On One-Loop Corrections to the Mass Sum Rule for the Neutral Higgs Bosons in the MSSM

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Ich versichere, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate kenntlich gemacht habe.

Referent: Prof. Hans–Peter Nilles
Koreferent: Prof. Herbert Dreiner

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Contents

Chapter 1

Introduction

The Minimal Supersymmetric Extension of the Standard Model (MSSM) contains three neutral Higgs bosons (h,H,A) and a charged Higgs pair ($H^{\pm}$). At tree level, the mass of the lightest Higgs, h, is bounded from above by the mass of the Z boson of 91 GeV, a region which is already excluded by experiment. However, large self-energy loop corrections give rise to a considerabe shift in the h mass.

Following the argumentation of [10], a formula for the correction to the neutral Higgs boson mass sum rule of the MSSM is derived. The correction is calculated explicitly to one loop, evaluating only diagrams which include top/stop and bottom/sbottom loops. These are expected to give the major contribution due to their large Yukawa couplings. In the limit of a large A mass the correction of the mass sum rule can be used to estimate the correction to the mass of the lightest Higgs. Graphical evaluation shows that the mass of the lightest Higgs is shifted to approximately 156 GeV in the best case. This includes the assumption that all MSSM parameters with the dimension of a mass do not exceed 1 TeV by large. It is demonstrated that the formula for the corrected h mass could be used to give an upper bound on the SUSY scale if the h mass was known. As an example we find an upper bound of 950 GeV for a h mass of 175$\pm$2 GeV.

This work is organized as follows:

In chapter 2 we try to motivate the Minimal Supersymmetric Extension of the Standard Model (MSSM). In chapters 3 to 5 the mass spectrum of the MSSM is derived at tree level, the neutral Higgs boson mass sum rule appears as a byproduct. Chapter 6 presents the derivation of the correction to the mass sum rule. Chapter 7 explains the explicit calculation of the correction (the result can be found in appendix F), contains the graphical analysis for the maximum search of the h mass and concludes with the bound on the SUSY scale.

Chapter 2

The Minimal Supersymmetric Standard Model (MSSM)

2.1 Motivations for Supersymmetry

The Standard Model

The Standard Model of Elementary Particle Physics (SM) of the strong and electroweak interactions is a remarkably successful description of present day high energy physics. However, physicists believe that at energy scales higher than those probed until today, new physics will appear, so that the SM will then turn out to be an effective low energy theory within a more complete underlying framework. One of these scales is the so called GUT scale of Grand Unified Theories, where one believes that the gauge couplings of the SM could unify. A second scale at even higher energies is set by the Planck scale, where gravity becomes comparably strong to the other forces and should thus be incorporated into the model if possible. One also expects that with these new theories, the particle spectrum will be enhanced and particles with very high masses, related to the order of magnitude of the new scales will enter the theory.

The Hierarchy Problem

One of the strongest motivations for supersymmetry comes from the expectation that it will solve the *hierarchy problem* [1, 6]: The SM contains one elementary scalar particle, the Higgs field, which is a necessary ingredient, because it gives masses to fermions in a gauge invariant way via the Higgs mechanism. The problem with scalar particles is that they are subject to large loop corrections which are quadratic in the cutoff scale of the theory. This does not pose a problem by itself, because physicists invented a renormalization programme to fight these divergences so that they are absorbed in the parameters of the Lagrangian. Accompanying these divergences, however, are finite terms which are quadratic in the masses of the particles in the loops. If there are now very heavy particles in the theory with masses of the order of some new scale, then the resulting Higgs mass can only be kept small when the couplings of the Higgs to the new particles are made very tiny. This is then called *fine-tuning*. Also the fine-tuning need not be regarded as a problem by itself. But after the inclusion of corrections at higher loop order the fine-tuning has to be redone and one could find this procedure somehow *unnatural*. Supersymmetry, a new symmetry which relates bosons and fermions, is one approach to solve the problems in a very elegant way. Supersymmetry introduces new particles into the theory which then would also give rise to loop corrections, but in such a way that these new graphs exactly cancel the SM graphs and thus offer a solution to the hierarchy and naturalness problems.

Gauge Coupling Unification

There is another motivation for supersymmetry: In Grand Unified Theories (GUTs) one can observe that the running values of the coupling constants approach each other at high energies, but they do not meet in a point. In fact, the trajectories span a triangle which is then called the GUT triangle. It was realized that supersymmetric Grand Unified Theories (SUSY GUTs) have the feature that the coupling constants unify in exactly one point. This discovery resulted in an enormous increase of interest in supersymmetry.

2.2 The field content of the MSSM

The Minimal Supersymmetric Standard Model (MSSM) [1] is defined as the Standard Model enhanced by Supersymmetry with 1 Supersymmetry generator (so called N=1 SUSY). In short terms it can be said that every particle from the SM gets a SUSY partner with a spin which differs by 1/2 from the SM particle. The new fields are called supersymmetric particles or, in short, *sparticles*. For reasons discussed later the introduction of a second Higgs field (and its SUSY counterpart) is also necessary. Let us first display the MSSM field content.

The MSSM Particle Spectrum

In the gauge sector the partners of the gauge bosons are spin 1/2 fermions and are called *gauginos* (*gluinos, winos, bino*). In the matter sector the partners of the spin 1/2 fermions are spin 0 bosons: The partners of the leptons are called *sleptons* (*sneutrino, selectron*) and the partners of the quarks are called *squarks* (*s-up, s-down* or

