{L} , H denote scalar doublets and \widetilde{u}_R , \widetilde{d}_R , \widetilde{e}_R singlets. To avoid confusion all family indices are suppressed and subscripts are unbarred. The first line in eq. (3.5) describes gaugino mass terms. M_1 , M_2 and M_3 are the bino, wino and gluino mass parameters, respectively. The second line contains trilinear scalar couplings (\mathcal{A} -terms). The matrices \mathcal{A} are complex 3×3 matrices in family space of dimension (mass)¹. The third line consists of squark and slepton mass terms. \mathcal{M}^2 are 3×3 hermitean matrices, given in family space. The fourth line contains Higgs boson mass parameters. The $B\mu$ -term in the fifth line represents the soft SUSY breaking contribution to the Higgs sector⁵. Given all these parameters, the SUSY breaking part in the MSSM Lagrangian appends 105 new parameters to the already existing 19 of the SM⁶. As mentioned in section 2.2, the masses of the superpartners should be roughly of order $M_{\text{SOFT}} \sim \mathcal{O}(1 \text{ TeV})$ to provide a solution to the hierarchy problem. Thus:

$$\begin{aligned}
M_1, M_2, M_3, \mathcal{A}_u, \mathcal{A}_d, \mathcal{A}_e & \sim M_{\text{SOFT}}, \\
\mathcal{M}_Q^2, \mathcal{M}_u^2, \mathcal{M}_d^2, \mathcal{M}_L^2, \mathcal{M}_e^2, m_{H_u}^2, m_{H_d}^2, B\mu & \sim M_{\text{SOFT}}^2.
\end{aligned}$$

3.2.1 Soft Breaking Universality

Most of the new parameters in eq. (3.5) introduce new sources of flavor changing neutral currents (FCNC) and CP violation [6, 7, 35]. This is originated by flavor dependence of the sfermion mass matrices and new complex phases. There is also the possibility of individual lepton number violation. After the EW breakdown, the matrices \mathcal{A}_u , \mathcal{A}_d and \mathcal{A}_e unleash off-diagonal squark and slepton mixings, leading

⁵This corresponds to the \mathcal{B} -term in eq. (3.3). $B\mu$ is thought of as a product of the SUSY preserving μ parameter and the soft breaking parameter B .

⁶Simplifying assumptions may reduce the number of free parameters, but without any organizing entity, the softly broken MSSM would be hidden in a labyrinth of arbitrariness.

to CP violating effects. However, the experimental constraints on processes like $\mu \rightarrow e\gamma$, $K^0 \leftrightarrow \bar{K}^0$, $b \rightarrow s\gamma$, etc. set boundaries on these parameters.

Hence, in order to render the MSSM realistic, one has to add essential constraints. One strategy often used is the so-called *universality* which is based on three premises:

- 1] *Propose flavor-blind sfermion mass matrices*

$$\mathcal{M}_{Q,u,d}^2 = m_{Q,u,d}^2 \mathbb{1}, \quad \mathcal{M}_{L,e}^2 = m_{L,e}^2 \mathbb{1}. \quad (3.6-1)$$

This assumption leads to mass degeneracy among sfermions with the same EW gauge quantum numbers. In practice, it suffices to consider mass degeneracy between sfermions of the first two generations [7].

- 2] *Claim the soft breaking A-terms to be proportional to the Yukawa matrices*

$$\mathcal{A}_{u,d,e} = A_{u,d,e} \mathcal{Y}_{u,d,e}. \quad (3.6-2)$$

Together with the assumption above eq. (2.14), this ensures that only third generation sfermions can have sizeable (scalar)³ couplings.

- 3] *Avoid new CP violating phases through*

$$\arg(M_{1,2,3}) = \arg(A_{u,d,e}) = 0 \vee \pi. \quad (3.6-3)$$

In this manner only the ordinary CKM phase of the SM remains.

An important feature of the RG equations is that they do not introduce any new non-negligible FCNC or CP violating effects [6] once universality is applied at the input scale, e. g. M_{GUT} . Therefore, the universality conditions are supposed to be constraints on the running soft parameters at the input scale. Alternative approaches in suppressing FCNC/CP violation are the so-called *alignment* [36] and *irrelevancy* [37] of the soft masses. In what follows we stick to the universality.

3.2.2 Electroweak Symmetry Breaking

So far, we have only considered the breakdown of supersymmetry within the MSSM. To describe our world, we have to include electroweak symmetry breaking (EWSB). The breakdown $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{EM}}$ is initiated through the VEV of the Higgs field. It is necessary that only electrically neutral Higgs field components acquire non-zero VEVs. Using an $SU(2)_L$ gauge transformation one can set $\langle H_u^+ \rangle = \langle H_d^- \rangle = 0$ at the minimum of the Higgs potential. Thus EWSB is characterized by non-zero VEVs of the neutral Higgs fields

$$\langle H_d^0 \rangle \equiv \frac{v_d}{\sqrt{2}}, \quad \langle H_u^0 \rangle \equiv \frac{v_u}{\sqrt{2}}, \quad \tan \beta \equiv \frac{v_u}{v_d}, \quad v \equiv \sqrt{v_d^2 + v_u^2}. \quad (3.7)$$

Given this, the tree level Higgs scalar potential reduces to

$$\begin{aligned}
V_{\text{HIGGS}} = & \frac{1}{8} (g^2 + g'^2) (|H_u^0|^2 - |H_d^0|^2) \\
& + (|\mu|^2 + m_{H_u}^2) |H_u^0|^2 + (|\mu|^2 + m_{H_d}^2) |H_d^0|^2 \\
& - B\mu H_u^0 H_d^0,
\end{aligned} \tag{3.8}$$

with g and g' being the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively. In order for the potential to be bounded from below and also that one linear combination of H_u^0 and H_d^0 has a negative squared mass near $H_u^0 = H_d^0 = 0$ gives the necessary condition

$$(|\mu|^2 + m_{H_u}^2) (|\mu|^2 + m_{H_d}^2) < (B\mu)^2. \tag{3.9}$$

If this condition, valid at or below M_{EW} is not satisfied, then $H_u^0 = H_d^0$ will not be a stable minimum of the potential and EWSB cannot occur. To have EWSB one has to assure $m_{H_u}^2 < m_{H_d}^2$. If one starts with $m_{H_u}^2 = m_{H_d}^2$ at the input scale, $m_{H_u}^2$ will be driven by RG (see eq. (B.15)) to (large) negative values at the EW scale. This circumstance is also known as the *radiative electroweak breaking* since EWSB is accomplished by quantum corrections. Note that a negative $m_{H_u}^2$ is not a necessary condition for having correct EWSB, since a too large $|\mu|$ or a too small B can counteract.

Minimizing the potential eq. (3.8), one arrives at

$$\frac{m_Z^2}{2} = -|\mu|^2 + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1}, \tag{3.10}$$

given at the EW scale with m_Z being the Z boson mass. Correct EWSB is ensured if eq. (3.10) is satisfied. Keep in mind that this is a tree level statement.

The μ -term within the MSSM causes a puzzle, which goes under the name *μ -problem*. Eq. (3.10) tells us that if the terms on the right hand side are all of the order $(10^2 \dots 10^3 \text{ GeV})^2$, then the left hand side can be obtained without unnatural cancellations. Otherwise one has to *fine-tune* the Higgs masses in order to reduce cancellations and obtain a small μ . It is however a good question why the μ parameter should be of the order of M_{EW} or M_{SOFT} and not M_{P} . The scalar potential of the MSSM contains SUSY preserving (μ) as well as SUSY violating (B) parts. The observed value of the EW scale suggests that these two parameters should be of order $(10^2 \dots 10^3 \text{ GeV})^2$. There are actually several approaches having been proposed in order to solve the μ -problem e. g. [38, 39]. In the MSSM μ is treated as a free parameter.

The scalar Higgs sector of the MSSM consists of 4 complex scalar fields with altogether 8 DOF. After EWSB takes place, 3 Goldstone bosons are generated and constitute the longitudinal helicity states of the W^\pm and Z^0 bosons. The remaining 5 DOF mix to form the following mass eigenstates: two charged scalars H^\pm , two CP even neutral scalars h^0, H^0 and one CP odd neutral scalar A^0 .

The mass of h^0 is bounded from above and corresponds to the lightest Higgs boson. The masses of the other 4 scalars are significantly larger. The upper mass bound on h^0 at tree level reads [40]

$$m_h < |\cos 2\beta| m_Z, \quad (3.11)$$

foreshadowing that it is lighter than the Z boson. This is, of course, tabooed by the current experimental lower bound [41] $m_h \gtrsim 114$ GeV. The remedy comes from loop corrections. The most important 1-loop contribution coming from top-stop loops reads [6, 42]

$$\delta m_h^2 \simeq \frac{3y_t^2 m_t^2}{4\pi^2} \sin^2 \beta \log \left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^2} \right], \quad (3.12)$$

with y_t^2 being the top-Yukawa coupling. Taking the limit $m_{\tilde{t}_1}, m_{\tilde{t}_2} > m_t$ and including some other smaller corrections [6], the upper bound can be elevated to

$$m_h < 135 \text{ GeV}.$$

3.2.3 Mixing

Once SUSY and EW symmetry are broken, a certain mixing in the bosonic gauge sector appears. Supersymmetric influences within the MSSM provide additional mixing patterns in the gaugino and sparticle sector.

The EW gauginos and higgsinos mix to hold new eigenstates. Charged EW gauginos \tilde{W}^\pm and charged higgsinos $\tilde{H}_u^\pm, \tilde{H}_d^\pm$ form two mass eigenstates with charges ± 1 called *charginos* $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$. Superpositions of the neutral EW gauginos \tilde{B}^0, \tilde{W}^0 and neutral higgsinos $\tilde{H}_u^0, \tilde{H}_d^0$ give four neutral fermions $\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$, called *neutralinos*. Usually they are ordered in the direction of increasing mass. The lightest neutralino can be decomposed as

$$\tilde{\chi}_1^0 = Z_{11} \tilde{B}^0 + Z_{12} \tilde{W}^0 + Z_{13} \tilde{H}_d^0 + Z_{14} \tilde{H}_u^0, \quad (3.13)$$

where Z_{ij} are components of an orthogonal matrix used to diagonalize the neutralino mass matrix. $\tilde{\chi}_1^0$ is called *gaugino-like* if $P \equiv |Z_{11}|^2 + |Z_{12}|^2 > 0.9$, *higgsino-like* if $P < 0.1$ and *mixed* otherwise [18]. The lightest neutralino often happens (or is arranged) to be the LSP. It can be considered as a good cold Dark Matter candidate since it is a weakly interacting particle, capable to produce reasonable relic abundance [35]. Note that the gluino, being a color octet fermion, do not have adequate quantum numbers to mix with any other particle.

In the slepton and squark sector, mixing can occur between left- and right-handed⁷ particles of same electric-, color- and P_R -charge. It turns out that only third family squarks and sleptons have non-negligible mixing angles due to large Yukawa couplings. Tab. 3.1 enfold the (relevant) mass eigenstates in the MSSM. The RG equations are listed in appendix B.1.

⁷Note that the handedness corresponds to fermionic superpartners.

Superfields	Mass Eigenstates						P_R
MATTER SECTOR							
Squarks	\tilde{u}_L	\tilde{u}_R	\tilde{c}_L	\tilde{c}_R	\tilde{t}_1	\tilde{t}_2	-1
	\tilde{d}_L	\tilde{d}_R	\tilde{s}_L	\tilde{s}_R	\tilde{b}_1	\tilde{b}_2	
Sleptons			\tilde{e}_L	$\tilde{\mu}_L$	$\tilde{\tau}_1$		
			\tilde{e}_R	$\tilde{\mu}_R$	$\tilde{\tau}_2$		
HIGGS SECTOR							
Higgs bosons	h^0	H^0	H^+	H^-	A	+1	
GAUGE SECTOR							
Charginos			$\tilde{\chi}_1^\pm$	$\tilde{\chi}_2^\pm$			
Neutralinos	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$			
Gluino					\tilde{g}	-1	

Tab. 3.1 :: Physical mass eigenstates in the MSSM.

3.3 Spontaneously broken SUGRA

We have already seen that in global SUSY a non-vanishing vacuum expectation value of the auxiliary fields, associated with a positive non-zero vacuum energy $E_{\text{vac}} > 0$, leads to a spontaneous breakdown of SUSY. This was the direct consequence of the algebra of global supersymmetry eq. (2.3). Spontaneous breakdown of supergravity, however, brings new facets.

To find the condition for spontaneously broken SUGRA we have to look at the auxiliary fields. The transformation properties of the component fields change in SUGRA [5], but the essential result remains the same: the fermionic component of a superfield transforms into the auxiliary component. For chiral fermions the relevant part of the auxiliary field is given by

$$F_i \simeq e^{G/2} (K^{-1})_i^{\bar{j}} G_{\bar{j}}, \quad (3.14)$$

where $(K^{-1})_i^{\bar{j}}$ is the inverse Kähler metric and $G_{\bar{j}} = \partial G / \partial \bar{\varphi}^{\bar{j}}$. Here and from now on (if not stated otherwise), we will use the SUGRA units $M_{\text{P}} \equiv 1$. In analogy to the global case, SUGRA is spontaneously broken if and only if the auxiliary field develops a non-zero VEV. The SUGRA breaking scale is given by $\langle 0 | F_i | 0 \rangle = M_{\text{S}}$. Whether F acquire a VEV or not depends on the function G . Suppose that F has a non-zero VEV. Then

$$\langle 0 | e^{G/2} (K^{-1})_i^{\bar{j}} G_{\bar{j}} | 0 \rangle \neq 0.$$

Since $(K^{-1})_i^{\bar{j}}$ determines the kinetic terms, it must be non-zero to provide a well-defined theory. Thus, the quantity that controls spontaneous breakdown is⁸

$$G_{\bar{j}} = \frac{\partial G}{\partial \bar{\varphi}^{\bar{j}}} = \partial_{\bar{j}} G, \quad (3.15)$$

⁸This is also true if one discusses spontaneous breaking by D -terms. As mentioned above, we will focus on the phenomenologically relevant F -type breaking.

flagging that SUGRA is preserved if $G_{\bar{j}} = 0$ and spontaneously broken if $G_{\bar{j}} \neq 0$. Again here, the fermionic partner of the auxiliary field is the goldstino. In SUGRA, mixing between the gravitino ψ and goldstino χ becomes possible [5]

$$e^{G/2} G^i \bar{\psi}_\mu \gamma^\mu \chi_i.$$

In other words, the gravitino “eats” the goldstino and becomes massive thereby. The goldstino appears as longitudinal polarization states of the gravitino⁹. This procedure is the so-called *super Higgs mechanism*. It is analogous to the ordinary Higgs mechanism.

The distinct feature of the SUGRA scalar potential is the way it relates the vacuum energy to the SUGRA breakdown. If SUGRA is preserved, the scalar potential eq. (2.23) simplifies to $V = -3e^G$, which is in general non-zero! We see that in unbroken SUGRA the vacuum energy is negative semidefinite, unlike in global SUSY. In broken SUGRA the scalar potential at the minimum is

$$E_{\text{vac}} = -3e^{G_0} + e^{G_0} G_{0i} G_0^i,$$

where we used $K_{\bar{j}}^i = -\delta_{\bar{j}}^i$ (minimal kinetic terms) and the subscript 0 denotes that the quantity was evaluated at the minimum of the potential. In contrast to global SUSY, in SUGRA we can “adjust” the vacuum energy (= cosmological constant) to a positive/negative value or to zero, due to $G_0^i \neq 0$, without affecting the breakdown. This was impossible in global SUSY. Even though it is not understood why the cosmological constant (CC) has a very small positive value, with SUGRA we have a tool where we at least can fine-tune it to zero [5].

The gravitino mass, $m_{3/2}$, representing the mass splitting in the SUGRA multiplet is given by

$$m_{3/2} = e^{(G)/2} = e^{G_0/2} \quad (3.16)$$

and is related to the scale of spontaneously broken SUGRA through

$$M_{\text{S}} = \langle 0 | F_i | 0 \rangle \simeq \langle 0 | M_{\text{P}} e^{G_0/2} G_{0j} | 0 \rangle.$$

Thus

$$m_{3/2} \sim \frac{M_{\text{S}}^2}{M_{\text{P}}}. \quad (3.17)$$

A special meaning of the gravitino mass is that it represents the scale of SUSY breaking. This can be easily seen in SUGRA units, where $m_{3/2} \sim M_{\text{S}}$ as evident from eq. (3.17).

⁹The transverse components ($\pm 3/2$) of the gravitino have only gravitational interactions, whereas its longitudinal components ($\pm 1/2$) are able to have non-gravitational couplings.

Chapter 4

Mediations

In this chapter we would like to consider how soft terms can emerge in the observable sector. Our starting point will be spontaneously broken SUGRA in the hidden sector by the VEV of a non-SM gauge singlet field. There are different possibilities to transmit the spontaneous breakdown of SUGRA from the hidden to the observable sector. We will pick up two of them. The first one, called *gravity mediation*, is primarily concerned with the transmission of the SUGRA breakdown to the observable sector at tree level by M_{P} suppressed operators. The other possibility, known as *anomaly mediation*, uses superconformal anomaly to communicate the SUGRA breakdown to the observable sector at loop level.

4.1 Gravity Mediation

We assume that gravity couples to every form of matter, independently of whether it is observable or hidden. Therefore we can dare to consider gravity can as the carrier of SUSY breakdown. This breakdown is encoded in the VEV $\langle F^X \rangle$ of some hidden sector field X . The scale of the soft terms in the observable sector should be roughly of order

$$M_{\text{SOFT}} \sim \frac{\langle F^X \rangle}{M_{\text{P}}}.$$

This is because we know that $M_{\text{SOFT}} \rightarrow 0$ in the limit of restored SUSY $\langle F^X \rangle \rightarrow 0$, and also when gravity becomes irrelevant ($M_{\text{P}} \rightarrow \infty$). Moreover, naturalness requires $M_{\text{SOFT}} \sim \mathcal{O}(1 \text{ TeV})$ and thus $\langle F^X \rangle \sim (10^{11} \text{ GeV})^2$.

For further discussion, it is useful to define a chiral superfield Θ_M with a multiindex $M \in \{m, \alpha\}$ containing both, the observable sector fields Φ_m and the hidden sector fields¹ $\widehat{\Phi}_\alpha$:

$$\Theta_M = \left\{ \Phi_m, \widehat{\Phi}_\alpha \right\}, \text{ with scalar components } \vartheta_M = \{Q_m, \mathfrak{h}_\alpha\}.$$

Latin indices denote the observable fields and greek indices run over the hidden sector fields. The hidden sector fields can acquire large VEVs, $\langle \mathfrak{h}_\alpha \rangle \sim \mathcal{O}(M_{\text{P}})$. When the auxiliary field

$$F_\alpha = e^{G/2} (K^{-1})_\alpha^{\bar{\beta}} G_{\bar{\beta}} \tag{4.1}$$

¹In our notation hidden sector quantities are always hatted.

of at least one of the hidden sector fields develops a non-zero VEV, SUGRA gets spontaneously broken, leading to soft breaking terms in the observable sector. The substantial elements of SUGRA, namely the Kähler potential, the superpotential and the gauge kinetic function depend on both types of fields:

$$G(\Theta_M, \Theta_M^\dagger) = K(\Theta_M, \Theta_M^\dagger) + \log |W(\Theta_M)|^2, \\ \mathbb{f}_a = \mathbb{f}_a(\Theta_M).$$

This also applies to the scalar potential

$$V(\Theta_M, \Theta_M^\dagger) = e^G \left(G^M G_{\bar{N}} (K^{-1})_{\bar{M}}^{\bar{N}} - 3 \right) = \left(\bar{F}^{\bar{N}} F_M K_{\bar{N}}^M - 3e^G \right), \quad (4.2)$$

where we have used eq. (4.1) to obtain the last equality. We will follow the ansatz

$$W_{\text{TOT}} = W + \widehat{W},$$

assuming that the superpotential is additively decomposed into an observable part W and a hidden sector part \widehat{W} . The procedure to obtain the soft breaking terms in the observable sector is as follows [8]

Soft Terms in Gravity Mediation

- Contemplate a proper normalization scheme in order to respect phenomenological constraints due to FCNC and CP violation.
- Replace the hidden sector fields and their auxiliary fields by their VEVs in the SUGRA Lagrangian.
- Take the *flat* limit: $M_{\text{P}} \rightarrow \infty$ & $m_{3/2}$ fixed.
- Non-renormalizable gravitational contributions melt down, leaving a global SUSY Lagrangian with soft breaking terms.

The normalization scheme we will design is framed as follows. Observable MSSM fields have a diagonal Kähler metric

$$K_j^i \longrightarrow \mathcal{K}_i \equiv \frac{\partial^2 K}{\partial Q^i \partial \bar{Q}^i} \quad (4.3)$$

and their normalization is chosen to be²

$$(\mathcal{K}^{1/2})_j \delta^{ji} Q^i \longrightarrow Q^i, \\ \sqrt{\Re \mathbb{f}_b} \delta^{ba} \lambda^a \longrightarrow \lambda^a.$$

²Normalization schemes for the trilinear and bilinear terms can be found in [8, 43].

Expansion of the Kähler potential to lowest order in the observable fields yields:

$$K_{\text{TOT}} = \widehat{K} + \mathcal{K}_i Q^i \overline{Q}^i + (ZH_1 H_2 + \text{h.c.}) + \dots$$

and for the superpotential

$$W = \widehat{W} + \mu H_1 H_2 + Y_{ijk} Q^i Q^j Q^k + \dots,$$

where the ellipsis indicates terms of higher dimensional operators suppressed by inverse powers of M_{P} . Recall that the observable fields Q^i are those listed in tab. 2.1. Moreover, we sum over contracted type indices ijk . For simplicity, Yukawa couplings Y_{ijk} are assumed to be diagonal. Note that the coefficients \mathcal{K}_i , Z , μ and Y_{ijk} may in general depend on the hidden sector fields.

Integrating out the hidden sector fields in the SUGRA Lagrangian leads us to an effective Lagrangian of the shape

$$\begin{aligned} \mathcal{L}_{\text{EFF}} = & -\mathbf{c}_1 (M_a \lambda^a \lambda^a + \text{h.c.}) - \mathbf{c}_2 m_i^2 Q^i \overline{Q}^i \\ & - \left(\mathbf{c}_3 A_{ijk} Y_{ijk} Q^i Q^j Q^k + \mathbf{c}_4 B \mu H_1 H_2 + \text{h.c.} \right), \end{aligned} \quad (4.4)$$

with $\mathbf{c}_{1,2,3,4}$ in general being complex prefactors. This is nothing else than the soft breaking Lagrangian eq. (3.5) of the MSSM. The soft breaking parameters given at the input scale are

$$M_a = \frac{1}{2 \Re \mathbf{f}_a} F^\alpha \partial_\alpha \mathbf{f}_a, \quad (4.5a)$$

$$A_{ijk} = F^\alpha \left[\widehat{K}_\alpha + \partial_\alpha \log Y_{ijk} - \partial_\alpha \log (\mathcal{K}_i \mathcal{K}_j \mathcal{K}_k) \right], \quad (4.5b)$$

$$m_i^2 = (m_{3/2}^2 + V_0) - \overline{F}^{\overline{\alpha}} F^\beta \partial_{\overline{\alpha}} \partial_\beta \log \mathcal{K}_i, \quad (4.5c)$$

where α and β run over the hidden sector SUGRA breaking fields, $\widehat{K}_\alpha = \partial_\alpha \widehat{K}$ and V_0 denotes vacuum energy (= CC). In practice, the soft parameters μ and B are treated in a special way [7]. The requirement for M_Z having the experimental value of 91.1876 GeV and eq. (3.10) determine the size of $|\mu|$. Thus, the sign of μ remains as a free parameter. The B parameter can be reexpressed in terms of $\tan \beta$ using $-2B\mu = (m_{H_d}^2 - m_{H_u}^2) \tan 2\beta + M_Z^2 \sin 2\beta$.

The soft terms in eq. (4.5) were derived in the *flat* limit. The arising Lagrangian eq. (4.4) thus can only be valid at energies below M_{P} . Eqs. (4.5) should be understood as *boundary conditions* at a scale where the effects of M_{P} fade away, typically M_{GUT} . As mentioned earlier in section 3.1, this is where the MSSM fields start to perceive the SUSY breakdown coming from the hidden sector. Using RG equations, one can then run down the value of these parameters to a low energy scale and make predictions.

Eqs. (4.5) bare the specific meaning of the gravitino in gravity mediated SUSY breakdown. Its mass sets the overall scale of the soft terms. Note that the soft terms, although all of same order $\mathcal{O}(m_{3/2})$, are not universal!

As evident from eqs. (4.5), gravity mediated soft terms depend strongly on the particular type of the SUGRA model, that is, on the shape of the Kähler potential K and the gauge kinetic function \mathbb{f}_a . There is a plethora of possibilities how to frame these two functions. One possible model that has been extensively studied (e. g. [7]) merits a very short review.

4.1.1 mSUGRA

In the so-called minimal supergravity (mSUGRA) model, the Kähler metric takes the simple form

$$\mathcal{K}_i \equiv \mathbb{1},$$

yielding minimal/canonical kinetic terms. The gauge kinetic function is universal for all gauge groups

$$\mathbb{f}_a = f.$$

Finally, the superpotential parameters $Y_{\alpha\beta\gamma}$ and μ are assumed to be independent of hidden sector fields. These three assumptions lead to a very simple pattern of universal soft breaking parameters, namely

$$M_a = m_{1/2}, \tag{4.6a}$$

$$A_{ijk} = A_0, \tag{4.6b}$$

$$m_i^2 = m_{3/2}^2 \equiv m_0^2, \tag{4.6c}$$

where we have considered a vanishing CC. Each type of soft parameters gets the same value at the input scale, denoted by the subscript 0. With these considerations, the 105 free parameters of the MSSM reduce to a set of 5 parameters

$$\{m_{1/2}, A_0, m_0, \tan \beta, \text{sign } \mu\}.$$

Demanding correct EWSB and several experimental constrains, like mass bounds from the Large Lepton-Proton Collider (LEP), shrinks the parameter space.

4.1.2 Properties of mSUGRA

The main aspects in mSUGRA mediated supersymmetry breakdown are

- The soft parameters are universal at the input scale. Due to different RG equations (see appendix B.1) they will obtain non-universal values at a lower scale.
- The RG equations imply the ordering of the gaugino masses

$$M_3 : M_2 : M_1 = g_3^2 : g_2^2 : g_1^2.$$

If the gaugino masses are universal at the GUT scale the ordering at the electroweak scale is typically $M_1 : M_2 : M_3 \simeq 1 : 2 : 7$.

- The LSP is the lightest neutralino $\tilde{\chi}_1^0$. In mSUGRA one typically has a gaugino-like LSP [7].

4.2 Anomaly Mediation

In this section we derive heuristically the soft breaking terms at quantum (loop) level. The trick we will use is to introduce a new pseudo-symmetry, which is preserved at classical level but will be violated at quantum level. As we know from section 2.5 the SUGRA multiplet consists of the graviton, the gravitino and the corresponding auxiliary field. It is convenient to house this auxiliary field inside a non-dynamical chiral superfield ϕ called *chiral conformal compensator*

$$\phi \equiv 1 + \theta^2 F^\phi. \quad (4.7)$$

This superfield is only a mnemonic device in order to implement the new symmetry. We can couple it (or rather F^ϕ) to the MSSM chiral and vector superfields through

$$\mathcal{L}_\phi = \int d^2\theta \frac{1}{g^2} \Xi^2 + \int d^4\theta \phi^\dagger \phi Q^\dagger N Q + \int d^2\theta \phi^3 W + \text{h.c.}, \quad (4.8)$$

which is a *rescaling* of the usual Lagrangian. Here $N = \exp(2g\Gamma^a T^a)$, Q and Γ are chiral and vector superfields, respectively. One can attempt to gauge away ϕ by the scale transformation

$$Q \longrightarrow \phi^{-1} Q.$$

The Lagrangian eq. (4.8) changes to

$$\mathcal{L}_\phi = \int d^2\theta \frac{1}{g^2} \Xi^2 + \int d^4\theta Q^\dagger N Q + \int d^2\theta (Q^3 + \mu\phi Q^2) + \text{h.c.}. \quad (4.9)$$

We see that eq. (4.8) would be scale invariant at loop level if the superpotential W would not contain the quadratic term. This μ -term needs a special treatment [9] and we will not consider it here. Anyhow, from this we see that an explicit mass term would spoil scale invariance.

What we have done so far was just a rewriting of the usual Lagrangian with an extra superfield ϕ . This rewriting introduced a new symmetry (scale invariance), called *superconformal symmetry*. We also saw that at tree level the Lagrangian was scale invariant indicating that a non-zero $\langle F^\phi \rangle$ is ineffective. So we can conclude that at classical level F^ϕ decouples from the visible sector.

This changes drastically if we include the loop level. Due to quantum corrections the gauge and Yukawa couplings become scale dependent. The scale dependence is given by the respective β -functions. Since the presence of ultraviolet divergences requires regularization, this introduces an explicit mass scale; the cut-off Λ_{UV} . As we saw above, an explicit mass term ruins the superconformal symmetry and one is left over with a *superconformal anomaly*. The coupling of $\langle F^\phi \rangle$ to the observable fields is enacted by these explicit mass terms. The soft breaking terms can be achieved by the following procedure of [9, 44, 45] :

Soft Terms in Anomaly Mediation

- The auxiliary field of the SUGRA multiplet acquires its VEV, $\langle F^\phi \rangle$, through the coupling to a SUSY breaking sector.
- Impose a superconformal symmetry (scale invariance).
- At quantum level $\langle F^\phi \rangle$ couples to the observable sector through the superconformal anomaly.
- The soft terms can be derived from eq. (4.8) [9].

It is worth to stress that in order to break SUSY we need a separate sector. Superconformal anomaly is merely responsible for transmitting the breakdown, which occurs in a hidden sector, to the observable sector. Once a SUSY breaking sector provides $\langle F^\phi \rangle \neq 0$, anomaly mediation is collateral present at loop level. Since it contains the auxiliary field of the SUGRA multiplet, the conformal compensator couples to the visible as well as to the hidden sector. The effect of the hidden sector dynamics manifests itself in ϕ .

Let us briefly discuss the soft breaking parameters. Explicit derivations [9] reveal that the gaugino masses are given by

$$M_a = \frac{\beta_{g_a}}{g} F^\phi, \quad (4.10a)$$

with $\beta_{g_a} = \partial g_a / \partial \log \varrho$. The A parameters, also arising at 1-loop, are

$$A_{ijk} = \frac{1}{2} \left(\frac{\partial \log Z_i}{\partial \log \varrho} + \frac{\partial \log Z_j}{\partial \log \varrho} + \frac{\partial \log Z_k}{\partial \log \varrho} \right) F^\phi, \quad (4.10b)$$

where Z_i , Z_j and Z_k denote the wavefunction renormalization. The soft scalar squared masses arise at 2-loop level

$$m_i^2 = -\frac{1}{4} \left(\frac{\beta_g}{16\pi^2} \frac{\partial \gamma_i}{\partial g} + \frac{\beta_y}{16\pi^2} \frac{\partial \gamma_i}{\partial y} \right) \left| F^\phi \right|^2. \quad (4.10c)$$

Here β_g and β_y are the β -functions for the gauge and Yukawa couplings, respectively. The so-called *anomalous dimension* γ_i describes the RG dependence of the wave function renormalization Z_i . It is defined by

$$\frac{1}{16\pi^2} \gamma_i \equiv \frac{\partial \log Z_i}{\partial \log \varrho^2}, \quad \frac{1}{16\pi^2} \dot{\gamma}_i \equiv \frac{\partial \gamma_i}{\partial \log \varrho^2}. \quad (4.11)$$

Eq. (4.10) are exact in the sense that they hold to all orders in perturbation theory. We also see that in anomaly mediation the soft terms are controlled by the MSSM couplings, β -functions and anomalous dimensions. To accentuate the magnitude of the soft terms, let us reexpress eqs. (4.10) using the RG equations

for the gauge couplings eq. (3.4) and the anomalous dimension eq. (4.11). We then obtain

$$M_a = b_a g_a^2 \frac{F^\phi}{16\pi^2}, \quad (4.12a)$$

$$A_{ijk} = (\gamma_i + \gamma_j + \gamma_k) \frac{F^\phi}{16\pi^2}, \quad (4.12b)$$

$$m_i^2 = -\dot{\gamma}_i \frac{|F^\phi|^2}{(16\pi^2)^2}. \quad (4.12c)$$

We see that the overall scale of the soft terms is the loop suppressed $\langle F^\phi \rangle$. If we assume $\langle F^X \rangle \simeq \langle F^\phi \rangle$, with X being a hidden sector field of gravity mediation, we see that anomaly mediation would be suppressed by a factor of $16\pi^2 \sim 160$.

4.2.1 Properties of Anomaly Mediation

The main aspects in anomaly mediated supersymmetry breakdown are:

- Anomaly mediation is non-universal at and below the GUT scale.
- From eq. (4.10c) it is clear that for fields with gauge interactions $\gamma_i \sim +g_a^2$ we get

$$m_i^2 \sim -g_a^4 b_a \left| \langle F^\phi \rangle \right|^2.$$

Therefore sleptons are tachyonic due to $b_a > 0$ for $SU(2)$.

- The gaugino masses depend on the β -functions (or strictly speaking on b_a).
- To have SUSY breaking of the order of the EW scale anomaly mediation requires $\langle F^\phi \rangle \sim 10^4$ GeV.
- Anomaly mediation is flavor-blind [9].

Chapter 5

Mirage™ Scale Formation

In the previous chapter we have discussed two possible ways of mediating the breakdown of supersymmetry from the hidden to the observable sector. We have also seen that both options suffer from different troubles. There are other mechanisms available like gauge mediation [6, 7] or gaugino mediation [46, 47] with their own defects. A next logical step would be a *mixture* of different mediation types in order to obtain an improved scenario. There are good reasons for considering mixed scenarios. Since its proposal, string theory [12] has been a very appealing candidate for a theory containing all fundamental forces. The first section serves as a short overview and motivation to shift the gear into a new format of mediating the breakdown of supersymmetry: *mirage mediation* [18–23]. This new scenario is a mixed modulus-anomaly mediation and exhibits a distinct pattern of soft parameters which differs from other scenarios. In section 5.1 we will consider an alternative set-up, which can also lead to mirage mediation. The soft breaking terms will be derived in section 5.2. Section 5.3 is dedicated to the analysis of low energy spectra arising in mirage mediation. Finally, section 5.4 deals with mirage mediation in heterotic string theory.

5.1 SUSY violating de Sitter Vacua

5.1.1 Joining String Theory

In superstring theory [12], which comprises 5 separate types¹, pointlike particles are replaced by 1-dimensional oscillating objects, called *strings*. Different excitations of strings can be interpreted as different particles. Strings can be open and closed. Open strings end on spatially extended objects of dimension p , called *Dirichlet branes* (Dp branes)².

Consistency requires superstring theory to have 10 spacetime dimensions. Since our world is observed to indwell a 4-dimensional spacetime, we have to do something with the unobserved 6 dimensions. One idea is to *compactify* the extra dimensions on a small manifold. To have supersymmetry in 4 dimensions, one needs to compactify on special manifolds called Calabi Yau (CY) manifolds [12]. One can

¹There are the following types of string theories: *TypeI*, *TypeIIA*, *TypeIIB*, *O-Heterotic* and *E-Heterotic*. These 5 types are different limits of a single underlying theory, called *M-Theory*.

²A $D0$ brane is a point, a $D1$ brane is a string, a $D2$ brane is a membrane, etc. A Dp brane sweeps out a $p + 1$ dimensional world-volume as it propagates through spacetime.

imagine compactification as rolling up the surplus dimensions. Generically, the scale of superstring theory M_{ST} is expected to be very large.

In a low energy approximation (4D SUGRA), we see only the massless excitations of the strings. The CY manifolds are parameterized by the so-called *moduli*. These are massless scalar fields which are flat directions of the scalar potential. The most relevant moduli are: the complex structure moduli Z_α parameterizing the shape of the CY manifold, the Kähler moduli T_i describing the volume of the various cycles and the dilaton S . The shape and size of the CY manifolds in turn is connected to the strength of the couplings. Thus, the moduli determine the values of the coupling constants. Moduli are gauge singlet fields and interact only gravitationally with ordinary matter—they live in the hidden sector.

If the moduli are not stabilized the theory is not able to make any prediction since all couplings depend on the value of the moduli. The stabilization of moduli can occur perturbatively (e. g. through fluxes³ [13]) as well as non-perturbatively (e. g. gaugino condensation⁴ [14]).

One more thing to add is that recent observational data [48] suggest that our universe has a de Sitter (dS) vacuum, with a small positive CC. Given all this, we are interested in solutions with a ($N = 1$) supersymmetry violating dS vacuum.

5.1.2 KKL T Model

Recently, Kachru, Kallosh, Linde and Trivedi (KKLT) presented an interesting set-up providing dS vacua [15]. This toy model is based on type IIB compactification on CY with matter fields originating from $D3$ and/or $D7$ branes. The moduli under consideration are the complex structure moduli Z_α , the dilaton S and one single Kähler modulus T . The analysis is carried out in the framework of low energy effective SUGRA with the Kähler potential

$$K = -3 \log(T + \bar{T}) - \log(S + \bar{S}) - K(Z_\alpha, \bar{Z}_\alpha) + \dots, \quad (5.1)$$

where the ellipsis denotes the omission of the Kähler potential for the observable sector fields. To arrive at the desired situation, the construction of KKLT consists of three steps:

KKLT Construction

- Stabilize complex structure moduli Z_α and the dilaton S with fluxes. The Kähler modulus T remains unstabilized. SUSY is broken.
- Use gaugino condensation to stabilize the T modulus. The outcome consists of an AdS vacuum and restored SUSY.
- Introduce *anti-branes* to uplift the AdS vacuum to a dS vacuum and to break SUSY.

³Fluxes are non-zero VEVs of certain field strengths.

⁴Hidden sector couplings become strong in the infrared and gauginos condensate.

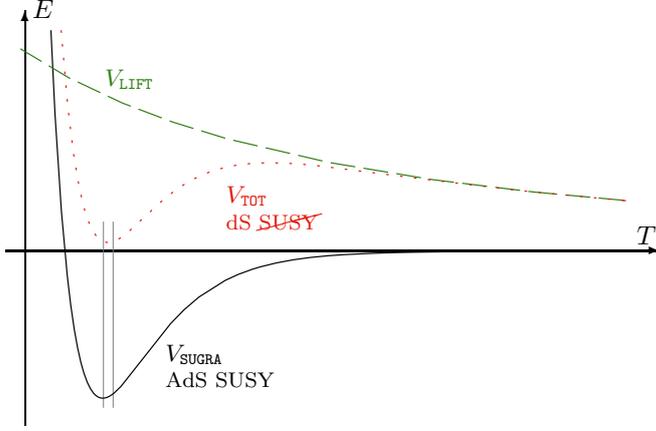


Fig. 5.1 :: Total scalar potential in KKLT set-up $V_{\text{TOT}} = V_{\text{SUGRA}} + V_{\text{LIFT}}$ (red/dotted) as a sum of the SUGRA potential (black/solid) and an uplifting potential (green/dashed). Vertical lines represent the displacement of the minima.