Superfield	Boson Field	Fermion Field	$\mathbf{SU(3)_C}$	$\mathbf{SU(2)_L}$	$\mathbf{U(1)_Y}$
		Gauge Sector			
$\widehat{G}$	g	$\tilde{g}$	8	0	0
$\widehat{V}$	W^a	$\tilde{W}^a$	1	3	0
$\widehat{V}'$	B	$\tilde{B}$	1	1	0
		Matter Sector			
$\widehat{L}$	$\tilde{L}^j = \begin{pmatrix} \tilde{\nu} \\ \tilde{e}_L \end{pmatrix}$	$L^j = \begin{pmatrix} \nu \\ e_L \end{pmatrix}$	1	2	-1
$\widehat{Q}$	$\tilde{Q}^j = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$Q^j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	1/3
$\widehat{E}$	$\tilde{E} = \tilde{e}_R^*$	$E = e_L^c$	1	1	2
$\widehat{U}$	$\tilde{U} = \tilde{u}_R^*$	$U = u_L^c$	3^*	1	-4/3
$\widehat{D}$	$\tilde{D} = \tilde{d}_R^*$	$D = d_L^c$	3^*	1	2/3
		Higgs Sector			
$\widehat{H}_1$	$H_1^j = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$	$\tilde{H}_1^j = \begin{pmatrix} \tilde{H}_1^0 \\ \tilde{H}_1^- \end{pmatrix}$	1	2	-1
$\widehat{H}_2$	$H_2^j = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$	$\tilde{H}_2^j = \begin{pmatrix} \tilde{H}_2^+ \\ \tilde{H}_2^0 \end{pmatrix}$	1	2	1

Table 2.1: MSSM particle spectrum

u-squark, d-squark). The scalar Higgs particles keep company with the spin 1/2 *higgsinos*. In table 2.1 we have displayed the interaction eigenstates. When we go to the mass eigenstates, we will see that a mixing of the interaction eigenstates occurs. It is a consequence of this that the neutral gauginos and higgsinos mix to new mass eigenstates which will then be called *neutralinos*, whereas the charged gauginos and higgsinos will mix to states called *charginos*.

Two Higgs Fields

There are two reasons why a second Higgs field is necessary: For the first reason have a look at the SM, where we introduce one scalar Higgs field Φ with the hypercharge assignment Y=+1 to give mass to the up-type squarks and electron-type leptons via a Yukawa coupling. These couplings are in agreement with the Lagrangian to have a total hypercharge of zero. To satisfy the same condition in the Yukawa couplings for the down-type squarks, we make use of the *hermitean conjugate* of the Higgs field, which carries a hypercharge of Y=−1. In the MSSM the Higgs field appears in an expression called the *superpotential* which supersymmetry requires to be a holomorphic function of the fields. Therefore, it cannot contain the hermitean conjugate of the Higgs field and we are forced to introduce a second Higgs field with a negative hypercharge to give mass to the down-type squarks. A second argumentation involves the SUSY partners of the Higgs: In the SM we have anomaly cancellation "by chance" in the sense that the fermionic fields match exactly to cancel the upcoming anomalies. When we now introduce supersymmetry, we have with the higgsino one additional fermion. Therefore, we need another fermion, the second higgsino, to ensure that the theory is anomaly-free again. By supersymmetry again the higgsinos must have a SUSY partner which is the second Higgs field.

Broken SUSY

If SUSY was an exact symmetry of nature, every particle and its partner sparticle would have to have the same mass. As in the experiments conducted so far no sparticles have been observed, SUSY must be *broken* somewhere above our energy scales. In our model we explicitly break supersymmetry by inserting SUSY violating terms into our Lagrangian.

2.3 The Lagrangian of the MSSM

The complete Lagrangian of the MSSM written out in the fields can be found in appendix B. Here we display the individual contributions which make up the Lagrangian.

2.3.1 The Superpotential of the MSSM

In supersymmetric field theories, the superpotential $\mathcal{W}$ specifies the theory. The MSSM superpotential is given by

$$\mathcal{W}_{\mathrm{MSSM}} = \epsilon_{ij}\left[h_\tau \widehat{H}_1^i \widehat{L}^j \widehat{E} + h_b \widehat{H}_1^i \widehat{Q}^j \widehat{D} - h_t \widehat{H}_2^i \widehat{Q}^j \widehat{U} - \mu \widehat{H}_1^i \widehat{H}_2^j\right], \tag{2.1}$$

where i, j are $SU(2)_L$ indices and the caret indicates a superfield. Superfields [5] can be seen as a collection of fields which all differ by half a unit of spin. The matter fields in our model [1] live in complex chiral multiplets $\hat{A} \equiv (A, \Psi_L, F)$ where A and F are complex scalar fields, while Ψ_L is a complex WEYL spinor. F is called an auxilliary field, because its equations of motion are purely algebraic and we can therefore integrate it out. The massless gauge fields belong to massless real superfields $\hat{V}^a \equiv (\lambda_L^a, V^{\mu a}, D^a)$, where a denotes the components of the adjoint representations of the gauge group. Here λ_L is a complex WEYL spinor, V^μ is a massless gauge field and D is a real auxiliary field. The symbols h_τ, h_b and h_t denote the 3×3 Yukawa coupling matrices, which in the model with one family are simply numbers proportional to the corresponding fermion masses:

$$h_\tau = \frac{g' m_\tau}{\sqrt{2} m_W \cos\beta}, \qquad h_b = \frac{g' m_b}{\sqrt{2} m_W \cos\beta}, \qquad h_t = \frac{g' m_t}{\sqrt{2} m_W \sin\beta}. \tag{2.2}$$

$\mathcal{W}_{\mathrm{MSSM}}$ is the most general gauge invariant renormalizable superpotential which respects R-parity (see below). In addition to that, also the following terms would in principle be allowed:

$$\mathcal{W}_{\mathrm{NR}} = \epsilon_{ij}\left[\lambda_L \widehat{L}^i \widehat{L}^j \widehat{E} + \lambda'_L \widehat{L}^i \widehat{Q}^j \widehat{D} - \mu' \widehat{L}^i \widehat{H}_2^j\right] + \lambda_B \widehat{U}\widehat{D}\widehat{D}. \tag{2.3}$$

Assigning the baryon quantum numbers $B = +1/3$ to the $\widehat{Q}$ superfield, $B = -1/3$ to the $\widehat{U}$ and $\widehat{D}$ superfields and $B = 0$ to all others and the lepton quantum numbers $L = +1$ to the $\widehat{L}$ superfield, $L = -1$ to the $\widehat{E}$ superfield and $L = 0$ to all others, we observe that the first three interactions in the brackets violate lepton number while the last interaction violates baryon number[2]. Therefore, we set $\mathcal{W}_{NR}$ to zero by the