In the first step, complex structure moduli Z_α and the dilaton S are stabilized by turning on the fluxes. These fluxes generate a superpotential \mathcal{W} for these moduli, but not for the Kähler modulus T . After Z_α and S have been stabilized, they can acquire a huge mass ($\simeq \mathcal{O}(M_{\text{ST}}) \simeq \mathcal{O}(10^{17} \text{ GeV})$) and can be integrated out, leaving a constant contribution $\mathcal{W}_0 = \mathcal{W}(Z_0, S_0)$ to the effective low energy theory⁵. The superpotential \mathcal{W} breaks SUSY, with the scale of SUSY breaking being characterized by \mathcal{W}_0 . The stabilization of the T modulus in the second step requires non-perturbative mechanisms (gaugino condensation on $D7$ branes) and is based on the ansatz

$$W = \mathcal{W}_0 + Ae^{-aT}, \quad (5.2)$$

with a, A being real constants and \mathcal{W}_0 is the remnant of flux stabilization. The magnitudes of these constants are typically $A \sim \mathcal{O}(1)$, $aT > 1$ and $\mathcal{W}_0 \ll 1$. Note that we are working in units with $M_{\text{P}} = 1$. The stabilization of T through gaugino condensation restores supersymmetry, since it leads to

$$\mathfrak{D}_T W = 0$$

and $W_0 \neq 0$ in the minimum. Eq. (4.2) reveals for the vacuum energy

$$V_0 = -3e^K |W_0|^2 < 0.$$

At this stage, all moduli are stabilized but we are left with an anti de Sitter (AdS) SUSY preserving vacuum (fig. 5.1). The remedy comes in the last step where KKLT introduce (ad hoc) non-supersymmetric objects: *anti-branes*. The presence of anti-branes ($\overline{D3}$) breaks supersymmetry and contributes an additional energy, which can be viewed as an *uplifting* potential

$$V_{\text{LIFT}} = \frac{D}{(T + \overline{T})^2},$$

⁵The constant \mathcal{W}_0 is quantized in the sense that it is fixed by the fluxes, which are quantized.

where D is a constant. By fine-tuning D , it is possible to obtain a dS vacuum close to zero. The uplifting procedure changes the value of the T modulus at the minimum only slightly (fig. 5.1) due to the fact that before uplifting the SUGRA potential is quite steep, unlike the uplifting potential. Thus the value of the T modulus in the minimum is nearly unaffected by the uplifting.

From fig. 5.1, we see that the dS vacuum can be destabilized by tunneling effects and one might worry about the lifetime. In [16] it was argued that the lifetime of the metastable vacuum is approximately $10^{10^{20}}$ years, so, for all practical purposes, the dS vacuum can be considered as completely stable.

5.1.3 Little Hierarchy

There is a lot of fine-tuning to be done in the KKLT model in order to obtain a reasonable low energy SUSY (breaking). As mentioned in the previous section, typical values for this scheme are

$$\begin{aligned} A &\sim \mathcal{O}(1) [\times M_{\text{P}}^3], \\ \mathscr{W}_0 &\sim 10^{-4} [\times M_{\text{P}}^3], \end{aligned}$$

leading for example to a gravitino mass of order

$$m_{3/2} = M_{\text{P}} e^{G_0/2} |W_0| \sim M_{\text{P}} \frac{1}{(2\Re T_0)^{3/2}} \mathscr{W}_0 \sim 10^{10} \dots 10^{14} \text{ GeV},$$

which is far away from the TeV region. Also in order to obtain a dS vacuum energy that be consistent with observation, one has to fine-tune the uplifting potential with an accuracy of order 10^{-120} [16].

Now let us briefly consider the derivation of the soft breaking terms in the low energy scenario caused by the KKLT model. As discussed above, the effective (low energy) potential contains two parts:

$$V = V_{\text{SUGRA}} + V_{\text{LIFT}}. \quad (5.3)$$

The first part V_{SUGRA} describes the situation after the heavy Z_α and S moduli have been integrated out and the T modulus was stabilized. In the effective theory, this can be described by the Lagrangian [20, 21]

$$\mathcal{L} = \int d^4\theta \bar{\phi}\phi \left[-3e^{-\hat{K}/3} \right] + \int d^2\theta \left[\left[\frac{1}{4} \mathbb{f}_a \Xi^2 + \phi^3 W \right] + \text{h.c.} \right], \quad (5.4)$$

where the $\phi = 1 + \theta^2 F^\phi$ is the conformal compensator, Ξ the spinorial field strength. The gauge kinetic function \mathbb{f}_a , the superpotential W and the Kähler potential K are given by the ansätze [19–21]

$$K = -3 \log(T + \bar{T}) + \sum_i (T + \bar{T})^{-n_i} Q^i \bar{Q}^i, \quad (5.5)$$

$$W = \mathscr{W}_0 - A e^{-aT} + \frac{1}{6} \lambda_{ijk} Q^i Q^j Q^k, \quad (5.6)$$

$$\mathbb{f}_a = T^{l_a}, \quad (5.7)$$

with Q^i being the visible sector fields. The so-called *effective modular weights* n_i denote the location of the visible fields. For MSSM fields living on $D3$ branes one has $n_i = 1$ and $l_a = 0$ whereas for MSSM fields originating from $D7$ branes $n_i = 0$ and $l_a = 1$. If the visible fields live on brane intersections, n_i take fractional values $n_i \in (0, 1)$ [20, 21]. The holomorphic Yukawa couplings λ_{ijk} are assumed to be moduli independent.

The low energy consequence of $\overline{D3}$ uplifting can be expressed through [21]

$$\mathcal{L}_{\text{LIFT}} = \int d^4\theta \phi^2 \bar{\phi}^2 \theta^2 \bar{\theta}^2 \mathcal{P}_{\text{LIFT}}, \quad (5.8)$$

where $\mathcal{P}_{\text{LIFT}} \equiv D(T + \bar{T})^{n_p}$ is the so-called *spurion* operator, which is related to the uplifting by $V_{\text{LIFT}} = \exp(2/3\hat{K})\mathcal{P}_{\text{LIFT}}$, where \hat{K} denotes the hidden sector part of the Kähler potential. In the original KKLT set-up $n_p = 0$.

The minimization of the total scalar potential eq. (5.3), under fine-tuning the CC to zero⁶, reveals [19–21]

$$a\Re T_0 \sim \log(A/\mathcal{W}_0) \sim \log(M_{\text{P}}/m_{3/2}), \quad (5.9a)$$

$$m_{3/2} \sim M_{\text{P}} \frac{\mathcal{W}_0}{(2\Re T_0)^{3/2}}, \quad (5.9b)$$

$$m_T \sim (a\Re T_0) m_{3/2}, \quad (5.9c)$$

and the F -term contributions to the soft terms are found to be

$$\frac{F^T}{T_0 + \bar{T}_0} \sim \frac{1}{(a\Re T_0)} m_{3/2}, \quad (5.9d)$$

$$F^\phi \sim m_{3/2}. \quad (5.9e)$$

From eqs. (5.9), we see the specific relations among the soft masses, the gravitino and the modulus masses. The soft masses from gravity/modulus⁷ mediation are suppressed by $\langle a\Re T_0 \rangle$ against the gravitino mass, while the mass of T is enhanced by the same factor.

To get a MSSM spectrum in the TeV range, \mathcal{W}_0 has to be of order 10^{-13} . This can be achieved by fine-tuning fluxes in the underlying string theory. After this fine-tuning has been performed, one finds

$$a\Re T_0 \sim \log(M_{\text{P}}/m_{3/2}) = \mathcal{O}(4\pi^2), \quad (5.10a)$$

$$m_T \sim 10^7 \dots 10^6 \text{ GeV}, \quad (5.10b)$$

$$m_{3/2} \sim 10^5 \dots 10^4 \text{ GeV}, \quad (5.10c)$$

$$m_{\text{SOFT}} \sim 10^2 \dots 10^3 \text{ GeV}. \quad (5.10d)$$

This result has some very interesting consequences, which we summarize below.

⁶Even though the cosmological constant is non-zero, its small value $\Lambda \sim [10^{-3} \text{ eV}]^4$ can be well approximated by 0 for our purposes.

⁷In gravity mediation soft terms are guided by the F component of certain hidden sector fields. Moduli are fields of that kind motivated by string theory, hence the name *modulus mediation*.

- Unlike other SUSY breaking scenarios, the gravitino mass lies in the multi-TeV range whereas the T modulus is heavier by a factor $4\pi^2$. This is interesting for cosmological considerations. In particular, the heavy moduli and gravitinos produced in the early Universe would decay before nucleosynthesis and thus would not affect the abundances of light elements [18]. But there is still an open discussion about the cosmological moduli and gravitino problems [24–26, 49]. In this work, however, we will not consider these problems.
- The contribution to the soft terms from modulus mediation is suppressed by $4\pi^2$ and becomes comparable to the loop suppressed contribution from anomaly mediation.

<i>Modulus</i>	<i>Anomaly</i>
$m_{\text{SOFT}} \sim \frac{F^T}{T_0 + \bar{T}_0} \sim \frac{m_{3/2}}{4\pi^2}$	$m_{\text{SOFT}} \sim \frac{F^\phi}{16\pi^2} \sim \frac{m_{3/2}}{16\pi^2}$

The soft terms receive comparable contributions from both mediation types. This *mixed modulus-anomaly mediation* leads to a distinct pattern of soft masses which we will study shortly.

- The moderately large parameter $(a\Re T_0) \sim 4\pi^2$ leads to a hierarchy among the modulus, the gravitino and the soft masses

$$m_T \sim 4\pi^2 m_{3/2} \sim (4\pi^2)^2 m_{\text{SOFT}},$$

which is called *the little hierarchy* [17]. This little hierarchy is given by the logarithm of the large hierarchy between M_{P} and $m_{3/2}$.

It was argued that such structures generically arise when moduli are stabilized by non-perturbative mechanisms close to a supersymmetric point [17, 18]. In that sense KKL_T is just one example. We will use this example as an encouragement to study the low energy phenomenology of the mixed modulus-anomaly mediation.

5.1.4 Mirage Unification

Now let us figure out the specific properties in the mixed modulus-anomaly mediated scenario. This can be done simply by looking at the gaugino masses. We will consider the 1-loop level. Schematically we have

$$M_a = M_{\text{MODULUS}} + M_{\text{ANOMALY}}$$

and both contributions are of comparable size. The contribution from anomaly mediation eq. (4.12a) is proportional to the respective β -functions⁸ or strictly speaking to the beta function coefficients b_a . Thus, anomaly mediation splits the masses of the gauginos at the GUT scale according to b_a . Furthermore, we know

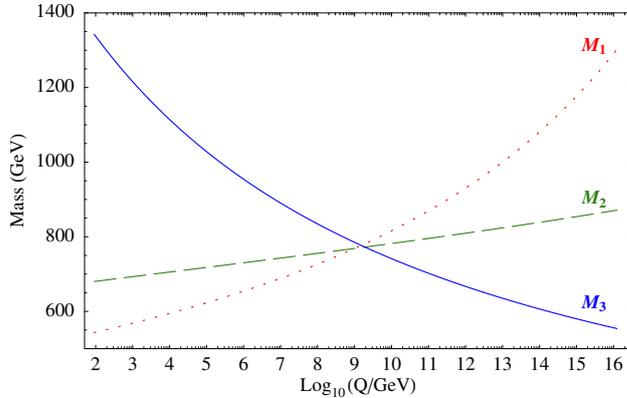


Fig. 5.2 :: RG evolution of the gauginos and mirage unification. At the GUT scale the gluino is the lightest gaugino. Above/below the mirage scale, the ordering of the masses flips. At the EW scale the gluino is the heaviest gaugino due to the positive RG contribution from its beta function coefficient b_a .

from section 4.2 that anomaly mediation is non-universal below the GUT scale. In the simplest case where the contribution from modulus mediation is universal at the GUT scale, it will not change the ordering of the gaugino masses at the GUT scale but it will change the relative ratios due to the universal shift. In any case, the gluino will be the lightest gaugino. The ratio of the gaugino masses at the GUT scale is typically $M_3 : M_2 : M_1 \simeq 1 : 1.6 : 2.4$.

Now the evolution of the gaugino masses from the GUT scale is governed by the same β -functions (i. e. the same b_a), so the gaugino masses will meet again at a given scale. This scale however depends on the ratio of anomaly to modulus mediation. If both contributions are comparable, the gauginos will meet at an intermediate scale. Then at the EW scale the ordering of the gauginos is inverted and typically $M_1 : M_2 : M_3 \simeq 1 : 1.2 : 2.6$.

Since there is no physical threshold associated with this scale, it is called *mirage scale* M_{MIR} . This form of mediating the SUSY breakdown is called *mirage mediation* because mirage unification⁹ is a generic feature of schemes where modulus and anomaly mediation are competitive [18, 21–23].

One can introduce a certain parameter, which measures the balance between modulus and anomaly contributions. In mirage mediation this parameter tells us where the gaugino masses coincide and thus the size of the mirage scale. The domination of modulus over anomaly mediation shifts the mirage scale towards the GUT scale (fig. 5.3 frame ①) and leads to mSUGRA.

On the other hand, when anomaly mediation prevails over modulus mediation, the gauginos will coincide at a very low energy scale (fig. 5.3 frame ②). Anyway, the most interesting case is where the modulus to anomaly ratio is $\mathcal{O}(1)$. One should note however that scalars differ. Even though the first and the second generation scalars unify at the mirage scale, the third generation scalars “deviate” from the mirage scale due to large Yukawa contributions (see fig. 5.4). Nevertheless, for gauginos this 1-loop statement is very robust.

⁸The β -function is negative for the gluino and positive for the bino/wino. See also appendix B.1.

⁹If we apply a GUT, e. g. $SU(5)$ or $SO(10)$, there would be a true unification for the gauginos above the GUT scale and a mirage unification at an intermediate scale.

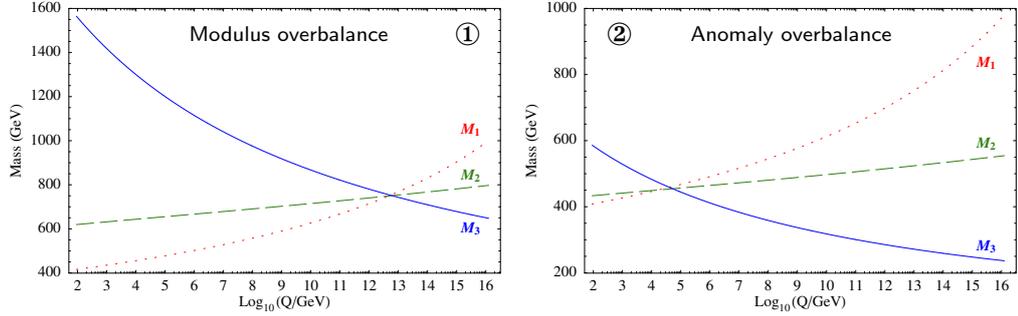


Fig. 5.3 :: Influence of modulus/anomaly ratio on M_{MIR} .

5.1.5 de Sitter Vacua from Spontaneously Broken SUGRA

Let us just pause for the moment and have some thoughts about the KKLТ model. The interesting feature in this scheme is that the resulting soft terms receive comparable contributions from modulus and anomaly mediation, leading to mirage mediation. On the other side, the way KKLТ arrived at SUSY violating dS vacua is rather “brute”. Recall from section 5.1.2 that in the first step of the construction SUSY is broken, then in the second step it is restored and it gets explicitly broken in the last step. The desired situation would be to achieve mirage mediation in the framework of spontaneously broken SUGRA.

Indeed, this idea was investigated by Lebedev et al. [50]. It was shown that the search for stable dS vacua within a spontaneously broken SUGRA can be successful if one introduces new (light) DOF. This new DOF can be represented in the effective low energy theory by additional hidden sector matter fields¹⁰. The simplest ansatz was suggested to be [50]

$$K = -3 \log(T + \bar{T}) + C\bar{C} + \dots, \quad (5.11)$$

with T being an overall Kähler modulus, C is a hidden sector matter field and the ellipsis denote the omission of the Kähler potential for the visible fields. The superpotential, after integrating out heavy moduli, is given by the ansatz

$$W = \sum_i \omega_i(C) e^{-a_i T_i} + \tau(C), \quad (5.12)$$

where the sum runs over gaugino condensates¹¹ and the functions ω and τ follow from integrating out heavy DOF. In what follows these functions will be treated as some generic functions. Given this set-up, the authors of [50] showed, that stable dS/Minkowski vacua in the framework of spontaneously broken SUGRA can be realized. In addition, as we will see later, new patterns of soft masses can arise.

¹⁰These hidden sector matter fields are assumed to be singlets under the unbroken gauge symmetries.

¹¹Here gaugino condensation breaks SUGRA spontaneously, setting the scale of the breakdown.

5.2 Soft Terms in Mirage Mediation

In this section we would like to derive the soft breaking terms in the framework of spontaneously broken SUGRA as suggested in [50]. We will then see that this framework offers several possibilities of which mirage mediation is just one option. The underlying string theory is type IIB.

To derive the soft terms in the mixed modulus anomaly scenario we use the ansätze [50]

$$K = -3 \log(T + \bar{T}) + C\bar{C} + Q^i \bar{Q}^i (T + \bar{T})^{-n_i} [1 + \xi_i C\bar{C}], \quad (5.13)$$

$$f_a = T^{l_a}, \quad (5.14)$$

where n_i are the effective modular weights, depending on the location of the visible MSSM fields Q^i . For matter fields on $D3$ branes $l_a = 0$ and $n_i = 1$ whereas for the MSSM fields living on $D7$ branes $l_a = 1$ and $n_i = 0$. If the visible fields live on the intersection of branes, the modular weights take fractional values $n_i \in (0, 1)$. As was pointed out in [21], the most phenomenologically attractive situation is to have gauge fields originated from $D7$ branes but to leave a freedom for the origin of matter fields. This gives $l_a = 1$ but n_i remains undetermined¹². Note that the last term in eq. (5.13) is suppressed by M_{P}^{-2} . The parameter ξ_i describes the coupling between visible and hidden matter fields. This is a new ingredient endowing the pattern of soft masses with new facets.

To derive the soft breaking terms, we first look at the scalar potential, which is given by eq. (4.2). Inserting K from eq. (5.13) we obtain

$$V = \frac{e^{C\bar{C}}}{(T + \bar{T})^3} \left[\frac{1}{3} |W_T(T + \bar{T}) - 3W|^2 + |W_C + W\bar{C}|^2 - 3|W|^2 \right], \quad (5.15)$$

where the subscripts on W denote derivatives with respect to the SUSY breaking fields T and C . The soft terms are obtained at the minimum of the potential which, without loss of generality, can be placed at¹³ [50]

$$V(T = T_0, C = 0) \simeq 0, \quad (5.16)$$

where we have tuned the cosmological constant to zero. Eq. (5.16) connects the SUSY breaking terms eq. (4.1) via

$$m_{3/2}^2 = \frac{|F^T|^2}{(T_0 + \bar{T}_0)^2} + \frac{1}{3} |F^C|^2. \quad (5.17)$$

To obtain the soft terms in the mixed modulus-anomaly scenario we have to be somewhat careful. For the pure modulus contribution we can use eq. (4.5) and for

¹²In particular, if MSSM fields are located on $D3$ branes, the resulting phenomenology is controlled by anomaly mediation. Phenomenology of anomaly mediation however was exhaustively studied in the literature, e. g. [51].

¹³That is, the SUSY breaking fields T and C can be stabilized at $C \simeq 0$ and $T \simeq T_0$ respectively.

the pure anomaly part eqs. (4.12) are suitable. The soft scalar masses, however, include mixed terms which can be found in [20, 21]. See also appendix B.2 for details. Taking all this into account the soft breaking terms become

$$M_a = \frac{F^T}{T_0 + \bar{T}_0} + b_a g_a^2 \frac{F^\phi}{16\pi^2}, \quad (5.18a)$$

$$A_{ijk} = (-3 + n_i + n_j + n_k) \frac{F^T}{T_0 + \bar{T}_0} + (\gamma_i + \gamma_j + \gamma_k) \frac{F^\phi}{16\pi^2}, \quad (5.18b)$$

$$m_i^2 = \Delta_i + (3\xi_i - n_i) \frac{|F^T|^2}{(T_0 + \bar{T}_0)^2} - \dot{\gamma}_i \frac{|F^\phi|^2}{(16\pi^2)^2} + \frac{2F^\phi F^T \partial_T}{16\pi} \gamma_i, \quad (5.18c)$$

with $\Delta_i = (1 - 3\xi_i)m_{3/2}^2$. Note that we have calculated the soft terms in the minimum of the potential eq. (5.16) using eq. (5.17) for $m_{3/2}^2$.

From eq. (5.17) we know that F^T and F^C are related to each other, but (in general) there is no relation to the conformal compensator of anomaly mediation F^ϕ . Thus, in principle, we can have different scenarios depending on the dosage of F^T and F^ϕ . In this work we are mainly interested in mirage mediation, so (motivated by KKL^T) we consider again

$$\frac{F^T}{T_0 + \bar{T}_0} \sim \frac{F^\phi}{4\pi^2} \sim \frac{m_{3/2}}{4\pi^2}.$$

As mentioned earlier, the phenomenology of mirage mediation is sensitive to the ratio between modulus and anomaly contribution. Therefore it is appropriate to introduce a new parameter which measures the balance between these two contributions. Actually two different definitions of this parameter have been used [18, 21], which are inverse proportional to each other. We will use the definition of [18], which is more descriptive for the study of low energy spectra:

$$\frac{F^T}{T_0 + \bar{T}_0} \equiv \alpha M_s, \quad M_s \equiv \frac{m_{3/2}}{16\pi^2}. \quad (5.19)$$

In the limit $\alpha \gg 1$ modulus mediation dominates, since the contribution from F^T would exceed F^ϕ . The case $\alpha = 0$ corresponds to pure anomaly mediation. The precise value of α depends on the details of the underlying model (e. g. shape of the uplifting potential). The original KKL^T model predicts $\alpha \sim 5$ [18]. However, here we can treat α and M_s (or respectively $m_{3/2}$) as free parameters and analyze the resulting phenomenology for $\alpha \sim \mathcal{O}(1)$. With eq. (5.19) the boundary condition eq. (5.18) can be recasted as

$$M_a = M_s \left[\alpha + b_a g_a^2 \right], \quad (5.20a)$$

$$A_{ijk} = M_s \left[(-3 + n_i + n_j + n_k) \alpha + (\gamma_i + \gamma_j + \gamma_k) \right], \quad (5.20b)$$

$$m_i^2 = M_s^2 \left[(3\xi_i - n_i) \alpha^2 - \dot{\gamma}_i + 2\alpha (T_0 + \bar{T}_0) \partial_T \gamma_i \right] + (1 - 3\xi_i) m_{3/2}^2. \quad (5.20c)$$

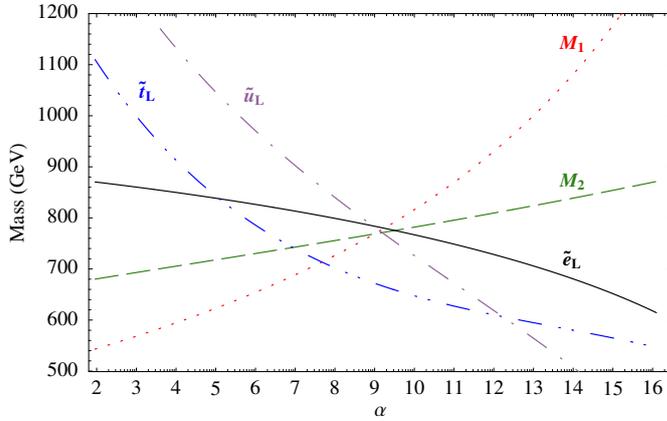


Fig. 5.4 :: Mirage unification of gauginos and the first and second generation sfermions. To retain clarity only some of the sfermions are plotted. The stop quark does not mirage unify.

The phenomenology of mirage mediation differs somewhat from that of pure modulus or anomaly mediation. That can be already read off from eqs. (5.20). The mixture of modulus and anomaly contributions is plainest present in the scalar squared masses, which contain pure modulus/anomaly parts as well as mixed ones. Then we have an additional DOF given by the modular weights n_i which act only on the A -terms and on the scalar squared masses. Changing n_i would not affect the mirage unification of the gauginos. Finally we have this new parameter ξ_i (special for the model of [50]) which only affects the scalar squared masses. Note, that this parameter controls the last term in (5.20c) which is enhanced by $(16\pi^2)^2$ relative to the other terms. $\xi = 1/3$ corresponds to the original version of mirage mediation from KKL_T. In the case $0 \leq \xi < 1/3$ the soft scalar squared masses receive an additional positive contribution that might be useful in order to cancel the “disturbing” negative contribution coming from anomaly mediation. One last comment is to add. Using the parameterization eq. (5.19) the mirage unification scale M_{MIR} can be written as [18]

$$M_{\text{MIR}} = M_{\text{GUT}} e^{-\frac{8\pi^2}{\alpha}}. \quad (5.21)$$

At this scale all gaugino masses unify as well as the first and second generation sfermions (fig. 5.4). The third generation sfermions feel the effect of larger Yukawa couplings and thus behave differently than their family members. From eq. (5.21) one sees that the mirage scale moves to lower energies when α decreases and it grows with increasing α . For $\alpha \sim 5$ the mirage scale is $M_{\text{MIR}} \simeq 10^9$ GeV. A mirage scale in the TeV region corresponds to $\alpha \sim 2$.

5.3 Low Energy Phenomenology

In the following sections we want to analyze the low energy spectra emerging in mirage mediation. Let us begin by looking at the parameter space of mirage mediation. First of all we have the parameter α , measuring the ratio between modulus/anomaly contributions, and the gravitino mass setting the scale of the

soft parameters. Then, as also common in other mediating scenarios, we have the μ parameter, whose absolute value is determined by requiring correct EWSB. The sign of μ remains a free parameter. Furthermore, the $B\mu$ -term, as mentioned in section 4.1 can be traded for $\tan\beta$. And at last we have the modular weights and the ξ_i parameter. So the parameter space is spanned by

$$\{\alpha, m_{3/2}, \tan\beta, \text{sign } \mu, n_i, \xi_i\}.$$

However, the parameters $\tan\beta$, $\text{sign } \mu$, n_i and ξ_i are usually fixed and the scheme is governed by α and $m_{3/2}$ only. So, compared to other schemes (e.g. mSUGRA), in mirage mediation one has just two free continuous parameters. The low energy spectra are subject to several experimental and theoretical constraints of which the most important ones are [18]:

- 1] absence of tachyons,
- 2] correct electroweak symmetry breaking,
- 3] uncharged/uncolored LSP,
- 4] Higgs mass bound $m_h > 114 \text{ GeV}$,
- 5] chargino mass bound $m_{\chi^+} > 103.5 \text{ GeV}$,
- 6] branching ratio for the decay $2.33 \times 10^{-4} \leq \text{BR}(b \rightarrow s\gamma) \leq 4.15 \times 10^{-4}$,
- 7] branching ratio for the decay $\text{BR}(B_s \rightarrow \mu^+\mu^-) < 2.9 \times 10^{-7}$,
- 8] muon $g - 2$ anomaly,
- 9] neutralino Dark Matter relic abundance respecting acroWMAP data.

In this work we will concentrate on the constraints 1–4.

The chargino mass constraint 5 happens to be weaker [18] than the Higgs mass constraint 4. That is the reason why we will neglect it here.

Our analysis of the type IIB inspired mirage mediation is divided in two parts. The first part deals with $\xi_i = 1/3$ and the second part with $\xi_i \neq 1/3$, for all i . For definiteness we decompose ξ as

$$\xi = \frac{1}{3} - \Delta\xi.$$

Throughout our analysis we use $m_t = 172 \text{ GeV}$ as input value and scan over the parameters α and $m_{3/2}$ in the range

$$0 \leq \alpha \leq 10, \quad 0 \leq m_{3/2} \leq 120 \text{ GeV},$$

for different values of $\tan\beta$. Taking only the constraints 1–4 into account the phenomenology with $\mu < 0$ is similar to that with $\mu > 0$. All low energy spectra are calculated using SOFTSUSY [52].

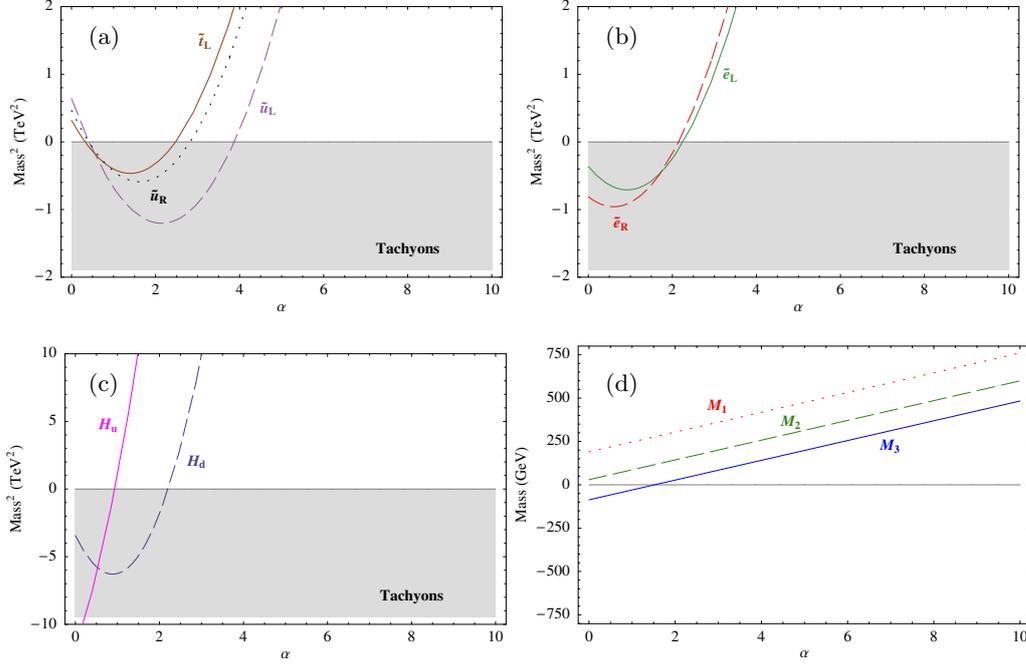


Fig. 5.5 :: Miscellaneous soft breaking parameters at the GUT scale plotted versus α .

5.3.1 Case $\Delta\xi = 0$ (Ordinary Mirage Mediation)

We begin our analysis by looking at the soft parameters at the GUT scale for zero modular weights $n_i = 0 \forall i$. Fig. 5.5 (a) – (c) shows soft breaking parameters at the GUT scale plotted vs. α . In the region $0 \leq \alpha \leq 4$ tachyonic squarks and sleptons are present due to anomaly overbalance. The limit $\alpha = 0$ corresponds to pure anomaly mediation. The boundary condition at the GUT scale, eq. (5.20), can be approximated by

$$M_1 \simeq (3.33 + \alpha) M_s, \quad M_2 \simeq (0.52 + \alpha) M_s, \quad M_3 \simeq (-1.52 + \alpha) M_s, \quad (5.22)$$

$$\begin{aligned} m_{L_1}^2 &\simeq (-0.91 - 1.85\alpha + \alpha^2) M_s^2, & m_{L_3}^2 &\simeq (-2.02 - 1.21\alpha + \alpha^2) M_s^2, \\ m_{e_1}^2 &\simeq (-2.03 - 1.22\alpha + \alpha^2) M_s^2, & m_{e_3}^2 &\simeq (-2.04 - 1.20\alpha + \alpha^2) M_s^2, \\ m_{Q_1}^2 &\simeq (1.59 - 4.29\alpha + \alpha^2) M_s^2, & m_{Q_3}^2 &\simeq (0.68 - 2.19\alpha + \alpha^2) M_s^2, \\ m_{u_1}^2 &\simeq (1.14 - 3.24\alpha + \alpha^2) M_s^2, & m_{u_3}^2 &\simeq (-0.66 + 0.94\alpha + \alpha^2) M_s^2, \\ m_{d_1}^2 &\simeq (1.82 - 2.83\alpha + \alpha^2) M_s^2, & m_{d_3}^2 &\simeq (1.82 - 2.83\alpha + \alpha^2) M_s^2, \end{aligned} \quad (5.23)$$

where we have used $\tan \beta = 10$. Although the squarks are non-tachyonic in pure anomaly mediation, they become tachyonic due to the mixture of anomaly and modulus mediation as α increases. Squarks are tachyonic for $2 < \alpha < 4$, sleptons and Higgs scalars in the region $0 \lesssim \alpha \lesssim 2$. Absence of tachyons requires $\alpha > 4$.

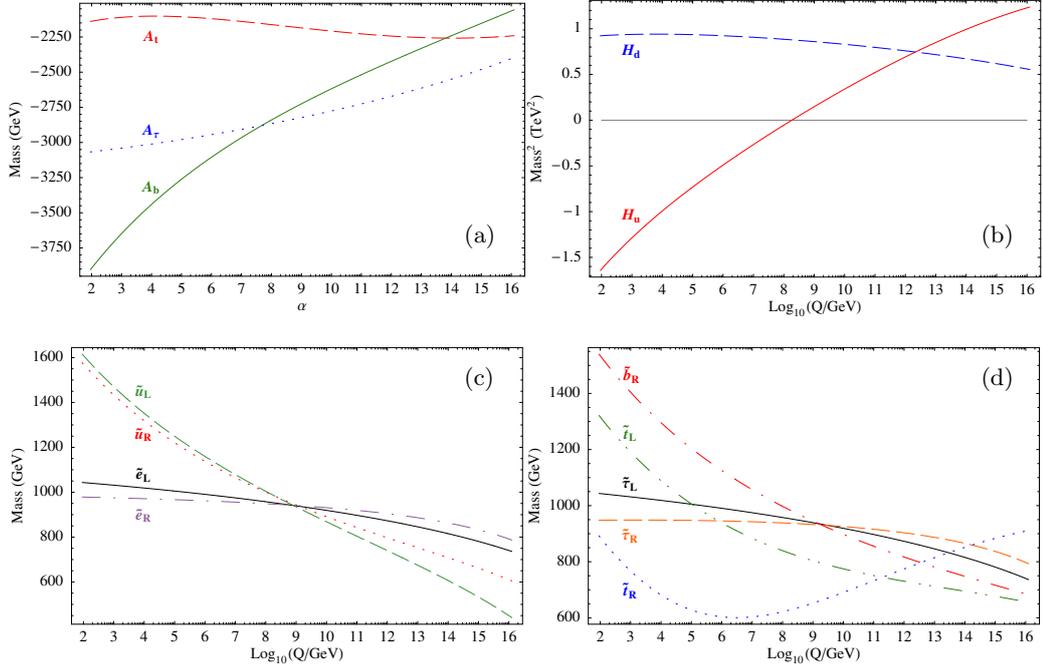


Fig. 5.6 :: Miscellaneous soft breaking parameters RG evolved to the EW scale for $\alpha = 5$, $m_{3/2} = 30$ TeV, $\mu > 0$ and $\tan \beta = 10$.

The gaugino masses (fig. 5.5 (d)) are ordered as $M_1 > M_2 > M_3$ at the GUT scale. As evident from eq. (5.22), they grow linearly with α . The ordering of the gauginos is not affected by α but the relative ratios, specified through the beta function coefficients b_a , change. For $\alpha = 0$ one obtains the familiar signature of anomaly mediation $M_1 > M_2$ and $M_3 < 0$. As α increases, the gluino M_3 crosses the α -axis at $\alpha \sim 1.5$. Thus, around $\alpha \sim 1.5$ the gaugino mass is very small. For large α values the gaugino masses unify, leading to pure modulus mediation. The behavior of the gauginos is independent of ξ_i and n_i .

Let us now evolve the input parameters from the GUT scale to the EW scale. The (1-loop) RG equations can be found in the appendix B.1. The A parameters are always negative. The RG running pushes the already (large) negative values at the GUT scale to more negative values. Thus at the EW scale $|A|$ is very large. A_t is the only exception. Its flat running is due to the large Yukawa coupling y_t , see eq. (B.7), which counterbalances the contribution from the gauge terms. Instead y_b and y_τ are too small to yield the same effect for A_b and A_τ . Fig. 5.6 (a) shows the RG flow of the A -terms.

The first and second generation of squarks and sleptons (fig. 5.6 (c)) behave in a similar way as the gauginos. For $\alpha = 5$ they unify at an intermediate scale $Q \sim 10^{10}$ GeV. Thus their mass ordering at the GUT scale becomes inverted at the EW scale. Colored particles receive larger positive contributions from RG running than uncolored.

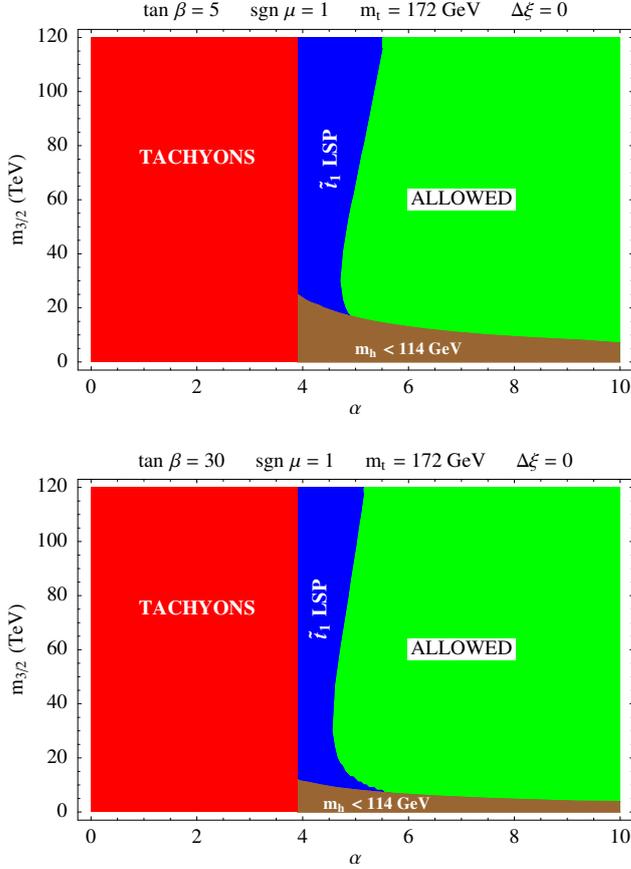


Fig. 5.7 :: Parameter space spanned by $(\alpha, m_{3/2})$. For fixed sign μ the constraints on the Higgs mass bound and an uncharged LSP are less restrictive if $\tan \beta$ is large. In the green/■ reef all constraints we consider are satisfied. The case with negative μ is identical to that of positive μ . However, the $\text{BR}(b \rightarrow s\gamma)$ and $(g - 2)_\mu$ constraints favor a positive μ and therefore the parameter space (if one includes these constraints) would be more restricted for the negative μ . Since we do not consider such constraints here we will stick to $\mu > 0$.

The third generation scalars (fig. 5.6 (d)) feel the effect of the Yukawa couplings and thus behave differently. Their RG evolution is governed by the A -terms. As evident from eqs. (B.17–B.19) the large $|A_t|$ parameters act to suppress the third generation squarks. This is most significant for the $m_{\tilde{t}_R}^2$ quark, where X_t enters with a larger coefficient. This effect, along with large mixing in the top squark sector, leads to $m_{\tilde{t}_1}$ being very close to the mass of the neutralino $m_{\tilde{\chi}_1}$. For low $\tan \beta$ the stop quark is the next to lightest supersymmetric particle (NLSP) whereas for large $\tan \beta$ values this role is played by the stau lepton.

The RG evolution of $m_{H_u}^2$ (fig. 5.6 (b)) is controlled to a large extent by $m_{Q_3}^2$ and $|A_t|$ (see eq. (B.15)). Since the gluino M_3 drives the squark $m_{Q_3}^2$, it also controls $m_{H_u}^2$. Both, M_3 and the large $|A_t|$ push $m_{H_u}^2$ to large negative values. This is the well-known mechanisms of radiative EWSB. The tree level condition eq. (3.10) can only be satisfied, if $m_{H_u}^2 \lesssim -(100 \text{ GeV})^2$. Thus, if the gluino is very light, $m_{H_u}^2$ will not reach this value and EWSB is not possible. On the other hand, a too negative $m_{H_u}^2$ would lead to a large μ -term. We can summarize the action of the large A_t parameter as to increase $-m_{H_u}^2$ and to suppress $m_{\tilde{t}_1}^2$.

Now let us consider the low energy spectrum (at the EW scale). Having fixed $\tan \beta$ and sign μ we scan over α and $m_{3/2}$. The obtained spectra are presented

in fig. 5.7. We first analyze the spectrum for $\tan\beta = 5$. The red/■ area is excluded by the presence of tachyons. In the blue/■ region the lightest stop quark \tilde{t}_1 happens to be the LSP. This is, as already discussed, due to the fact that the gluino in the region $4 \leq \alpha \leq 5.5$ is not heavy enough to counterbalance the suppression of the squark mass coming from the large A_t . Then, in the brown/■ part, the lightest Higgs scalar m_h is below the LEP mass bound. This is because the spectrum is relatively light in this region (since $m_{3/2}$ is very small) and thus the radiative correction eq. (3.12) does not give a sizeable contribution. In the green/■ region all constraints under consideration are satisfied.