[1] In this respect, the Higgses belong to the matter fields

[2] Note that in the SM all renormalizable gauge invariant interaction terms *automatically conserve* baryon and lepton number

introduction of a new discrete symmetry called *R-parity*. R-parity goes along with a multiplicative quantum number, defined as

$$R = (-1)^{3(B-L)+2S}, \tag{2.4}$$

where S is the spin of the particle. Then it can be seen that all terms from $\mathcal{W}_{\mathrm{NR}}$ are forbidden by R-parity while all those from $\mathcal{W}_{\mathrm{MSSM}}$ remain allowed. From the superpotential we will derive the F-term contribution to the scalar potential and the Yukawa coupling part.

2.3.2 D-Term Contribution to the Scalar Potential

The D-terms are

$$\begin{aligned} D^a &= -gT^a_{ij}A^*_iA_j, & (2.5)\\ D' &= -\frac{1}{2}g'y_iA^*_iA_i - \xi. & (2.6)\end{aligned}$$

There is one term D^a for every scalar particle A_i in the theory which is charged under an $SU(2)_L$ or $SU(3)_C$ factor of the gauge group with T^a_{ij} meaning the corresponding representation matrices (i.e. the PAULI matrices divided by 2 for $SU(2)_L$ and the GELL-MAN matrices divided by 2 for $SU(3)_C$). The indices i, j are then weak or colour indices and the coupling g is equal to g_2 or g_3, respectively. For every scalar field which is charged under $U(1)_Y$ there is an additional term D' with the coupling g' and y the hypercharge of this particle. The additive constant ξ is the so called FAYET-ILLIOPOLOUS term which is usually assumed to be small and therefore set to zero in this model. Every scalar gives the following contribution to the scalar potential

$$V_D = \frac{1}{2}[D^aD^a + (D')^2]. \tag{2.7}$$

This part of the Lagrangian gives rise to mass terms for all matter scalars (i.e. squarks, sleptons, Higgses).

2.3.3 F-Term Contribution to the Scalar Potential

The F-terms are

$$F^*_i = -\frac{\partial W}{\partial A_i}. \tag{2.8}$$

They are derived from the superpotential, where the derivative is here meant in such a way that the superfield in $\mathcal{W}$ has to be replaced by only its lowest scalar component.

A_i stands for any scalar field in the theory. Then the F-term contribution to the scalar potential is given by

$$V_F = F_i^* F_i. \tag{2.9}$$

Also from this part of the Lagrangian arise mass terms for all matter scalars (i.e. squarks, sleptons, Higgses).

2.3.4 Supersymmetric Interactions

The next contribution is called "Supersymmetric Interactions"

$$-\mathcal{L}_{SUSY} = -\sqrt{2} g T_{ij}^a \left(\overline{\tilde{V}^a} P_L \Psi_j A_i^* \right) + \text{h.c.} \tag{2.10}$$

It is meant that A and Ψ stand for every scalar/fermion matter pair related by supersymmetry. Then $\tilde{V}^a$ is the gaugino corresponding to the group under which the fields are charged and T_{ij}^a and g the generators and couplings as above. For the $U(1)_Y$ factor, gT_{ij}^a must be replaced by $g' \frac{y}{2} \delta_{ij}$ as above. From here we get mass terms for the Bino and the three Winos. The Bino and the neutral 3-component of the Wino form the neutralinos, whereas the first two components of the Winos mix to the so called charginos.

2.3.5 Yukawa Couplings

The last part of the interaction Lagrangian contains the Yukawa couplings

$$-\mathcal{L}_Y = \frac{1}{2} \left(\frac{\partial^2 W}{\partial A_i \partial A_j} \overline{\Psi_i^c} P_L \Psi_j \right) + \text{h.c.}. \tag{2.11}$$

It is meant that the second derivative of the superpotential is to be taken at the place of the scalar fields. Ψ_i and Ψ_j are then the supersymmetric partners of the fields A_i and A_j the derivative has been taken with respect to. This part of the Lagrangian contains mass terms for all matter fermions (i.e. quarks, leptons, higgsinos).

2.3.6 Explicit Soft Supersymmetry Breaking

As we want to construct a model of broken supersymmetry, we have to include SUSY-breaking terms. The criterion for these terms should be that they do not destroy the good ultraviolet behaviour of our theory, i.e. we do not want them to reintroduce quadratic divergences. Such a breaking mechanism which does not reintroduce quadratic divergences is called *soft*. All possible soft breaking terms have been classified by Girardello and Grisaru [7]. For the MSSM we add all these possible soft terms by hand into our Lagrangian. Simply inserting SUSY-breaking terms by hand is called *explicit* SUSY-breaking. Thus, our breaking mechanism is explicit and soft. All possible soft terms that respect R-parity are

$$\begin{aligned}
V_{\text{soft}} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_{12}^2(\epsilon_{ij}H_1^iH_2^j + \text{h.c.}) \\
&+M_{\tilde{Q}}^2\left[\tilde{t}_L^*\tilde{t}_L + \tilde{b}_L^*\tilde{b}_L\right] + M_{\tilde{U}}^2\tilde{t}_R^*\tilde{t}_R + M_{\tilde{D}}^2\tilde{b}_R^*\tilde{b}_R \\
&+M_{\tilde{L}}^2\left[\tilde{\nu}_L^*\tilde{\nu}_L + \tilde{\tau}_L^*\tilde{\tau}_L\right] + M_{\tilde{E}}^2\tilde{\tau}_R^*\tilde{\tau}_R \\
&+\frac{g}{\sqrt{2}m_W}\epsilon_{ij}\left[\frac{m_\tau A_\tau}{\cos\beta}H_1^i\tilde{l}_L^j\tilde{\tau}_R^* + \frac{m_b A_b}{\cos\beta}H_1^i\tilde{q}_L^j\tilde{b}_R^* - \frac{m_t A_t}{\sin\beta}H_2^i\tilde{q}_L^j\tilde{t}_R^*\right] \\
&+\frac{1}{2}\left[M_3\bar{\tilde{g}}\tilde{g} + M_2\bar{\tilde{W}}^a\tilde{W}^a + M_1\bar{\tilde{B}}\tilde{B}\right]. \qquad (2.12)
\end{aligned}$$

2.3.7 Kinetic Terms

The kinetic part of the Lagrangian has the same form as always: There is a kinetic term for every gauge boson, gaugino, matter boson and matter fermion. The covariant derivative specifies the couplings to the gauge bosons.

Chapter 3

The Gauge Sector and the Ghosts

In this chapter the Higgs Mechanism is applied to the Lagrangian of the MSSM. In the same way as in the SM the $SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}$ symmetry is broken down to $SU(3)_{\rm C} \times U(1)_{\rm EM}$ and the gauge bosons acquire a mass. The mass spectrum of the gauge bosons and ghost fields is derived.

3.1 Higgs Mechanism and Gauge Fixing

3.1.1 Higgs Mechanism

As noted above, the MSSM contains two $SU(2)$ Higgs Doublets. Both can get a vacuum expectation value (*vev*). In order not to break $U(1)_{\rm EM}$ only the neutral fields can get a vev, which means that

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \longrightarrow H_1 + \langle H_1 \rangle = H_1 + \begin{pmatrix} v_1/\sqrt{2} \\ 0 \end{pmatrix}, \tag{3.1}$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \longrightarrow H_2 + \langle H_2 \rangle = H_2 + \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix}. \tag{3.2}$$

(The conditions which v_1 and v_2 have to fulfill so that they are the real minimum, the so-called *minimization conditions*, are given in chapter 3, where also the Higgs spectrum is discussed.) It should be kept in mind for the next calculations, that H_1 and H_2 (and also their vevs $\langle H_1 \rangle$ and $\langle H_2 \rangle$) are still SU(2)-spinors with their entries as stated above. It will prove useful to define the quantity

$$\tan\beta := \frac{v_2}{v_1} \tag{3.3}$$

for the ratio of the two vacuum expectation values, which will furtheron very often appear. After symmetry breaking, the kinetic part for the first Higgs field (with the appropriate covariant derivatives) looks like

$$\begin{aligned}
&[D_\mu H_1]^\dagger[D^\mu H_1] \\
&= [\partial_\mu H_1^\dagger - \frac{ig'B_\mu}{2}[H_1 + \langle H_1\rangle]^\dagger + \frac{ig_2 W_\mu^a}{2}[H_1 + \langle H_1\rangle]^\dagger \tau^a] \\
&\quad \times [\partial^\mu H_1 + \frac{ig'B^\mu}{2}[H_1 + \langle H_1\rangle] - \frac{ig_2 W^{\mu b}}{2}\tau^b[H_1 + \langle H_1\rangle]] \\
&= (\partial_\mu H_1^\dagger)(\partial^\mu H_1) + \frac{ig'B_\mu}{2}[[\partial^\mu H_1^\dagger][H_1 + \langle H_1\rangle] - [H_1^\dagger + \langle H_1\rangle]\partial^\mu H_1] \\
&\quad + \frac{ig_2}{2}W_\mu^a[-[\partial^\mu H_1^\dagger]\tau^a H_1 - \partial^\mu H_1^\dagger \tau^a \langle H_1\rangle + H_1^\dagger \tau^a \partial^\mu H_1 + \langle H_1\rangle \tau^a \partial^\mu H_1] \\
&\quad - \frac{g' g_2}{2}B_\mu W^{\mu a}[H_1^\dagger \tau^a[H_1 + \langle H_1\rangle] + \langle H_1\rangle^\dagger \tau^a [H_1 + \langle H_1\rangle]] \\
&\quad - \frac{ig'^2 B^2}{4}[|H_1|^2 + H_1^\dagger\langle H_1\rangle + \langle H_1\rangle^\dagger H_1 + |\langle H_1\rangle|^2] \\
&\quad + \frac{g_2^2}{4}W_\mu^a W^{\mu b}[H_1^\dagger \tau^a \tau^b H_1 + H_1^\dagger \tau^a\tau^b\langle H_1\rangle + \langle H_1\rangle^\dagger \tau^a\tau^b H_1 + \langle H_1\rangle^\dagger \tau^a \tau^b \langle H_1\rangle],
\end{aligned} \tag{3.4}$$

where H_1 is now the shifted field. The first observation confirms that the mass terms for the B- and W-fields have been generated. But one also notices the appearance of some unwanted terms in which the gauge fields mix with the derivatives of the Higgs fields, namely

$$\frac{ig'}{2}B_\mu \partial^\mu H_1^\dagger \langle H_1\rangle - \frac{ig'}{2}B_\mu \langle H_1\rangle \partial^\mu H_1, \tag{3.5}$$

$$-\frac{ig_2}{2}W_\mu^a \partial^\mu H_1^\dagger \tau^a \langle H_1\rangle + \frac{ig_2}{2}W_\mu^a \langle H_1\rangle^\dagger \tau^a \partial^\mu H_1. \tag{3.6}$$

A similar expression for the second Higgs field contains the unwanted terms

$$-\frac{ig'}{2}B_\mu \partial^\mu H_2^\dagger \langle H_2\rangle + \frac{ig'}{2}B_\mu \langle H_2\rangle^\dagger \partial^\mu H_2, \tag{3.7}$$

$$-\frac{ig_2}{2}W_\mu^a \partial^\mu H_2^\dagger \tau^a \langle H_2\rangle + \frac{ig_2}{2}W_\mu^a \langle H_2\rangle^\dagger \tau^a \partial^\mu H_2. \tag{3.8}$$

These terms can be cancelled by fixing an appropriate gauge. The additional terms which are for this purpose introduced into the Lagrangian will then give rise to FADDEEV-POPOV ghost fields in the theory. We will now say some words about the R_ξ-gauge, which we are going to use here.