In the case of $\tan\beta = 30$ the constraints due to the stop LSP and the Higgs mass become less stringent. The stop quark sooner feels the positive contribution from the gluino and thus the wrong LSP realm shrinks. The contribution to the Higgs mass m_h , (cf. eq. (3.11)) increases with $\tan\beta$ and hence the Higgs mass bound can be exceeded in larger portions of the parameter space.

The Higgs mass bound $m_h > 114 \text{ GeV}$ sets a lower bound on the value of the gravitino mass, which is approximately 8 TeV for $\tan\beta = 5$ and 5 TeV for $\tan\beta = 30$. In this “corner” the spectrum is very light (see tab. 5.1). The LSP constraint, paired with the requirement of absence of tachyons sets a lower bound on the α parameter. Viable spectra can be obtained for $\alpha \gtrsim 5$. Curiously, the KKLТ predicted value $\alpha \sim 5$ is consistent with our constraints. Even if we include the complete list of constraints, $\alpha \sim 5$ would be allowed [18]. In the allowed region the LSP is a bino-like neutralino. This is due to the fact that $|\mu| \gg M_1$, whereas $|\mu|$ is large because the large $|A_t|$ increases $-m_{H_u}^2$. We see that much of the phenomenology can be derived from the large A -terms.

In tab. 5.1 three representative points are listed. Point A describes a moderately heavy spectrum. The stop quark is the NLSP. In point B the spectrum is quite heavy. This is obvious, since α and $m_{3/2}$ are large. Also here the stop is the NLSP. Point C lies at the lower bound for $m_{3/2} = 5 \text{ TeV}$. Thus the spectrum is rather light. For large $\tan\beta$, as usual, the stau lepton is the NLSP. We see that by an appropriate choice of α and $m_{3/2}$ it is possible to obtain a spectrum below 1 TeV. The main features of the spectrum can be summarized as:

Mirage Spectra @ $n_i = 0$ & $\Delta\xi_i = 0 \forall i$

- Allowed parameter space (depending on $\tan\beta$) obtainable for $\alpha \gtrsim 5.5$ and $m_{3/2} \gtrsim 5 \text{ GeV}$.
- Phenomenology is derivable from the large magnitude of the A -terms.
- The μ -term is large, because the large A_t drives $-m_{H_u}^2$ to large values.
- The LSP is a bino-like $\tilde{\chi}_1^0$, due to $M_1 \ll |\mu|$
- The NLSP is either a \tilde{t}_1 (low $\tan\beta$) or a $\tilde{\tau}_1$ (large $\tan\beta$).
- For some low α values the gluino is not heavy enough to drive the \tilde{t} heavier than $\tilde{\chi}_1^0$.

	$\Delta\xi = 0$			$\Delta\xi = 10^{-3.5}$		
	A	B	C	A'	B'	C'
$\tan\beta$	5	10	30	5	10	30
α	5.5	7	9	5.5	7	9
$m_{3/2}$ (TeV)	30	50	5	30	50	5
M_1	729	1466	160	732	1469	161
M_2	910	1918	234	911	1919	234
M_3	1649	2582	564	1637	3567	563
m_h	119	127	116	119	127	116
m_A	1816	3754	430	2041	4020	448
m_H	1816	3753	429	2041	4020	447
μ	1370	2918	453	1367	2877	452
$m_{\tilde{\chi}_1^0}$	723	1457	158	727	1463	158
$m_{\tilde{\chi}_1^+}$	929	1957	233	935	1983	233
$m_{\tilde{g}}$	1704	3691	582	1731	3727	586
$m_{\tilde{t}_1}$	830	1870	245	1006	2111	265
$m_{\tilde{t}_2}$	1432	3003	551	1607	3253	561
$m_{\tilde{u}_L}$	1730	3704	578	1923	3993	596
$m_{\tilde{u}_R}$	1665	3608	566	1892	3908	585
$m_{\tilde{b}_1}$	1384	2981	436	1571	3235	453
$m_{\tilde{b}_2}$	1648	3526	519	1876	3829	536
$m_{\tilde{d}_L}$	1705	3706	583	1926	3996	601
$m_{\tilde{d}_R}$	1657	3586	563	1885	3889	583
$m_{\tilde{\tau}_1}$	1072	2239	170	1412	2708	222
$m_{\tilde{\tau}_2}$	1148	2443	311	1469	2880	344
$m_{\tilde{e}_L}$	1151	2473	331	1472	2908	363
$m_{\tilde{e}_R}$	1080	2305	303	1420	2770	339

Tab. 5.1 :: Three points taken out of the parameter space for different $\tan\beta$ and $\mu > 0$. All masses (except for the gravitino) are given in GeV. A, B, C correspond to $\Delta\xi = 0$ whereas A', B', C' additionally have $\Delta\xi = 10^{-3.5}$.

Point A represents a moderately heavy spectrum whereas in B the spectrum is quite heavy. In point C the spectrum is very light due to the light gravitino. This spectrum lies on the border of the Higgs mass bound.

The difference between the “primed” and the “unprimed” spectra is quite small because the contribution from $\Delta\xi = 10^{-3.5}$ to the soft scalar masses at the GUT scale is of order 16%. In cases A and B, μ is quite large.

Fig. 5.8 depicts the masses of some sparticles at the EW scale, depending on α . Let us close this section by looking at the modular weights. These are constants in the region $n_i \in [0, 1]$. As can be seen from eq. (5.20) non-zero modular weights would decrease the modulus mediated part in the GUT scale boundary condition, leading to larger tachyonic regions. Thus we are not going to consider non-zero modular weights, but instead we will try to remove the tachyonic curtain and see what may be behind it.

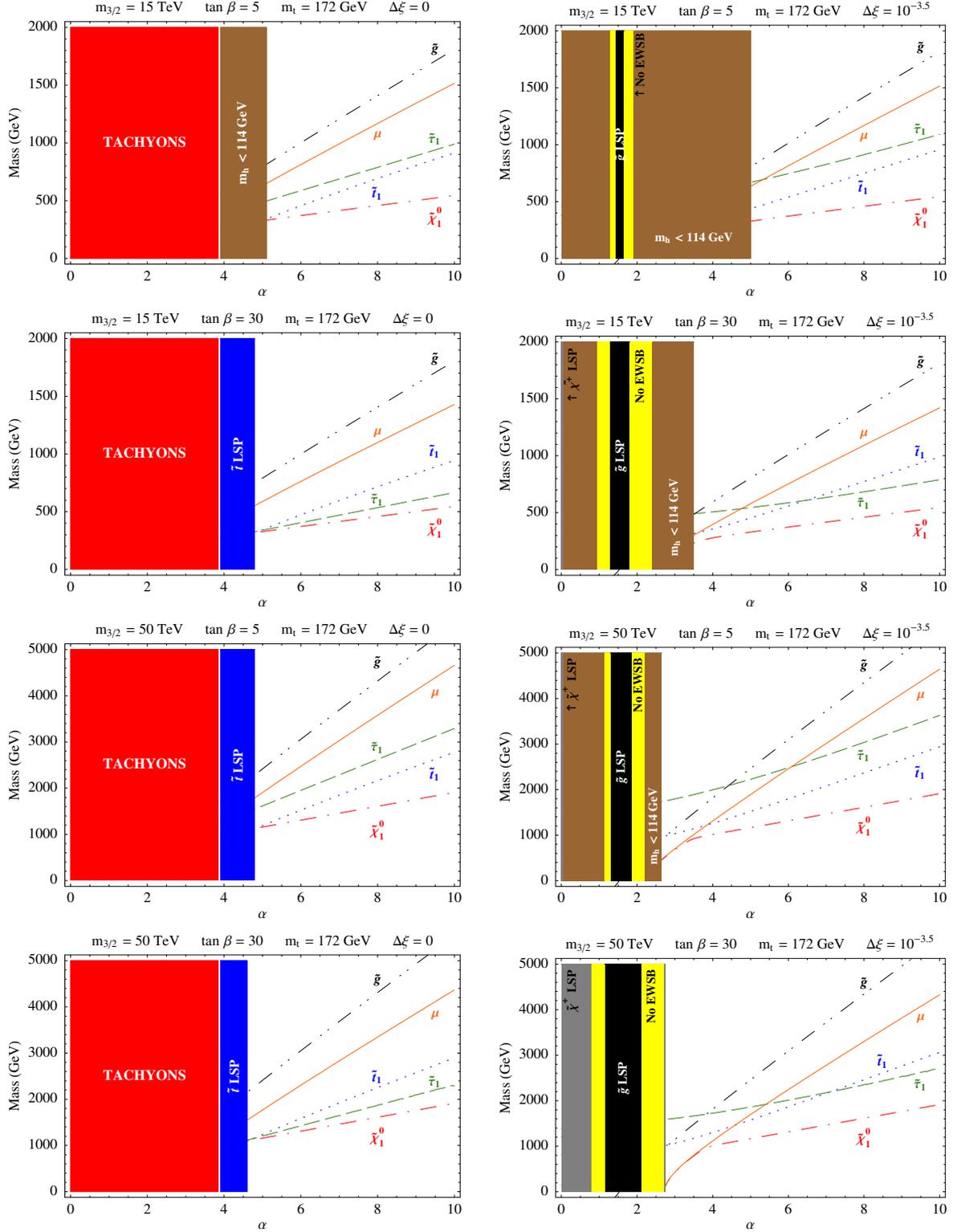


Fig. 5.8 :: Low energy mass spectra versus α for low/large $m_{3/2}$ as well as $\tan\beta$. Figures on the left correspond to $\Delta\xi = 0$ whereas those on the right to $\Delta\xi = 10^{-3.5}$. For non-zero $\Delta\xi$ the spectra are not significantly heavier compared to $\Delta\xi = 0$. For low $\tan\beta$ values the stop quark is very close to the neutralino, whereas for large $\tan\beta$ the stau lepton shows this behavior. Here sign $\mu = +1$.

5.3.2 Case $\Delta\xi \neq 0$ (Extended Mirage Mediation)

We saw that mirage mediation offers a distinct pattern of soft masses. It also ameliorates some issues of pure modulus/anomaly mediation. However, the presence of tachyonic sleptons and squarks—a notorious touch of anomaly mediation, is not removed and indicates that the boundary conditions are ill-defined. The simplest possibility to push the tachyons into the void is to add a positive constant to soft scalar squared masses at the GUT scale. With the ξ_i parameter we do have this possibility, motivated by the set-up in [50]. If $\xi_i < 1/3$ then the last term in eq. (5.20c) pushes the scalar squared masses to positive values. But we have to be careful with ξ_i since the last term in eq. (5.20c) is enhanced by $(16\pi^2)^2$ relative to the other terms. As in the previous section, we impose an universal $\xi_i = \xi \forall i$.

Removing Tachyons

Our strategy will be to slowly increase $\Delta\xi$ at the GUT scale and look at the resulting low energy spectrum i. e. perform scans over $(\alpha, m_{3/2})$. We repeat this procedure until all tachyons at the GUT scale vanish.

The obtained results are shown in fig. 5.9. In frame (a) we see that $\Delta\xi = 10^{-5}$ is too small to cause any change. The spectrum is just a copy of that for $\Delta\xi = 0$. By choosing $\Delta\xi = 10^{-4.5}$ in frame (b) the sleptons, which are tachyonic for $\alpha < 2$, have now positive mass squares and thus the region $\alpha < 2$ is non-tachyonic. Also for $3 \lesssim \alpha \lesssim 4$ the tachyons disappear. The stop LSP extends down to $\alpha \sim 3$. Even though we have relieved one part of the spectrum, we see that we have discovered new problematic regions that were “covered” by the tachyons. In particular, we see regions with chargino and gluino LSP and parts with no EWSB. In frame (c), $\Delta\xi = 10^{-4}$ is large enough that all tachyons do completely disappear. The region with \tilde{t}_1 LSP remains and is now smaller than with $\Delta\xi = 0$. If we proceed to increase $\Delta\xi$, the scalars gain more and more mass and, at a given point, the stop quark cannot be lighter than the neutralino. The problem with the stop LSP is solved for $\Delta\xi = 10^{-3.5}$ (see frame (d)).

Let us try to figure out how the regions with the other wrong LSPs and the no-EWSB region come about. We begin with the chargino LSP. A typical feature of anomaly mediation is that the (lightest) chargino is only slightly heavier than the (lightest) neutralino, $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0} \sim 200 \text{ MeV}$ [6]. This small mass degeneracy can lead in some cases to long lived charginos, as long as anomaly dominates significantly over modulus mediation¹⁴. The chargino LSP disappears for $\alpha > 0.3$.

A somewhat deeper issue is the presence of the gluino LSP which has subtle consequences. In fig. 5.5 (c) we see that the gluino starts off negative at $\alpha = 0$ at the GUT scale and crosses the α -axis at $\alpha \sim 1.5$. Consequently, it is very light around the crossing point. The RG evolution is positive for the gluino, but since

¹⁴It might be also possible that SOFTSUSY is not stable in this region. For $\mu < 0$ the problem with the chargino LSP does not appear (see fig. 5.10).

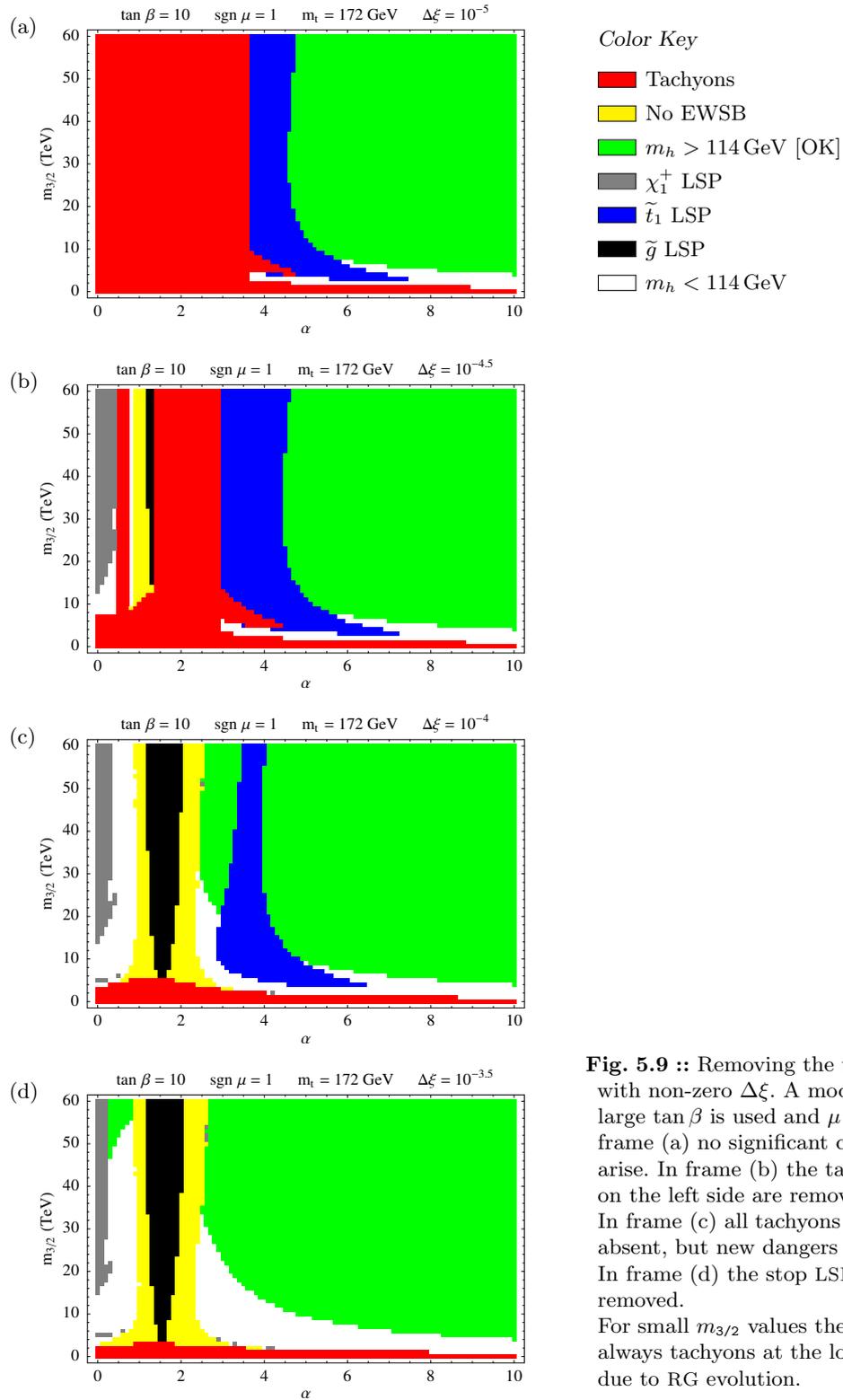
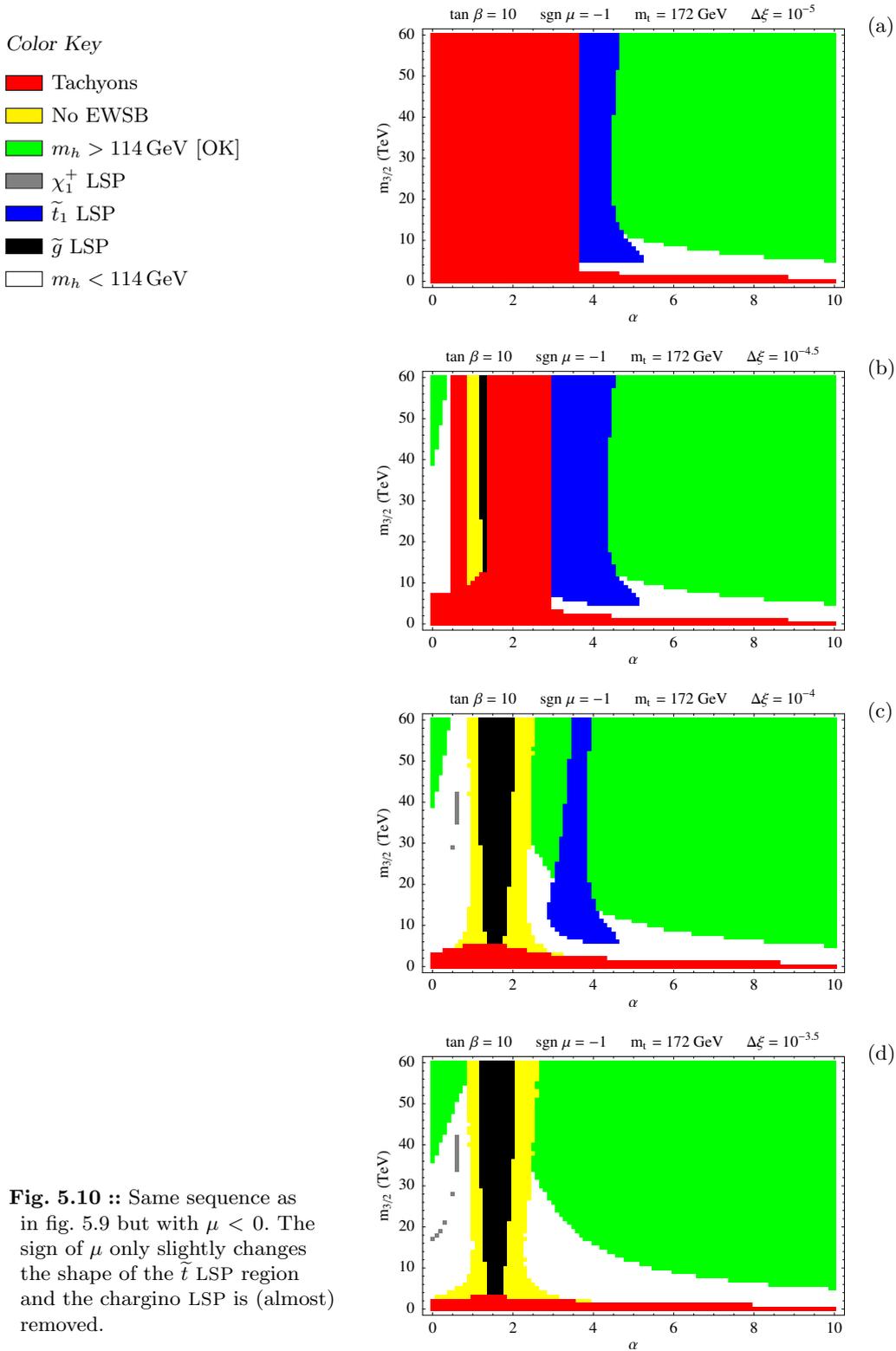


Fig. 5.9 :: Removing the tachyons with non-zero $\Delta\xi$. A moderately large $\tan\beta$ is used and $\mu > 0$. In frame (a) no significant changes arise. In frame (b) the tachyons on the left side are removed, In frame (c) all tachyons are absent, but new dangers appears. In frame (d) the stop LSP is removed. For small $m_{3/2}$ values there are always tachyons at the low scale due to RG evolution.