3.1.2 R_ξ Gauge Fixing

Gauge Field Quantization

If one wants to quantize a gauge theory with the path-integral approach, one sees that the path integral over the gauge field is divergent (cf. [12]). The reason is an overcounting due to the fact that one has integrated also over all classes of gauge fields, which are related by a gauge transformation. To avoid this problem, we can *fix a gauge*, which is done by a trick due to FADEEV and POPOV: We insert a delta function into the path integral to pick out one orbit over which we want to integrate. To compensate for the delta function, one has to introduce also the determinant of the response function. The integration over the group space then gives a 1, which can be inserted into the path integral.

$$\int [d\theta(x)]\delta[f_a(\mathbf{A}_\mu^\theta)]\det\frac{\delta f_a}{\delta\theta_b} = 1, \tag{3.9}$$

where θ is a group element and the gauge function specifying the hypermanifold in group space fulfills

$$f_a[\mathbf{A}_\mu] = 0. \tag{3.10}$$

Then the delta function and the determinant are written in an exponential form so that their contributions to the action can be seen. The delta function gives rise to the gauge fixing term

$$S_{\mathrm{gf}} = -\frac{1}{2\xi}\int d^4x\,[f_a[\mathbf{A}_\mu(x)]]^2\,. \tag{3.11}$$

In order to convert the FADEEV-POPOV determinant, one uses that any functional determinant can be written as an integral over GRASSMANN variables of the form

$$S_{\mathrm{FPG}} = \int d^4x d^4y \sum_{a,b} c_a^\dagger(x)[M_f(x,y)]_{ab}c_b(y). \tag{3.12}$$

The GRASSMANN variables c and $c^\dagger$ can then be interpreted as unphysical *ghost fields*.

R$_\xi$ Gauge

We will now motivate the particular choice of gauge that we are taking: Theories with massive spin-1 particles that contain a mass term of the form $m^2A_\mu A^\mu$ have problems with renormalizability. The vector field propagator then has the asymptotic behaviour that it approaches a constant value when the momentum is sent to large scales. A power counting check then reveals that the theory is not renormalizable. However, in theories in which the vector fields acquire mass by the Higgs mechanism, the situation is different. Starting from the observation that *before* symmetry breaking the Lagrangian is renormalizable by power counting, 'T HOOFT proved that one can find a gauge (or better a class of gauges, depending on the parameter ξ) in which renormalizability is explicit also *after* symmetry breakdown [8, 9]. We define the *gauge functions*

$$\begin{aligned} f' &:= \partial_\mu B^\mu + \frac{ig'\xi}{2}[-H_1^\dagger\langle H_1\rangle + \langle H_1\rangle^\dagger H_1 + H_2^\dagger\langle H_2\rangle - \langle H_2\rangle^\dagger H_2], \qquad (3.13)\\ f^a &:= \partial_\mu W^{\mu a} + \frac{ig_2\xi}{2}[-\langle H_1\rangle^\dagger\tau^a H_1 + H_1^\dagger\tau^a\langle H_1\rangle + H_2^\dagger\tau^a\langle H_2\rangle - \langle H_2\rangle^\dagger\tau^a H_2]. \qquad (3.14)\end{aligned}$$

Then the gauge fixing part of the Lagrangian is given by

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\xi}f'^2 - \frac{1}{2\xi}f^a f^a. \qquad (3.15)$$

The parameter ξ remains undefined, but one can recover the usual gauges, that is the FEYNMAN gauge in the limit $\xi \to 1$ and the LANDAU gauge in the limit $\xi \to 0$. The unitary gauge is found for $\xi \to \infty$. In this limit the masses of the unphysical particles (which are proportional to ξ as can be seen later) become infinitely large, which makes the propagators zero, and the graphs including these states vanish. Of course, one can do calculations in general R_ξ gauge, too. Then the physical result must in the end no longer depend on ξ, so this delivers a first checkpoint for the validity of a calculation.

Back to our calculation we can see that, after multiplying $\mathcal{L}_{\mathrm{gf}}$ out, the gauge terms carry out their duty and cancel exactly the unwanted terms from above, whereas the quadratic terms in B, W and ξ remain, so that the kinetic part of the Higgs-Lagrangian is found to be

$$\begin{aligned} \mathcal{L}_{\mathrm{Higgs}}^{\mathrm{kin}} = \; & (\partial_\mu H_1^\dagger)(\partial^\mu H_1) + (\partial_\mu H_2^\dagger)(\partial^\mu H_2) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ & \frac{ig'B_\mu}{2}[(\partial^\mu H_1^\dagger)H_1 - H_1^\dagger(D_\mu H_1)] + \frac{ig_2W_\mu^a}{2}[H_1^\dagger\tau^a(\partial_\mu H_1^\dagger)\tau^a H_1]\\ & + \frac{g'^2B_\mu B^\mu}{4}[H_1^\dagger(H_1 + \langle H_1\rangle) + (H_1 + \langle H_1\rangle)^\dagger]\\ & + \frac{g_2^2W_\mu^aW^{\mu b}}{4}[H_1^\dagger\tau^a\tau^b(H_1 + \langle H_1\rangle) + \langle H_1\rangle^\dagger\tau^a\tau^b(H_1 + \langle H_1\rangle)]\\ & + \frac{g'g_2B_\mu W^{\mu a}}{2}[H_1^\dagger\tau^a(H_1 + \langle H_1\rangle) + \langle H_1\rangle^\dagger\tau^a(H_1 + \langle H_1\rangle)]\\ & + \frac{ig'B_\mu}{2}[(-\partial^\mu H_2^\dagger)H_2 + H_2^\dagger(D_\mu H_2)] + \frac{ig_2W_\mu^a}{2}[H_2^\dagger\tau^a(\partial_\mu H_2^\dagger)\tau^a H_2]\\ & + \frac{g'^2B_\mu B^\mu}{4}[H_2^\dagger(H_2 + \langle H_2\rangle) + (H_2 + \langle H_2\rangle)^\dagger] \end{aligned}$$

$$
\begin{aligned}
&+\frac{g_2^2 W_\mu^a W^{\mu b}}{4}[H_2^\dagger \tau^a \tau^b (H_2 + \langle H_2\rangle) + \langle H_2\rangle^\dagger \tau^a \tau^b (H_2 + \langle H_2\rangle)] \\
&+\frac{g' g_2 B_\mu W^{\mu a}}{2}[H_2^\dagger \tau^a (H_2 + \langle H_2\rangle) + \langle H_2\rangle^\dagger \tau^a (H_2 + \langle H_2\rangle)] \\
&-\frac{1}{2\xi}(\partial_\mu B^\mu)^2 + \frac{g'^2 \xi}{8}[-H_1^\dagger \langle H_1\rangle + \langle H_1\rangle^\dagger H_1 + H_2^\dagger \langle H_2\rangle - \langle H_2\rangle^\dagger H_2]^2 \\
&-\frac{1}{2\xi}(\partial_\mu W^{\mu a})^2 + \frac{g_2^2 \xi}{8}[H_1^\dagger \tau^a \langle H_1\rangle - \langle H_1\rangle^\dagger \tau^a H_1 + H_2^\dagger \tau^a \langle H_2\rangle - \langle H_2\rangle^\dagger \tau^a H_2]^2,
\end{aligned}
\tag{3.16}
$$

where

$$
F_{\mu\nu} := \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c. \tag{3.17}
$$

3.2 Mass Spectrum of Gauge Bosons

From the above Lagrangian the mass terms for the gauge bosons are

$$
\begin{aligned}
\mathcal{L}_{\text{g.bos.}}^{\text{mass}} &= \frac{g'^2 B_\mu B^\mu}{4}[\langle H_1\rangle^\dagger \langle H_1\rangle + \langle H_2\rangle^\dagger \langle H_2\rangle] \\
&\quad + \frac{g' g_2 B_\mu W^{\mu a}}{2}[-\langle H_1\rangle^\dagger \tau^a \langle H_1\rangle + \langle H_2\rangle^\dagger \tau^a \langle H_2\rangle] \\
&\quad + \frac{g_2^2 W_\mu^a W^{\mu b}}{4}[\langle H_1\rangle^\dagger \tau^a \tau^b \langle H_1\rangle + \langle H_2\rangle^\dagger \tau^a \tau^b \langle H_2\rangle] \\
&= \frac{g'^2 (v_1^2 + v_2^2)}{4} B_\mu B^\mu - \frac{g' g_2 (v_1^2 + v_2^2)}{4} B_\mu W^{\mu 3} + \frac{g_2^2 (v_1^2 + v_2^2)}{4} W_{\mu a} W^{\mu a} \\
&= (W_\mu^1, W_\mu^2) \begin{pmatrix} \frac{g_2^2 v^2}{4} & 0 \\ 0 & \frac{g_2^2 v^2}{4} \end{pmatrix} \begin{pmatrix} W^{\mu 1} \\ W^{\mu 2} \end{pmatrix} + (W_\mu^3, B_\mu) \begin{pmatrix} \frac{g_2^2 v^2}{4} & -\frac{g' g_2 v^2}{4} \\ -\frac{g' g_2 v^2}{4} & \frac{g_2^2 v^2}{4} \end{pmatrix} \begin{pmatrix} W^{\mu 1} \\ B^\mu \end{pmatrix}.
\end{aligned}
\tag{3.18}
$$

Here we can see that the mass terms for the gauge bosons in the MSSM are absolutely the same as in the SM if only one replaces

$$
v_{SM}^2 \longrightarrow v_1^2 + v_2^2 =: v^2. \tag{3.19}
$$

So the diagonalization of these matrices gives the well known results

$$
W_\mu^\pm := \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2), \tag{3.20}
$$

$$
M_{W^\pm}^2 = \frac{g_2^2 v^2}{4} \tag{3.21}
$$

for the W^+ and W^- and

$$\begin{aligned} A_\mu &:= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu, & (3.22)\\ Z_\mu &:= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu & (3.23)\end{aligned}$$

for the photon field A_μ and the Z-boson and

$$m_\gamma^2 = 0, \tag{3.24}$$

$$m_Z^2 = \frac{g'^2 + g_2^2}{4} v^2 \tag{3.25}$$

for their masses.

3.3 Mass Spectrum of Ghost Fields

In the FADDEEV-POPOV determinant we need the functional derivatives of the gauge functions with respect to the group elements. We display first, how the fields behave under infinitesimal $U(1)_Y$ transformations.

$$\begin{aligned} \delta_{U(1)} B_\mu &= -\frac{1}{g'}\partial_\mu \alpha(x), & (3.26)\\ \delta_{U(1)} H_1 &= +\frac{i\alpha}{2}(H_1 + \langle H_1\rangle), & (3.27)\\ \delta_{U(1)} H_2 &= -\frac{i\alpha}{2}(H_2 + \langle H_2\rangle). & (3.28)\end{aligned}$$

and thus

$$\begin{aligned} \delta_{U(1)} f &= [-\partial_\mu\partial^\mu - \frac{g'^2\xi}{4}[H_1^\dagger\langle H_1\rangle + 2|\langle H_1\rangle|^2 + \langle H_1\rangle^\dagger H_1 \\ &\quad + H_2^\dagger\langle H_2\rangle + 2|\langle H_2\rangle|^2 + \langle H_2\rangle^\dagger H_1]]\alpha'(x), & (3.29)\\ \delta_{U(1)} f^a &= -\frac{g' g_2 \xi}{4}[-\langle H_1\rangle^\dagger \tau^a H_1 - 2\langle H_1\rangle^\dagger\tau^a\langle H_1\rangle - H_1^\dagger\tau^a\langle H_1\rangle \\ &\quad + H_2^\dagger\tau^a\langle H_2\rangle + \langle H_2\rangle^\dagger\tau^a\langle H_2\rangle + \langle H_2\rangle^\dagger\tau^a H_2]\alpha'(x), & (3.30)\end{aligned}$$