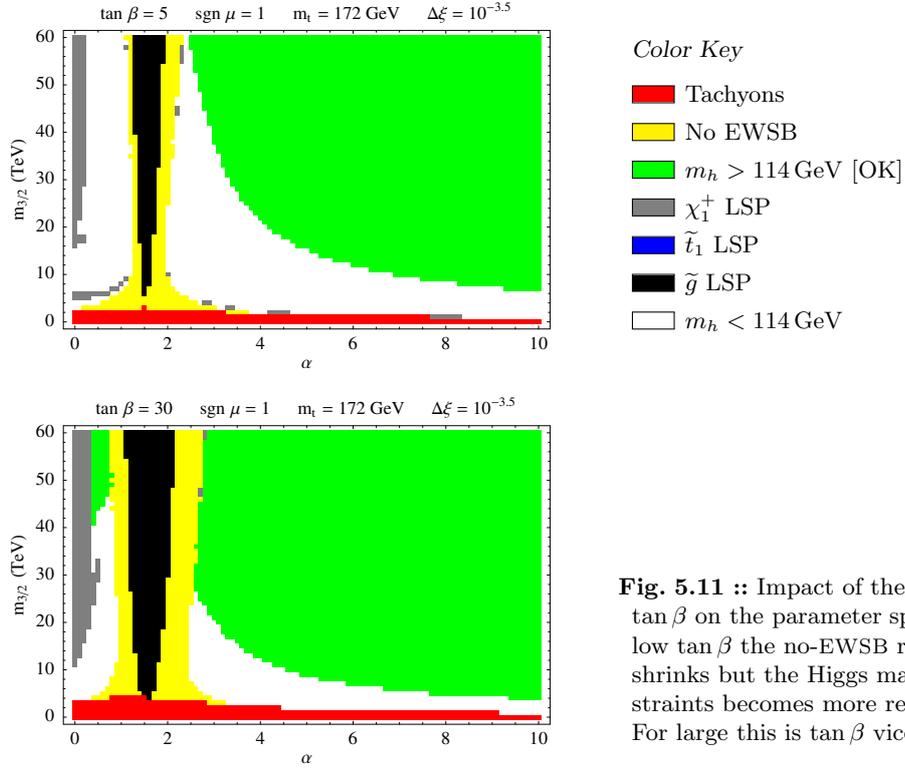


Fig. 5.11 :: Impact of the value of $\tan\beta$ on the parameter space. For low $\tan\beta$ the no-EWSB region shrinks but the Higgs mass constraints becomes more restrictive. For large this is $\tan\beta$ vice versa.

it is very light at the GUT scale, it will also be very light at the EW scale, even lighter than the neutralino. This happens in the region $1 \lesssim \alpha \lesssim 2$. Such a light gluino also affects the EWSB. We have shifted all scalar squares by a constant positive value at the GUT scale. This was necessary to avoid tachyons. On the other hand, in order to have EWSB, $-m_{H_u}^2$ must be sufficiently large in magnitude. If we push $m_{H_u}^2$ to large positive values at the GUT scale, it will be difficult for $-m_{H_u}^2$ to reach the appropriate value at the EW scale. We also know that the gluino mass acts to drive $-m_{H_u}^2$ to large values. But if the gluino is very light, and this is the case around $\alpha \sim 1.5$, $-m_{H_u}^2$ will not reach the required value for small α . This is what happens in the yellow/ region. However, for α close to zero, the gluino provides a sufficiently large $|M_3|$ in order to assure correct EWSB. If we increase $\Delta\xi$ further, all scalars become very heavy and then the no-EWSB region will cover larger portions of the parameter space because $m_{H_u}^2$ is too positive at the GUT scale and its value at the EW scale will be sufficient negative only if we go to very large α .

The Higgs mass constraint remains nearly unchanged. The heavier \tilde{t}_1, \tilde{t}_2 enter the radiative correction eq. (3.12) only logarithmically. One can improve the Higgs mass constraint by choosing larger $\tan\beta$ values. This however leads to larger regions with no EWSB, since the evolution of $-m_{H_u}^2$ favors low $\tan\beta$ values. Fig. 5.11 presents the difference between a low and a large $\tan\beta$ value. Note that the gauginos and the A -terms are not affected by the ξ -uplifting procedure.

Even based on the minimal ansatz presented in this chapter, mirage mediation can lead to interesting pattern of soft masses. However, there are some regions in the parameter space that come into conflict with phenomenology. In this work the region behind the tachyons were analyzed in this simple version of mirage mediation. We have seen that it is possible to remove the tachyons, but the result we obtained poses an open about discussion what might be done in order to get rid of the emerging difficulties. The main challenge is the presence of the gluino LSP. This issue could be remedied if one modifies the gaugino boundary condition eq. (5.20a) for example by extending the gauge kinetic function \mathbb{f}_a .

Finally, tab. 5.1 compares three points at the EW scale before and after the removal of tachyons. The value $\Delta\xi = 10^{-3.5}$ does not make the mass spectrum significantly heavier since its contribution to the soft scalar squared masses at the GUT scale eq. (5.20c) is roughly 16%. Fig. 5.8 compares the masses of the soft parameters at the EW scale before and after tachyons have been removed. We can summarize the removal of tachyons as follows:

Mirage Spectra @ $n_i = 0$ & $\Delta\xi_i \neq 0 \forall i$

- Scalars can be non-tachyonic due to the universal shift $\Delta\xi \neq 0$. Increase $\Delta\xi$ until all scalar are non-tachyonic.
- Soft gaugino masses and the A-terms are unaffected.
- The very light gluino around $\alpha \sim 1.5$ prohibits EWSB.
- Access into the lower α region enabled but further modifications are needed in order to avoid a gluino LSP and to guarantee correct EWSB.

5.4 Heterotic Mirage Mediation

In this section, we consider a heterotic string motivated mirage mediation scenario. The procedure in deriving the soft breaking terms is similar to that in the type IIB case and is carried out in the framework of spontaneously broken SUGRA. We start with the complex structure moduli Z_α , one single Kähler modulus T and a dilaton S . The complex structure moduli and the Kähler modulus can be stabilized by fluxes. They become very heavy and can be integrated out [53]. The remaining dilaton can be stabilized by non-perturbative mechanisms such as gaugino condensation. The effective low energy theory, in analogy to section 5.2, is described by

$$K = -\log(S + \bar{S}) + C\bar{C} + Q^i \bar{Q}^i a_i [1 + \xi_i C\bar{C}], \quad (5.24)$$

$$\mathbb{f}_a = S, \quad (5.25)$$

where we call a_i *trivial weights*, Q^i are the observable fields and C is a hidden sector matter field. We consider gauge fields on $D3$ branes. In contrast to the

type IIB case, the logarithm comes with a smaller numerical coefficient. Recall that the last term in eq. (5.24) is suppressed by M_{P}^{-2} .

To derive the soft breaking terms we first determine the scalar potential. Inserting eqs. (5.24) and (5.25) in eq. (4.2) gives

$$V = \frac{e^{C\bar{C}}}{S + \bar{S}} \left[|W_S(S + \bar{S}) - W|^2 + |W_C + W\bar{C}|^2 - 3|W|^2 \right], \quad (5.26)$$

with subscripts on W denoting derivatives with respect to S and C . This result looks similar to eq. (5.15) but with different numerical coefficients. In analogy to [50], without loss of generality, one can stabilize S and C at

$$V(S = S_0, C = 0) \simeq 0, \quad (5.27)$$

tuning the CC to zero. Using eq. (5.27) one can easily derive the relation among the SUSY breaking terms from eq. (5.26)

$$\frac{|F^S|^2}{(S_0 + \bar{S}_0)^2} + |F^C|^2 = 3m_{3/2}^2. \quad (5.28)$$

The soft breaking terms are obtained by using eqs. (4.5) and (4.12). For the mixed modulus-anomaly part in the scalar squared masses, one needs the modular dependence of the gauge couplings. This can be found in appendix B.2. Using all these ingredients the soft breaking terms at the GUT scale are

$$M_a = \frac{F^S}{S_0 + \bar{S}_0} + b_a g_a^2 \frac{F^\phi}{16\pi^2}, \quad (5.29a)$$

$$A_{ijk} = -\frac{F^S}{S_0 + \bar{S}_0} + (\gamma_i + \gamma_j + \gamma_k) \frac{F^\phi}{16\pi^2}, \quad (5.29b)$$

$$m_i^2 = \Delta_i + \xi_i \frac{|F^S|^2}{(S_0 + \bar{S}_0)^2} - \dot{\gamma}_i \frac{|F^\phi|^2}{(16\pi^2)^2} + \frac{2F^\phi F^S \partial_S \gamma_i}{16\pi}, \quad (5.29c)$$

Note that the trivial weights do not appear in the soft terms boundary condition and, in contrast to the type IIB case, we do not have the DOF that were given by the modular weights. Mirage mediation appears if the contributions from modulus and anomaly mediation become comparable, i. e.

$$\frac{F^S}{S_0 + \bar{S}_0} \sim \frac{F^\phi}{4\pi^2} \sim \frac{m_{3/2}}{4\pi^2}.$$

Also here we impose the parameterization

$$\frac{F^S}{S_0 + \bar{S}_0} \equiv \alpha M_s, \quad M_s \equiv \frac{m_{3/2}}{16\pi^2}, \quad (5.30)$$

in order to study the phenomenology depending on the ratio of modulus to anomaly mediation. If $\alpha \gg 1$, modulus mediation dominates, and the limit $\alpha = 0$

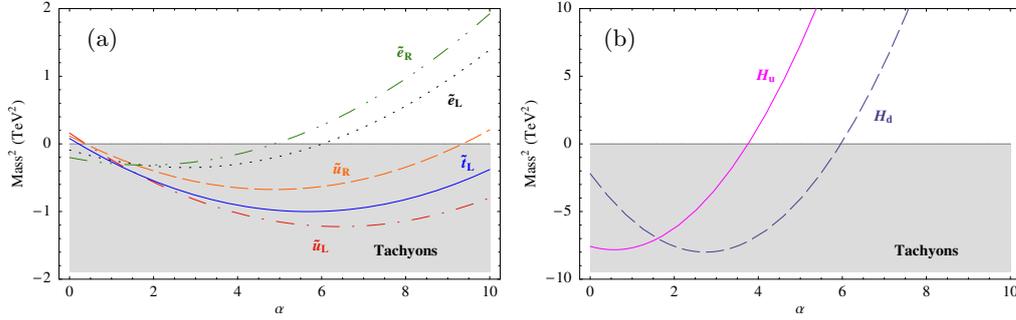


Fig. 5.12 :: Miscellaneous soft breaking parameters at the GUT scale plotted versus α in the heterotic case. The tachyonic region is enlarged compared to fig. 5.5.

corresponds to pure anomaly mediation. Using this parameterization, the GUT scale boundary condition eq. (5.29) takes the form

$$M_a = M_s \left[\alpha + b_a g_a^2 \right], \quad (5.31a)$$

$$A_{ijk} = M_s \left[-\alpha + (\gamma_i + \gamma_j + \gamma_k) \right], \quad (5.31b)$$

$$m_i^2 = M_s^2 \left[\xi_i \alpha^2 - \dot{\gamma}_i + 2\alpha (S_0 + \bar{S}_0) \partial_S \gamma_i \right] + (1 - 3\xi_i) m_{3/2}^2, \quad (5.31c)$$

This result is similar to eq. (5.20), but the numerical coefficients in the A -terms and in the scalar squared masses differ. In the case of *ordinary* mirage mediation, $\xi_i = 1/3$, the part from modulus mediation in the scalar squared masses comes with a smaller numerical coefficient compared to the type IIB situation eqs. (5.20). This results in larger tachyonic regions since here the balance is pushed towards anomaly mediation. Also the A -terms are now reduced. Only the gaugino masses remain unchanged. In fig. 5.12 we look at the soft scalar squared masses at the GUT scale. The whole region of interest, i. e. $\alpha \sim \mathcal{O}(1)$, is filled by the tachyons. Here again the first and second generation squarks are the problematic pieces. Thus, unlike the type IIB case, in the heterotic case we need right from the start $\Delta\xi_i \neq 0$ in order to survive in this scheme.

5.4.1 Phenomenological Aspects

To study the phenomenology in the region $0 \leq \alpha \leq 10$ we have to remove the tachyons. We will proceed in the same manner as in the previous section. First we impose a universal $\Delta\xi_i = \Delta\xi$ for all scalars and then we slowly increase it until all tachyons are gone. Fig. 5.13 summarized the procedure.

In frame (a) of fig. 5.13 $\Delta\xi = 10^{-5}$ is too small to cause any change in the whole region $0 \leq \alpha \leq 10$. In frame (b) $\Delta\xi$ suffices to remove the tachyons only in the region $0 \lesssim \alpha \lesssim 1.5$. Then in frame (c) with $\Delta\xi = 10^{-4}$ we still have tachyons. Recall that in the type IIB case, $\Delta\xi = 10^{-4}$ was sufficient to remove all tachyons. Here the problem with tachyons is solved for $\Delta\xi = 10^{-3.5}$. Surprisingly this is the

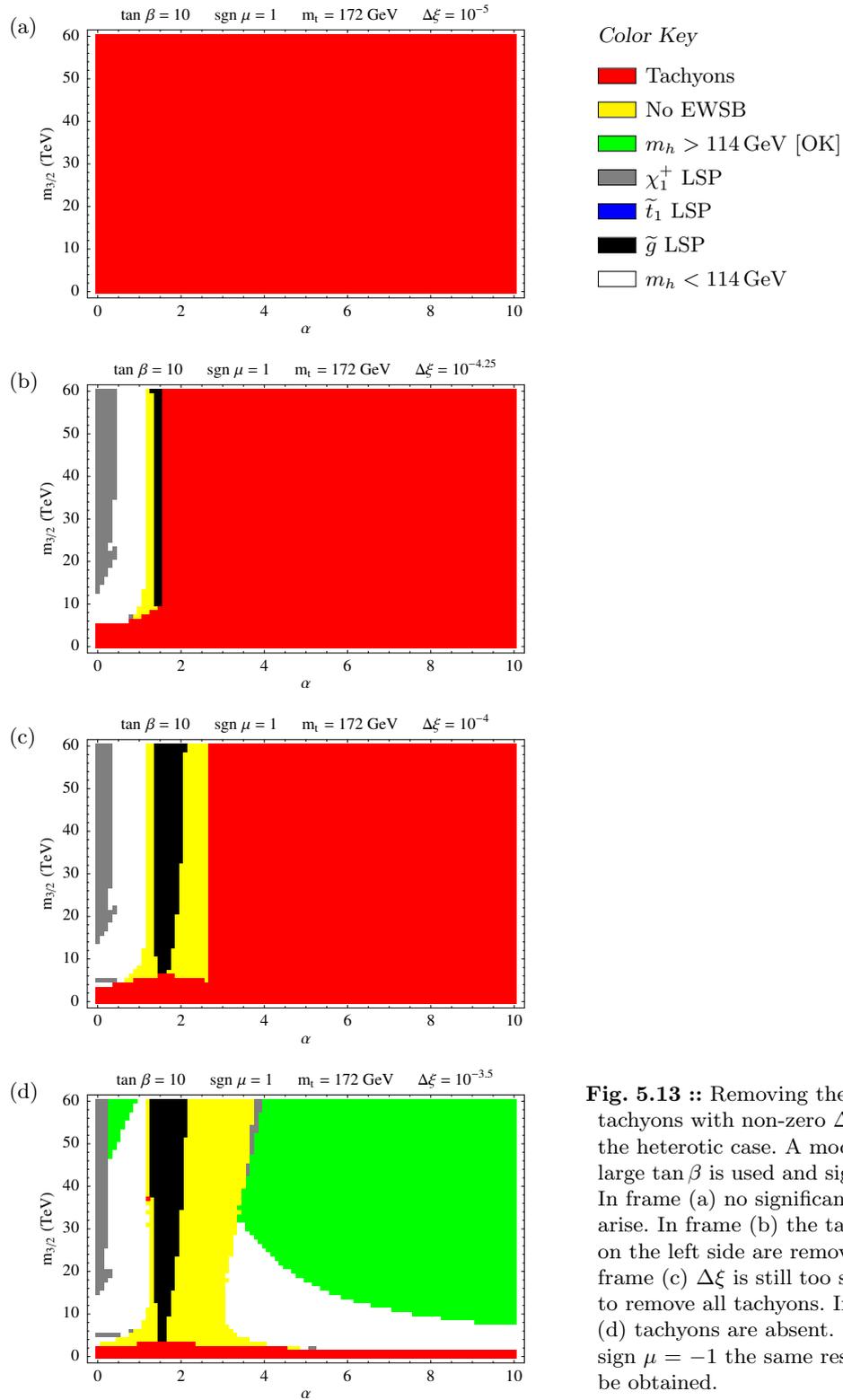


Fig. 5.13 :: Removing the tachyons with non-zero $\Delta\xi$ in the heterotic case. A moderately large $\tan \beta$ is used and $\text{sgn } \mu = 1$. In frame (a) no significant changes arise. In frame (b) the tachyons on the left side are removed. In frame (c) $\Delta\xi$ is still too small to remove all tachyons. In frame (d) tachyons are absent. For $\text{sgn } \mu = -1$ the same result can be obtained.

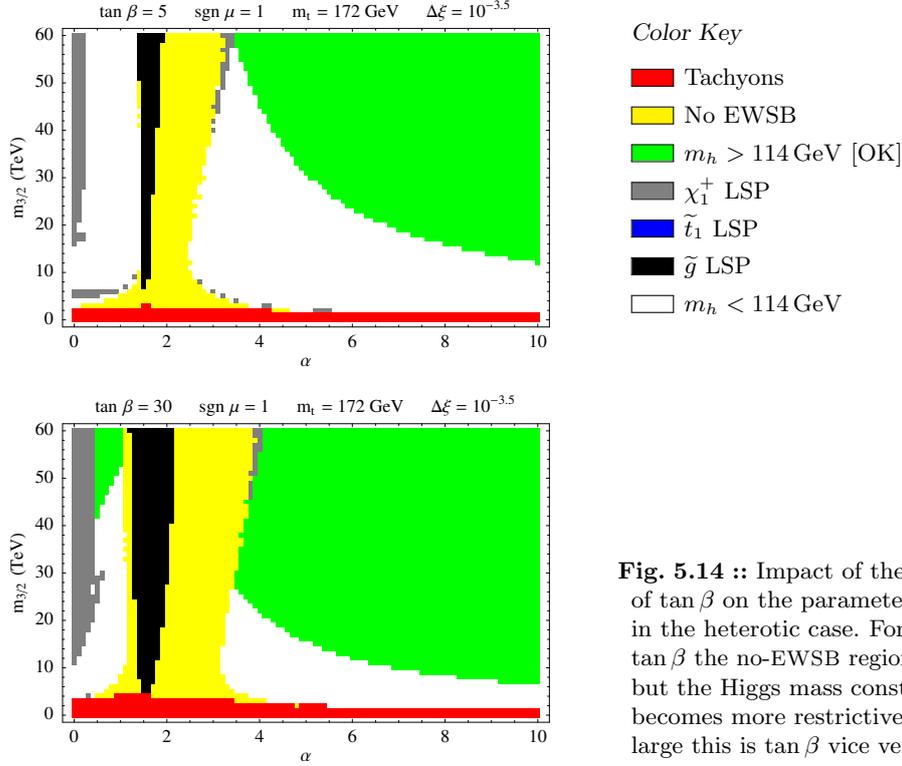


Fig. 5.14 :: Impact of the value of $\tan \beta$ on the parameter space in the heterotic case. For low $\tan \beta$ the no-EWSB region shrinks but the Higgs mass constraints become more restrictive. For large $\tan \beta$ this is vice versa.

same value as in the type IIB situation, but in the type IIB case we have chosen $\Delta\xi = 10^{-3.5}$ in order to make the spectrum heavy enough so that the stop LSP disappear. Here, we need such a “high” value because we have “more” tachyons than in the type IIB case.