where

$$\alpha'(x) = \frac{\alpha(x)}{g'}. \tag{3.31}$$

For the infinitesimal $SU(2)$ transformations we have

$$\delta_{SU(2)} W^a_\mu = -\frac{1}{g_2}\partial_\mu u^a(x) + \epsilon^{abc} u^b W^c_\mu, \tag{3.32}$$

$$\delta_{SU(2)} H_1 = -\frac{i\tau^a}{2} u^a (H_1 + \langle H_1 \rangle), \tag{3.33}$$

$$\delta_{SU(2)} H_2 = -\frac{i\tau^a}{2} u^a (H_2 + \langle H_2 \rangle) \tag{3.34}$$

and therefore

$$\begin{aligned} \delta_{SU(2)} f &= -\frac{g' g_2 \xi}{4}[-\langle H_1\rangle^\dagger \tau^a H_1 - 2\langle H_1\rangle^\dagger \tau^a \langle H_1\rangle - H_1^\dagger \tau^a \langle H_1 \rangle \\ &\quad + H_2^\dagger \tau^a \langle H_2\rangle + \langle H_2\rangle^\dagger \tau^a \langle H_2\rangle + \langle H_2\rangle^\dagger \tau^a H_2] u'^a(x), \qquad (3.35) \\ \delta_{SU(2)} f^a &= [-\partial_\mu \partial^\mu \delta^a_b + g_2 \epsilon^{abc} \partial_\mu W^{\mu c}] u'^b(x) \\ &\quad - \frac{g_2^2 \xi}{4}[\langle H_1\rangle^\dagger \tau^a \tau^b H_1 + 2\langle H_1\rangle^\dagger \delta^b_a \langle H_1\rangle + H_1^\dagger \tau^b \tau^a \langle H_1\rangle \\ &\quad + H_2^\dagger \tau^b \tau^a \langle H_2\rangle + 2\langle H_2\rangle^\dagger \delta_{ab} \langle H_2\rangle + \langle H_2\rangle^\dagger \tau^a \tau^b \langle H_2\rangle] u'^b(x), \qquad (3.36) \end{aligned}$$

where

$$u'^b(x) = \frac{u^b(x)}{g_2}. \tag{3.37}$$

The Faddeev-Popov determinant is now given by det M_f, with the following structure for M_f

$$(M_f) = \left(\begin{array}{c|c} M_{ab} & M_{a4} \\ \hline M_{4b} & M_{44} \end{array} \right), \tag{3.38}$$

where a and b run from 1 to 3. We now display from the matrix entries only those terms which contribute to the masses

$$\begin{aligned} M_{ab} &:= \frac{\delta_{SU(2)} f^a(x)}{\delta u'^b(y)} = -\frac{g_2^2 \xi}{2}[\langle H_1\rangle^\dagger \delta_{ab} \langle H_1\rangle + \langle H_2\rangle^\dagger \delta_{ab}\langle H_2\rangle + \ldots]\delta^4(x-y), \\ M_{a4} &:= \frac{\delta_{U(1)} f^a(x)}{\delta \alpha'(y)} = -\frac{g' g_2 \xi}{2}[-\langle H_1\rangle^\dagger \tau^a \langle H_1\rangle + \langle H_2\rangle^\dagger \tau^a \langle H_2\rangle + \ldots]\delta^4(x-y), \\ M_{4b} &:= \frac{\delta_{SU(2)} f(x)}{\delta u'^b(y)} = -\frac{g' g_2 \xi}{2}[-\langle H_1\rangle^\dagger \tau^b \langle H_1\rangle + \langle H_2\rangle^\dagger \tau^b \langle H_2\rangle + \ldots]\delta^4(x-y), \\ M_{44} &:= \frac{\delta_{U(1)} f(x)}{\delta \alpha'(y)} = -\frac{g'^2 \xi}{2}[\langle H_1\rangle^\langle H_1\rangle + \langle H_2\rangle^\dagger \langle H_2\rangle + \ldots]\delta^4(x-y). \qquad (3.39) \end{aligned}$$

So we get as an additional contribution to the action

$$\det M_{ab} \det M_{4b} \det M_{a4} \det M_{44} = \tag{3.40}$$

$$\int d[\chi_a^\dagger] d[\chi_a] d[\lambda^\dagger] d[\lambda] \exp[i \int d^4x d^4y (\chi_a^\dagger, \lambda^\dagger) \left(\begin{array}{c|c} M_{ab} & M_{a4} \\ \hline M_{4b} & M_{44} \end{array} \right) \begin{pmatrix} \chi_b \\ \lambda \end{pmatrix}.$$

Considering only the mass terms from above in the M-Matrix, we get after a calculation similar to that of the gauge bosons

$$\mathcal{L}_{\text{gh.}}^{\text{mass}} = \tag{3.41}$$

$$(\chi_1^\dagger, \chi_2^\dagger, \chi_3^\dagger, \lambda^\dagger) \begin{pmatrix} -\frac{g_2^2 \xi}{2}(v_1^2 + v_2^2) & 0 & 0 & 0 \\ 0 & -\frac{g_2^2 \xi}{2}(v_1^2 + v_2^2) & 0 & 0 \\ 0 & 0 & -\frac{g_2^2 \xi}{2}(v_1^2 + v_2^2) & +\frac{g' g_2 \xi}{2}(v_1^2 + v_2^2) \\ 0 & 0 & +\frac{g' g_2 \xi}{2}(v_1^2 + v_2^2) & -\frac{g'^2 \xi}{2}(v_1^2 + v_2^2) \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \lambda \end{pmatrix}.$$

This is diagonalized with the same mixing angle as the gauge boson matrices by introducing the mass eigenstates

$$G^\pm = \frac{1}{\sqrt{2}}(\chi_1 \mp i\chi_2), \tag{3.42}$$

$$G_Z = \cos\theta_W \chi_3 - \sin\theta_W \lambda, \tag{3.43}$$

$$G_\gamma = \sin\theta_W \chi_3 + \cos\theta_W \lambda, \tag{3.44}$$

with masses

$$m_{G^\pm}^2 = -\xi m_W^2, \tag{3.45}$$

$$m_{G_Z}^2 = -\xi m_Z^2, \tag{3.46}$$

$$m_{G_\gamma}^2 = 0. \tag{3.47}$$

Note that the ghost fields are unphysical degrees of freedom, which in the unitary gauge would not have appeared. Any calculation involving the ghost terms must therefore in the end be independent of ξ.