The gluino LSP and the realm of no EWSB can be explained in the same way as in section 5.3.2. The very light gluino around $\alpha \sim 1.5$ at the GUT scale results in a very light gluino at the EW scale and consequently cannot provide the necessary contribution for $-m_{H_u}^2$. This means that EW symmetry is not properly broken around $\alpha \sim 1.5$. In addition, the A -terms are reduced, and thus they provide a smaller RG contribution to $-m_{H_u}^2$ (see eq. (B.15)).

To summarize, in the heterotic set-up the situation is a bit difficult due to the reduced modulus contribution in the boundary condition eq. (5.31). Nevertheless, the missing modulus contribution in the scalar squared masses can be reintroduced through a non-zero $\Delta\xi$. Then, like in the type IIB case, a modification of the gaugino terms could solve the gluino LSP issue. Note also that the situation with only the dilaton in the effective theory is a simplification.

5.5 Conclusion

In this work a new form of mediating supersymmetry breakdown was considered. It is a mixed modulus-anomaly scenario, where the contributions to the soft parameters from gravity/anomaly mediation are of comparable size. The encouragement to study such a scenario comes directly from string theory. In string theory the stabilization of moduli can, under rather general circumstances, lead to such a mixture of modulus and anomaly mediation.

An interesting feature of this scenario is the occurrence of a mirage unification. That is, even though the gaugino and the scalar masses are non-universal at the GUT scale, they unify (due to the RG running) at some intermediate scale which depends on the ratio of modulus to anomaly mediation. Mirage mediation provides a distinct and interesting pattern of soft masses. A robust feature of this scheme is a hierarchy among the soft, the gravitino and the moduli masses $m_T \gg m_{3/2} \gg m_{\text{SOFT}}$, which is advantageous from the cosmological perspective. Special emphasis is given on the parameter space. The low energy phenomenology can be described by just two parameters $(\alpha, m_{3/2})$.

The contribution from anomaly mediation in this scheme leads to tachyonic squarks and sleptons, making the region $0 \leq \alpha \leq 4$ in the parameter space inaccessible. The aim of this work was to find out what is behind the tachyonic wall. To do so, an alternative set-up (carried out in the framework of spontaneously broken SUGRA [50]) was introduced. This set-up provides mirage mediation, as it is known from the KKLT example, but with additional new pattern of soft masses. In particular, the soft scalar squared masses receive a positive contribution which guarantees non-negative mass squares.

After the tachyons have been removed, there are still zones in the parameter space that are phenomenologically excluded. This is due to the fact that the gluino is very light around $\alpha \sim 1.5$ and consequently prevents correct EWSB. We should however not forget that this discussion is based on a simple version of mirage mediation, which nevertheless provides interesting physics. A proper modification (e. g. rearrangement of the gauge kinetic function f_a) may change rigorously the situation. Thus, the discovered issues in the region $0 \leq \alpha \leq 4$ open a new discussion and should be considered as a challenge in finding an interesting mechanism of supersymmetry breaking.

Appendix A

Notations and Conventions

A.1 Two-Component Spinors

Two-component notation is advantageous for theories containing chiral fermions. Our notations and conventions follow basically those in [7]. Here the most important ones are summarized.

- 4-component indices are presented by greek letters $\mu, \nu, \rho \dots$.
- 4-derivatives are defined by $\partial_\mu = \frac{\partial}{\partial x^\mu}$ and $\partial^\mu = \frac{\partial}{\partial x_\mu}$.
- We always sum over contracted indices, like ${}^\mu{}_\mu$.
- For γ matrices the following representation is used:

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

with σ_μ being the Pauli matrices

$$\sigma_0 = \bar{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = -\bar{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_2 = -\bar{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = -\bar{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

- In this basis, a 4-component Dirac spinor is shortly written in terms of two 2-component, complex, anticommuting Weyl spinors ξ_α and $\chi^{\dot{\alpha}}$ with two distinct types of indices $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$

$$\psi_D = \begin{pmatrix} \xi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix}.$$

The undotted (dotted) indices are used for the first (last) two components of a Dirac spinor. The heights of the indices are important. Spinors with undotted indices transform in the $(\frac{1}{2}, 0)$ representation of the Lorentz group whereas those with dotted indices in the $(0, \frac{1}{2})$ representation.

- Spinor indices are raised and lowered using the antisymmetric symbol $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$, $\epsilon_{11} = \epsilon_{22} = \epsilon^{11} = \epsilon^{22} = 0$, according to $\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta$, $\xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta$, $\chi_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\chi^{\dot{\beta}}$, $\chi^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\chi_{\dot{\beta}}$, with $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon_{\gamma\beta}\epsilon^{\beta\alpha} = \delta_\alpha^\gamma$ and $\epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} = \epsilon_{\dot{\gamma}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\alpha}} = \delta_{\dot{\alpha}}^{\dot{\gamma}}$.
By convention, undotted indices are contracted like $^\alpha_\alpha$ and dotted like $_{\dot{\alpha}}^{\dot{\alpha}}$. Note that $\xi^2 = \xi^\alpha\xi_\alpha = \xi^\alpha\epsilon_{\alpha\beta}\xi^\beta = -\xi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \xi^\beta\epsilon_{\beta\alpha}\xi^\alpha = \xi^2 \neq 0$.
- The field ξ is called a *left-handed Weyl spinor* and χ^\dagger is a *right-handed Weyl spinor*. The hermitean conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor $(\xi_\alpha)^\dagger = (\xi^\dagger)_{\dot{\alpha}}$ and vice versa $(\xi^\dagger)^\dagger = \xi^\alpha$.
- The bar on a 4-component spinor is defined by $\bar{\psi} = \psi^\dagger\gamma_0 = (\chi^\alpha, \xi_{\dot{\alpha}}^\dagger)$.
- The bar on a 2-component object is defined by $\bar{\xi}^\alpha = \xi^{\dagger\dot{\alpha}}$.
- The bar on a scalar field denotes its complex conjugate $\bar{\varphi} = \varphi^*$.
- The bars on the fields represent a left-handed CP conjugate of right-handed fields. Thus for particles and sparticles one has

$$\begin{pmatrix} e \\ \bar{e} \end{pmatrix} = \begin{pmatrix} e_L \\ e_R^\dagger \end{pmatrix}, \quad \begin{pmatrix} \tilde{e} \\ \bar{\tilde{e}} \end{pmatrix} = \begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R^* \end{pmatrix}.$$

- Some useful relations are:

$$\begin{aligned} \xi\sigma^\mu\bar{\chi} &= (\chi\sigma^\mu\bar{\xi})^\dagger, & \bar{\chi}\bar{\sigma}^\mu\xi &= (\bar{\xi}\bar{\sigma}^\mu\chi)^\dagger, \\ \sigma_{\alpha\dot{\beta}}^\mu &= \epsilon_{\alpha\gamma}\epsilon_{\dot{\beta}\dot{\delta}}\bar{\sigma}^{\mu\dot{\delta}\gamma}, & \bar{\sigma}^{\mu\dot{\alpha}\beta} &= \epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon^{\beta\delta}\bar{\sigma}_{\dot{\delta}\gamma}^\mu, \\ (\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu)_\alpha^\beta &= 2\eta^{\mu\nu}\delta_\alpha^\beta, & (\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu)_{\dot{\beta}}^{\dot{\alpha}} &= 2\eta^{\mu\nu}\delta_{\dot{\beta}}^{\dot{\alpha}}, \\ \xi\sigma^{\mu\nu}\chi &= -\chi\sigma^{\mu\nu}\xi, & \bar{\xi}\bar{\sigma}^{\mu\nu}\bar{\chi} &= -\bar{\chi}\bar{\sigma}^{\mu\nu}\bar{\xi}, \end{aligned} \quad (\text{A.1})$$

where $\eta^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ and $\text{Tr}(\sigma^\mu\bar{\sigma}^\nu) = 2\eta^{\mu\nu}$. Furthermore, $\sigma^{\mu\nu} \equiv \frac{i}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$, $\bar{\sigma}^{\mu\nu} \equiv \frac{i}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)$ and $\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu = 2\eta^{\mu\nu}$.

A.2 Superfield Language

In supersymmetric theories, it is appropriate to use the elegant superfield formalism. Superfields are functions that live in the so-called *superspace* which is spanned by the four spacetime coordinates and by anticommuting Grassmann variables θ_α , $\bar{\theta}^{\dot{\alpha}}$. A finite supersymmetry transformation can be represented by

$$S(x, \theta, \bar{\theta}) = e^{i(\theta Q + \bar{Q}\bar{\theta} - x_\mu P^\mu)},$$

where P^μ is the 4-momentum operator. Applying infinitesimal transformations to a superfield Φ , one can find the following representations of the supercharges

$$Q_\alpha = -i\frac{\partial}{\partial\theta^\alpha} + \sigma_{\alpha\dot{\beta}}^\mu\bar{\theta}^{\dot{\beta}}\partial_\mu, \quad (\text{A.2})$$

$$\bar{Q}_{\dot{\alpha}} = i\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \theta^\beta\sigma_{\beta\dot{\alpha}}^\mu\partial_\mu. \quad (\text{A.3})$$

The corresponding covariant derivatives, which commute with Q, \bar{Q} and generate translations in the superspace, are

$$\mathcal{D}_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu, \quad (\text{A.4})$$

$$\bar{\mathcal{D}}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu. \quad (\text{A.5})$$

It is usual to define the supersymmetric transformations in the left- and right-chiral representations

$$S_L = e^{i(\theta Q - x_\mu P^\mu)} e^{i\bar{Q}\bar{\theta}}, \quad (\text{A.6})$$

$$S_R = e^{i(\bar{Q}\bar{\theta} - x_\mu P^\mu)} e^{i\theta Q}, \quad (\text{A.7})$$

which simplify the shape of the supercharges and the covariant derivatives. The covariant derivatives in these representations look like

$$\begin{aligned} \mathcal{D}_{L\alpha} &= \frac{\partial}{\partial \theta^\alpha} - 2i\sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu, & \mathcal{D}_{R\alpha} &= \frac{\partial}{\partial \theta^\alpha}, \\ \bar{\mathcal{D}}_{L\dot{\alpha}} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}, & \bar{\mathcal{D}}_{R\dot{\alpha}} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + 2i\theta^\beta \sigma_{\beta\dot{\alpha}}^\mu \partial_\mu. \end{aligned} \quad (\text{A.8})$$

There is a simple relation between the three representations we have introduced:

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \Phi_L(x_\mu + i\theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}) \\ &= \Phi_R(x_\mu - i\theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}). \end{aligned} \quad (\text{A.9})$$

The superfields Φ_L (Φ_R) are called left(right)-handed superfields if they satisfy

$$\bar{\mathcal{D}}_L \Phi_L \equiv 0, \quad \mathcal{D}_R \Phi_R \equiv 0.$$

According to eq. (A.8), Φ_L depends only on (x, θ) whereas Φ_R depends on $(x, \bar{\theta})$. The quantity that remains invariant under (non-)abelian supergauge transformations is the so-called (left-chiral) spinorial field strength superfield

$$\begin{aligned} \Xi_\alpha &= \frac{1}{4} \bar{\mathcal{D}} \bar{\mathcal{D}} e^{-\Gamma} \mathcal{D}_\alpha e^\Gamma, \\ \bar{\Xi}^{\dot{\alpha}} &= \frac{1}{4} \mathcal{D} \mathcal{D} e^{\Gamma} \bar{\mathcal{D}}^{\dot{\alpha}} e^{-\Gamma}, \end{aligned}$$

where Γ denotes a vector superfield.

A.3 Dimensions

$[W] = 3$	$[\Phi] = 1$	$[\varphi] = 1$
$[K] = 2$	$[\Gamma] = 0$	$[\psi] = 3/2$
$[\mathbb{f}_{ab}] = 0$	$[\mathcal{L}] = 4$	$[\theta] = -1/2$
$[G] = 0$	$[\mathcal{Y}] = 0$	$[F] = 2$
$[V] = 4$	$[\mathcal{M}] = 2$	$[D] = 2$

Appendix B

Renormalization Group

B.1 RGE for the MSSM

The RG evolution is an essential tool in order to evolve the boundary conditions on the soft parameters from the high input scale down to a scale where low energy observation can take place. It is noted, that the framework of the MSSM requires supersymmetry respecting regularization and renormalization schemes. The suitable regularization scheme is the so-called *regularization by dimensional reduction* (DRED). For the renormalization one uses the *modified minimal subtraction* ($\overline{\text{DR}}$) [54]. The non-renormalization theorem [30, 31] influences the RG equations. In particular, it implies that all divergent contributions can always be absorbed into the wave function renormalization.

Although RG equations up to 3-loop order are available [55, 56], it is sufficient to consider the 1-loop contribution. We will use the approximation that only the third generation Yukawa couplings are significant and make use of the universality constraint eq. (3.6-2). As usual, the normalization

$$g_1 = \sqrt{5/3}g', \quad g_2 = g, \quad g_3 = g_s,$$

is used in order to interpret the generators of the SM gauge group as generators of a larger simple gauge group such as $SU(5)$ or $SO(10)$. Recall that $\varrho = Q/Q_0$, with Q being the RG scale and Q_0 the input scale.

Gauge couplings

$$\frac{dg_a}{d \log \varrho} = \frac{1}{16\pi^2} b_a g_a^3 \quad \text{with } b_a = (33/5, 1, -3) \quad (\text{B.1})$$

Yukawa Couplings

$$\frac{dy_t}{d \log \varrho} = \frac{y_t}{16\pi^2} \left[6 |y_t|^2 + |y_b|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right] \quad (\text{B.2})$$

$$\frac{dy_b}{d \log \varrho} = \frac{y_b}{16\pi^2} \left[6 |y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right] \quad (\text{B.3})$$

$$\frac{dy_\tau}{d \log \varrho} = \frac{y_\tau}{16\pi^2} \left[4 |y_\tau|^2 + 3 |y_b|^2 - 3g_2^2 - \frac{3}{5} g_1^2 \right] \quad (\text{B.4})$$

μ Term

$$\frac{d\mu}{d\log\varrho} = \frac{\mu}{16\pi^2} \left[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right] \quad (\text{B.5})$$

Gaugino Mass Parameters

$$\frac{dM_a}{d\log\varrho} = \frac{1}{8\pi^2} b_a g_a^2 M_a \quad (\text{B.6})$$

A Terms ($A_t = A_{Q_3 H_u u_R}$, $A_b = A_{Q_3 H_d d_R}$, $A_\tau = A_{L_3 H_d e_R}$)

$$\begin{aligned} \frac{dA_t}{d\log\varrho} &= A_t \left[18|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right] + 2A_b y_b^* y_t \\ &\quad + y_t \left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1 \right] \end{aligned} \quad (\text{B.7})$$

$$\begin{aligned} \frac{dA_b}{d\log\varrho} &= A_b \left[18|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right] \\ &\quad + 2A_t y_t^* y_b + 2A_\tau y_\tau^* y_b \\ &\quad + y_b \left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 \right] \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} \frac{dA_\tau}{d\log\varrho} &= A_\tau \left[12|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2 \right] + 6A_b y_b^* y_\tau \\ &\quad + y_\tau \left[6g_2^2 M_2 + \frac{18}{15}g_1^2 M_1 \right] \end{aligned} \quad (\text{B.9})$$

B Term

$$\begin{aligned} \frac{dB}{d\log\varrho} &= B \left[3|y_\tau|^2 + 3|y_b|^2 + |y_t|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right] \\ &\quad + \mu \left[6A_t y_t^* + 6A_b y_b^* + 2A_\tau y_\tau^* + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1 \right] \end{aligned} \quad (\text{B.10})$$

The soft scalar squared masses can be written in a more recognizable form using the following abbreviations

$$X_t = 2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + 2|A_t|^2, \quad (\text{B.11})$$

$$X_b = 2|y_b|^2 (m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) + 2|A_b|^2, \quad (\text{B.12})$$

$$X_\tau = 2|y_\tau|^2 (m_{H_d}^2 + m_{L_3}^2 + m_{e_3}^2) + 2|A_\tau|^2, \quad (\text{B.13})$$

as well as

$$S = \frac{1}{2} \sum_i Y_i m_i^2, \quad (\text{B.14})$$

where the sum runs over all scalars of the MSSM with hypercharge Y_i .

Higgs Mass Squared Parameters

$$\frac{dm_{H_u}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 + \frac{3}{5}g_1^2 S \right] \quad (\text{B.15})$$

$$\frac{dm_{H_d}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[3X_b + X_\tau - \frac{6}{5}g_1^2 |M_1|^2 + \frac{3}{5}g_1^2 S \right] \quad (\text{B.16})$$

Squark Mass Squared Parameters

$$\frac{dm_{Q_3}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[X_t + X_b - \frac{32}{3}g_3^2 |M_3|^2 - 6g_2^2 |M_2|^2 - \frac{2}{15}g_1^2 |M_1|^2 + \frac{1}{5}g_1^2 S \right] \quad (\text{B.17})$$

$$\frac{dm_{u_3}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[2X_t - \frac{32}{3}g_3^2 |M_3|^2 - \frac{32}{15}g_1^2 |M_1|^2 - \frac{4}{5}g_1^2 S \right] \quad (\text{B.18})$$

$$\frac{dm_{d_3}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[2X_b - \frac{32}{3}g_3^2 |M_3|^2 - \frac{8}{15}g_1^2 |M_1|^2 - \frac{2}{5}g_1^2 S \right] \quad (\text{B.19})$$

Sleptons Mass Squared Parameters

$$\frac{dm_{L_3}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5}g_1^2 |M_1|^2 - \frac{3}{5}g_1^2 S \right] \quad (\text{B.20})$$

$$\frac{dm_{e_3}^2}{d \log \varrho} = \frac{1}{16\pi^2} \left[2X_\tau - \frac{24}{5}g_1^2 |M_1|^2 - \frac{6}{5}g_1^2 S \right] \quad (\text{B.21})$$

The main properties of the RG evolution can be summarized as follows [6, 7]:

- The β -functions for each supersymmetric parameter are proportional to the parameter itself (as a result of the non-renormalization theorem).
- The β -functions of the soft breaking parameters are not proportional to the parameters itself (no protection by the non-renormalization theorem). Being zero at the input scale, RG will make them non-zero at a lower scale.
- If the constraint eq. (3.6-1) is applied at the input scale, then the sfermion masses will remain almost diagonal at any other RG scale. Unlike the first and second generations, the third generation sfermions feel the effect of the larger Yukawa couplings and thus will be renormalized differently.
- The quantities $X_{t,b,\tau}$ are generally positive. Thus, started from the input scale, they will decrease the Higgs masses. If y_t is the largest Yukawa coupling then X_t will dominate and $m_{H_u}^2$ will be driven to negative values near the EW scale; this is a crucial point in discussing EWSB.
- The gluino is the only gaugino that receives positive RG contribution.
- The gluino provides a positive contribution to the third generation squarks.

B.2 Anomalous Dimension

The anomalous dimension describes the scale dependence of the wavefunction renormalization Z_i . It is defined by

$$\frac{1}{16\pi^2} \gamma_i = \frac{\partial \log Z_i}{\partial \log \varrho}. \quad (\text{B.22})$$

Quadratic Casimirs

The quadratic Casimirs $\mathcal{C}^a(\Phi_i) = \mathcal{C}_i^a$ are group theory invariants for the superfields Φ_i defined in terms of Lie algebra generators T^a by $(T^a T^a)_i^j = \mathcal{C}_i^a \delta_i^j$. For the MSSM superfields one has [6]

Superfields	\mathcal{C}^3	\mathcal{C}^2	\mathcal{C}^1
Q_p	4/3	3/4	1/60
\bar{u}_p	4/3	0	4/15
\bar{d}_p	4/3	0	1/15
L_p	0	3/4	3/20
\bar{e}_p	0	0	3/5
H_u	0	3/4	3/20
H_d	0	3/4	3/20

Tab. B.1 :: Quadratic Casimirs for the MSSM fields.

Anomalous Dimension for MSSM Fields

The 1-loop anomalous dimensions for the MSSM fields are easily obtained via [6]

$$\gamma_i = 2 \sum_a g_a^2 \mathcal{C}_i^a - \sum_{y_i} |y_i|^2. \quad (\text{B.23})$$

Thus:

$$\gamma_{Q_p} = \frac{8}{3} g_3^2 + \frac{3}{2} g_2^2 + \frac{1}{30} g_1^2 - (y_t^2 + y_b^2) \delta_{3p}, \quad (\text{B.24})$$

$$\gamma_{u_p} = \frac{8}{3} g_3^2 + \frac{8}{15} g_1^2 - 2y_t^2 \delta_{3p}, \quad (\text{B.25})$$

$$\gamma_{d_p} = \frac{8}{3} g_3^2 + \frac{2}{15} g_1^2 - 2y_b^2 \delta_{3p}, \quad (\text{B.26})$$

$$\gamma_{L_p} = \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 - y_\tau^2 \delta_{3p}, \quad (\text{B.27})$$

$$\gamma_{e_p} = \frac{6}{5} g_1^2 - 2y_\tau^2 \delta_{3p}, \quad (\text{B.28})$$

$$\gamma_{H_u} = \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 - 3y_t^2, \quad (\text{B.29})$$

$$\gamma_{H_d} = \frac{3}{2} g_2^2 + \frac{3}{10} g_1^2 - 3y_b^2 - y_\tau^2. \quad (\text{B.30})$$

Running of the Anomalous Dimension

For the running of the anomalous dimension one simply has

$$\frac{1}{16\pi^2} \dot{\gamma}_i = \frac{\partial \gamma_i}{\partial \log \varrho}. \quad (\text{B.31})$$

Inserting (B.23) in (B.31) and using (B.1-B.4) one easily obtains

$$\dot{\gamma}_i = 2 \sum_a g_a^4 b_a C_i^a - \sum_{y_i} |y_i|^2 \beta_{y_i},$$

with β_{y_i} being the Yukawa β -functions and b_a as defined in eq. (B.1). For the MSSM this gives in particular

$$\dot{\gamma}_{Q_p} = -8g_3^4 + \frac{3}{2}g_2^4 + \frac{11}{50}g_1^4 - (y_t^2 \beta_{y_t} + y_b^2 \beta_{y_b}) \delta_{3p}, \quad (\text{B.32})$$

$$\dot{\gamma}_{u_p} = -8g_3^4 + \frac{88}{25}g_1^4 - 2y_t^2 \beta_{y_t} \delta_{3p}, \quad (\text{B.33})$$

$$\dot{\gamma}_{d_p} = -8g_3^4 + \frac{22}{25}g_1^4 - 2y_b^2 \beta_{y_b} \delta_{3p}, \quad (\text{B.34})$$

$$\dot{\gamma}_{L_p} = \frac{3}{2}g_2^4 + \frac{99}{50}g_1^4 - y_\tau^2 \beta_{y_\tau} \delta_{3p}, \quad (\text{B.35})$$

$$\dot{\gamma}_{e_p} = \frac{198}{25}g_1^4 - 2y_\tau^2 \beta_{y_\tau} \delta_{3p}, \quad (\text{B.36})$$

$$\dot{\gamma}_{H_u} = \frac{3}{2}g_2^4 + \frac{99}{50}g_1^4 - 3y_t^2 \beta_{y_t}, \quad (\text{B.37})$$

$$\dot{\gamma}_{H_d} = \frac{3}{2}g_2^4 + \frac{99}{50}g_1^4 - 3y_b^2 \beta_{y_b} - y_\tau^2 \beta_{y_\tau}. \quad (\text{B.38})$$

Modular Dependence of the Anomalous Dimension

The gauge couplings (in the anomalous dimension) are moduli dependent. The modular dependence of the anomalous dimension is given by

$$\partial_\alpha \gamma_i = - \sum_{jk} \frac{y_{ijk}^2}{2} \partial_\alpha \log \left[\frac{\lambda_{ijk}}{e^{-\widehat{K}} \mathcal{K}_i \mathcal{K}_j \mathcal{K}_k} \right] + 2 \sum_a g_a^2 C_i^a \partial_\alpha \log(\Re \mathbf{f}_a), \quad (\text{B.39})$$

where α runs over the SUSY breaking fields, \widehat{K} is the pure hidden sector part of the Kähler potential, \mathcal{K}_i is the diagonal Kähler metric for the observable fields as defined in eq. (4.3). The holomorphic Yukawa couplings λ_{ijk} are assumed to be moduli independent.

In type IIB as the underlying string theory one has

$$\Re T = \frac{1}{g_{\text{GUT}}^2}, \quad \widehat{K} = -3 \log(T + \bar{T}), \quad \mathcal{K}_i = (T + \bar{T})^{-n_i}.$$

Then, one obtains

$$\partial_T \gamma_i = \frac{1}{T + \bar{T}} \sum_{jk} \frac{y_{ijk}^2}{2} (3 - n_i - n_j - n_k) - \frac{2}{T + \bar{T}} \sum_a g_a^2 \mathcal{C}_i^a. \quad (\text{B.40})$$

For the MSSM fields this gives

$$(T + \bar{T}) \partial_T \gamma_{Q_p} = -\frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2 + (c_t y_t^2 + c_b y_b^2) \delta_{3p}, \quad (\text{B.41})$$

$$(T + \bar{T}) \partial_T \gamma_{u_p} = -\frac{8}{3} g_3^2 - \frac{8}{15} g_1^2 + 2 c_t y_t^2 \delta_{3p}, \quad (\text{B.42})$$

$$(T + \bar{T}) \partial_T \gamma_{d_p} = -\frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 + 2 c_b y_b^2 \delta_{3p}, \quad (\text{B.43})$$

$$(T + \bar{T}) \partial_T \gamma_{L_p} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + c_\tau y_\tau^2 \delta_{3p}, \quad (\text{B.44})$$

$$(T + \bar{T}) \partial_T \gamma_{e_p} = -\frac{6}{5} g_1^2 + 2 c_\tau y_\tau^2 \delta_{3p}, \quad (\text{B.45})$$

$$(T + \bar{T}) \partial_T \gamma_{H_u} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + 3 c_t y_t^2, \quad (\text{B.46})$$

$$(T + \bar{T}) \partial_T \gamma_{H_d} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + 3 c_b y_b^2 + c_\tau y_\tau^2, \quad (\text{B.47})$$

with $c_t = 3 - n_{Q_3} - n_{H_u} - n_{u_R}$, $c_b = 3 - n_{Q_3} - n_{H_d} - n_{d_R}$ and $c_\tau = 3 - n_{L_3} - n_{H_d} - n_{e_R}$.

In the heterotic case one usually has the ansätze

$$\begin{aligned} \widehat{K} &= -\log(S + \bar{S}), \\ \mathcal{K}_i &= \text{const}, \\ \mathfrak{f}_a &= S. \end{aligned}$$

This leads to

$$\partial_S \gamma_i = \frac{1}{S + \bar{S}} \sum_{jk} \frac{y_{ijk}^2}{2} - \frac{2}{T + \bar{T}} \sum_a g_a^2 \mathcal{C}_i^a \quad (\text{B.48})$$

and so for the MSSM fields

$$(S + \bar{S}) \partial_S \gamma_{Q_p} = -\frac{8}{3} g_3^2 - \frac{3}{2} g_2^2 - \frac{1}{30} g_1^2 + (y_t^2 + y_b^2) \delta_{3p}, \quad (\text{B.49})$$

$$(S + \bar{S}) \partial_S \gamma_{u_p} = -\frac{8}{3} g_3^2 - \frac{8}{15} g_1^2 + 2 y_t^2 \delta_{3p}, \quad (\text{B.50})$$

$$(S + \bar{S}) \partial_S \gamma_{d_p} = -\frac{8}{3} g_3^2 - \frac{2}{15} g_1^2 + 2 y_b^2 \delta_{3p}, \quad (\text{B.51})$$

$$((S + \bar{S}) \partial_S \gamma_{L_p} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + y_\tau^2 \delta_{3p}, \quad (\text{B.52})$$

$$(S + \bar{S}) \partial_S \gamma_{e_p} = -\frac{6}{5} g_1^2 + 2 y_\tau^2 \delta_{3p}, \quad (\text{B.53})$$

$$(S + \bar{S}) \partial_S \gamma_{H_u} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + 3 y_t^2, \quad (\text{B.54})$$

$$(S + \bar{S}) \partial_S \gamma_{H_d} = -\frac{3}{2} g_2^2 - \frac{3}{10} g_1^2 + 3 y_b^2 + y_\tau^2. \quad (\text{B.55})$$

Glossary

AdS	Anti de Sitter
CC	Cosmological Constant
CKM	Cabibbo Kobayashi Maskawa
CY	Calabi Yau
DOF	Degrees of Freedom
DRED	Regularization by Dimensional Reduction
dS	de Sitter
EW	Electroweak
EWSB	Electroweak Symmetry Breaking
FCNC	Flavor Changing Neutral Currents
GUT	Grand Unified Theory
KKLT	Kachru Kallosh Linde Trivedi
LEP	Large Electron-Proton Collider
LSP	Lightest Supersymmetric Particle
MSSM	Minimal Supersymmetric Standard Model
NLSP	Next to Lightest Supersymmetric Particle
RG	Renormalization Group
SM	Standard Model
SSB	Spontaneous Supersymmetry Breaking
SUGRA	Supergravity
SUSY	Supersymmetry
VEV	Vacuum Expectation Value
WMAP	Wilkinson Microwave Anisotropy Probe

Bibliography

- [1] S. Weinberg, “*A model of leptons*”, Phys. Rev. Lett. **19** (1967), 1264–1266.
- [2] F. Halzen and A. D. Martin, “*Quarks and leptons: An introductory course in modern particle physics*”, New York, Usa: Wiley (1984), 396.
- [3] M. J. G. Veltman, “*Second threshold in weak interactions*”, Acta Phys. Polon. **B8** (1977), 475.
- [4] J. Wess and B. Zumino, “*Supergauge transformations in four-dimensions*”, Nucl. Phys. **B70** (1974), 39–50.
- [5] H. P. Nilles, “*Supersymmetry, supergravity and particle physics*”, Phys. Rept. **110** (1984), 1.
- [6] S. P. Martin, “*A supersymmetry primer*”, hep-ph/9709356.
- [7] M. Drees, R. Godbole, and P. Roy, “*Theory and phenomenology of sparticles: An account of four- dimensional $n=1$ supersymmetry in high energy physics*”, Hackensack, USA: World Scientific (2004), 555.
- [8] A. Brignole, L. E. Ibáñez, and C. Muñoz, “*Soft supersymmetry-breaking terms from supergravity and superstring models*”, hep-ph/9707209.
- [9] L. Randall and R. Sundrum, “*Out of this world supersymmetry breaking*”, Nucl. Phys. **B557** (1999), 79–118, hep-th/9810155.
- [10] M. Y. Khlopov and A. D. Linde, “*Is it easy to save the gravitino?*”, Phys. Lett. **B138** (1984), 265–268.
- [11] B. A. Ovrut and P. J. Steinhardt, “*Supersymmetric inflation, baryon asymmetry and the gravitino problem*”, Phys. Lett. **B147** (1984), 263.
- [12] M. B. Green, J. H. Schwarz, and E. Witten, “*Superstring theory. vol. 1: Introduction*”,. Cambridge, Uk: Univ. Pr. (1987) 469 P. (Cambridge Monographs On Mathematical Physics).
- [13] M. Grana, “*Flux compactifications in string theory: A comprehensive review*”, Phys. Rept. **423** (2006), 91–158, hep-th/0509003.
- [14] M. Dine, R. Rohm, N. Seiberg, and E. Witten, “*Gluino condensation in superstring models*”, Phys. Lett. **B156** (1985), 55.

- [15] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, “*De sitter vacua in string theory*”, Phys. Rev. **D68** (2003), 046005, hep-th/0301240.
- [16] R. Kallosh and A. Linde, “*Landscape, the scale of susy breaking, and inflation*”, JHEP **12** (2004), 004, hep-th/0411011.
- [17] O. Loaiza-Brito, J. Martin, H. P. Nilles, and M. Ratz, “ *$\log(m_{Pl}/m_{3/2})$* ”, AIP Conf. Proc. **805** (2006), 198–204, hep-th/0509158.
- [18] A. Falkowski, O. Lebedev, and Y. Mambrini, “*Susy phenomenology of kkt flux compactifications*”, JHEP **11** (2005), 034, hep-ph/0507110.
- [19] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski, and S. Pokorski, “*Stability of flux compactifications and the pattern of supersymmetry breaking*”, JHEP **11** (2004), 076, hep-th/0411066.
- [20] K. Choi, A. Falkowski, H. P. Nilles, and M. Olechowski, “*Soft supersymmetry breaking in kkt flux compactification*”, Nucl. Phys. **B718** (2005), 113–133, hep-th/0503216.
- [21] K. Choi, K. S. Jeong, and K.-i. Okumura, “*Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models*”, JHEP **09** (2005), 039, hep-ph/0504037.
- [22] K. Choi, K. S. Jeong, T. Kobayashi, and K.-i. Okumura, “*Little susy hierarchy in mixed modulus-anomaly mediation*”, Phys. Lett. **B633** (2006), 355–361, hep-ph/0508029.
- [23] H. Baer, E.-K. Park, X. Tata, and T. T. Wang, “*Collider and dark matter searches in models with mixed modulus-anomaly mediated susy breaking*”, JHEP **08** (2006), 041, hep-ph/0604253.
- [24] M. Endo, K. Hamaguchi, and F. Takahashi, “*Moduli / inflaton mixing with supersymmetry breaking field*”, Phys. Rev. **D74** (2006), 023531, hep-ph/0605091.
- [25] S. Nakamura and M. Yamaguchi, “*Gravitino production from heavy moduli decay and cosmological moduli problem revived*”, Phys. Lett. **B638** (2006), 389–395, hep-ph/0602081.
- [26] M. Endo, M. Yamaguchi, and K. Yoshioka, “*A bottom-up approach to moduli dynamics in heavy gravitino scenario: Superpotential, soft terms and sparticle mass spectrum*”, Phys. Rev. **D72** (2005), 015004, hep-ph/0504036.
- [27] O. Lebedev, H. P. Nilles, and M. Ratz, “*A note on fine-tuning in mirage mediation*”, hep-ph/0511320.
- [28] A. Pierce and J. Thaler, “*Prospects for mirage mediation*”, JHEP **09** (2006), 017, hep-ph/0604192.

- [29] R. Haag, J. T. Łopuszański, and M. Sohnius, “All possible generators of supersymmetries of the s matrix”, Nucl. Phys. **B88** (1975), 257.
- [30] J. Wess and B. Zumino, “A lagrangian model invariant under supergauge transformations”, Phys. Lett. **B49** (1974), 52.
- [31] M. T. Grisaru, W. Siegel, and M. Rocek, “Improved methods for supergraphs”, Nucl. Phys. **B159** (1979), 429.
- [32] J. Wess and J. Bagger, “Supersymmetry and supergravity”, Princeton, USA: Univ. Pr. (1992), 259.
- [33] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, “Yang-mills theories with local supersymmetry: Lagrangian, transformation laws and superhiggs effect”, Nucl. Phys. **B212** (1983), 413.
- [34] L. Girardello and M. T. Grisaru, “Soft breaking of supersymmetry”, Nucl. Phys. **B194** (1982), 65.
- [35] J. Terning, “Modern supersymmetry: Dynamics and duality”, Oxford, UK: Clarendon (2006), 324.
- [36] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiral-resolution of naked singularities”, JHEP **08** (2000), 052, hep-th/0007191.
- [37] J. M. Maldacena and C. Nunez, “Towards the large n limit of pure $n = 1$ super yang mills”, Phys. Rev. Lett. **86** (2001), 588–591, hep-th/0008001.
- [38] J. E. Kim and H. P. Nilles, “Gaugino condensation and the cosmological implications of the hidden sector”, Phys. Lett. **B263** (1991), 79–85.
- [39] G. F. Giudice and A. Masiero, “A natural solution to the mu problem in supergravity theories”, Phys. Lett. **B206** (1988), 480–484.
- [40] K. Inoue, A. Kakuto, H. Komatsu, and S. Takeshita, “Low-energy parameters and particle masses in a supersymmetric grand unified model”, Prog. Theor. Phys. **67** (1982), 1889.
- [41] **LEP Higgs Working Group** Collaboration, “Searches for the neutral higgs bosons of the mssm: Preliminary combined results using lep data collected at energies up to 209-gev”, hep-ex/0107030.
- [42] H. E. Haber and R. Hempfling, “Can the mass of the lightest higgs boson of the minimal supersymmetric model be larger than $m(z)$?”, Phys. Rev. Lett. **66** (1991), 1815–1818.
- [43] C. Muñoz, “Soft supersymmetry breaking terms and the mu problem”, hep-th/9507108.

-
- [44] Y. Shadmi, “*Supersymmetry breaking*”, hep-th/0601076.
- [45] M. A. Luty, “*2004 tasi lectures on supersymmetry breaking*”, hep-th/0509029.
- [46] D. E. Kaplan, G. D. Kribs, and M. Schmaltz, “*Supersymmetry breaking through transparent extra dimensions*”, Phys. Rev. **D62** (2000), 035010, hep-ph/9911293.
- [47] Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, “*Gaugino mediated supersymmetry breaking*”, JHEP **01** (2000), 003, hep-ph/9911323.
- [48] **Supernova Search Team** Collaboration, A. G. Riess *et al.*, “*Observational evidence from supernovae for an accelerating universe and a cosmological constant*”, Astron. J. **116** (1998), 1009–1038, astro-ph/9805201.
- [49] M. Dine, R. Kitano, A. Morisse, and Y. Shirman, “*Moduli decays and gravitinos*”, Phys. Rev. **D73** (2006), 123518, hep-ph/0604140.
- [50] O. Lebedev, H. P. Nilles, and M. Ratz, “*de sitter vacua from matter superpotentials*”, Phys. Lett. **B636** (2006), 126, hep-th/0603047.
- [51] T. Gherghetta, G. F. Giudice, and J. D. Wells, “*Phenomenological consequences of supersymmetry with anomaly-induced masses*”, Nucl. Phys. **B559** (1999), 27–47, hep-ph/9904378.
- [52] B. C. Allanach, “*Softsusy: A c++ program for calculating supersymmetric spectra*”, Comput. Phys. Commun. **143** (2002), 305–331, hep-ph/0104145.
- [53] B. de Carlos, S. Gurrieri, A. Lukas, and A. Micu, “*Moduli stabilisation in heterotic string compactifications*”, JHEP **03** (2006), 005, hep-th/0507173.
- [54] D. M. Capper, D. R. T. Jones, and P. van Nieuwenhuizen, “*Regularization by dimensional reduction of supersymmetric and nonsupersymmetric gauge theories*”, Nucl. Phys. **B167** (1980), 479.
- [55] I. Jack, D. R. T. Jones, and C. G. North, “ *$N = 1$ supersymmetry and the three loop gauge beta function*”, Phys. Lett. **B386** (1996), 138–140, hep-ph/9606323.
- [56] P. M. Ferreira, I. Jack, and D. R. T. Jones, “*The three-loop ssm beta-functions*”, Phys. Lett. **B387** (1996), 80–86, hep-ph/9605440.

Index

A	
Anomalous dimension	70
Anomaly mediation	31
B	
Boundary conditions	29, 33, 44, 59
C	
Compactification	35
Cosmological constant ..	25, 39, 43, 58
G	
Gravity mediation	27, 28
GUT	19
H	
Hidden sector	17
K	
Kähler potential	13
heterotic model	57
in KKLT	36
in mSUGRA	30
type IIB model	38, 43
KKLT	2, 36
L	
Little Hierarchy	40
LSP	11
M	
Mirage mediation	
extended case	57
low energy spectra ..	49–56, 60, 61
ordinary case	47
parameter space	46
soft terms (heterotic)	59
soft terms (type IIB)	44
Mirage unification	40
Modular weights	39, 43
Moduli	36
Modulus mediation	39, 40
MSSM	
particle content	9, 24
softly broken	20
mSUGRA	30
N	
NLSP	49
Non-renormalization theorem	4, 67
O	
Observable sector	17
P	
Phenomenological constraints	46
R	
Renormalization group ..	18, 41, 48, 67
S	
Spurion	39
Standard Model	3
Supergravity	
definition of	12
spontaneously broken	24
Supersymmetry	
definition of	5
Lagrangians	7
multiplets	5
softly broken	17
spontaneously broken	16
superfields	6
T	
Trivial weights	57
W	
Wess-Zumino gauge	6