Chapter 4

The Higgs Sector

In this chapter we will find the Higgs mass spectrum and derive the mass sum rules. Let us see how many Higgs particles we can expect in the MSSM: The Higgs sector consists of two $SU(2)_L$-doublets with complex entries, so we have eight degrees of freedom before symmetry breaking. The Higgs mechanism then uses three would-be Goldstone bosons in order to give mass to the three gauge bosons of chapter 3. We will therefore end up with five physical Higgses.

4.1 The Higgs Potential

The Higgs part of the Lagrangian is distributed to three places in the MSSM Lagrangian: to the soft-breaking terms and to the F- and D-term contribution to the scalar potential. Then there are also ξ-dependent terms from the gauge fixed kinetic part of the Lagrangian from the would-be Goldstone bosons. These are discussed in section 4.3.

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} = & +m_1^2(|H_1^0|^2+|H_1^-|^2)+m_2^2(|H_2^+|^2+|H_2^0|^2) \\
& -m_{12}^2(H_1^0H_2^0-H_1^-H_2^+ + H_1^{0*}H_2^{0*}-H_1^{-*}H_2^{+*}) \\
& +|\mu|^2(|H_2^+|^2+|H_2^0|^2+|H_1^0|^2+|H_1^-|^2) \\
& +\frac{g'^2+g_2^2}{8}\left\{(|H_2^+|^2)^2+(|H_2^0|^2)^2+(|H_1^0|^2)^2+(|H_1^-|^2)^2+2|H_1^0|^2|H_1^-|^2\right. \\
& \left.+2|H_2^+|^2|H_2^0|^2-2|H_1^0|^2|H_2^+|^2-2|H_1^-|^2|H_2^+|^2-2|H_1^-|^2|H_2^0|^2-2|H_1^0|^2|H_2^0|^2\right\} \\
& +\frac{g_2^2}{2}\left\{|H_1^0|^2|H_2^+|^2+H_1^{0*}H_1^-H_2^{0*}H_2^+ + H_1^{-*}H_1^0H_2^{+*}H_2^0+|H_1^-|^2|H_2^0|^2\right\} \qquad (4.1)
\end{aligned}$$

Now the neutral Higgs fields are decomposed explicitly into their real and imaginary parts, while the charged Higgses remain in their given form.

$$H_1^0 = x_1 + iy_1, \qquad H_2^0 = x_2 + iy_2 \tag{4.2}$$

Plugging in the decomposition, we get

$$\begin{aligned}
\mathcal{L}_{\text{Higgs}} \quad = \quad & +(m_1^2+\mu^2)(x_1^2+y_1^2+|H_1^-|^2)+(m_2^2+\mu^2)(x_2^2+y_2^2+|H_2^+|^2) \\
& -m_{12}^2(2x_1x_2-2y_1y_2-H_1^-H_2^+-H_1^{-*}H_2^{+*}) \\
& +\frac{g'^2+g_2^2}{8}\left\{(x_1^2+y_1^2-x_2^2-y_2^2+|H_1^-|^2-|H_2^+|^2\right)^2 \\
& +\frac{g_2^2}{2}\left\{|H_1^0|^2|H_2^+|^2+H_1^{0*}H_1^-H_2^{0*}H_2^+ + H_1^{-*}H_1^0H_2^{+*}H_2^0+|H_1^-|^2|H_2^0|^2\right\}.
\end{aligned} \tag{4.3}$$

To find the correct vacuum expectation values v_1 and v_2, we look for a minimum of the potential along the directions of x_1 and x_2. From

$$\left.\frac{\partial V}{\partial x_1}\right|_{v_1} = 0, \qquad \left.\frac{\partial V}{\partial x_2}\right|_{v_2} = 0 \tag{4.4}$$

it follows that

$$m_1^2+\mu^2 \quad = \quad m_{12}^2\frac{v_2}{v_1}-\frac{1}{4}(g'^2+g_2^2)(v_1^2-v_2^2), \tag{4.5}$$

$$m_2^2+\mu^2 \quad = \quad m_{12}^2\frac{v_1}{v_2}-\frac{1}{4}(g'^2+g_2^2)(v_1^2-v_2^2). \tag{4.6}$$

These are the so called *minimization conditions* which have to be fulfilled by v_1 and v_2, so that the Higgs field is in the true vacuum after symmetry breakdown.

4.2 Higgs Mass Terms

Now the Higgs Mechanism is applied to $\mathcal{L}_{\text{Higgs}}$ and mass terms are kept. We can see directly that the mass terms split up into terms which belong to the neutral Higgses and terms which belong to the charged Higgses. We get the following matrix form for the mass terms of the neutral Higgses

$$\begin{aligned}
\mathcal{L}_{\mathrm{H}^0}^{\text{mass}} \quad = \quad & (x_1,x_2)\begin{pmatrix} m_1^2+|\mu|^2+\frac{g'^2+g_2^2}{4}(3v_1^2-v_2^2) & -m_{12}^2-\frac{g'^2+g_2^2}{2}v_1v_2 \\ -m_{12}^2-\frac{g'^2+g_2^2}{2}v_1v_2 & m_2^2+|\mu|^2+\frac{g'^2+g_2^2}{4}(3v_2^2-v_1^2) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\
& +(y_1,y_2)\begin{pmatrix} m_1^2+|\mu|^2+\frac{g'^2+g_2^2}{4}(v_1^2-v_2^2) & m_{12}^2 \\ m_{12}^2 & m_2^2+|\mu|^2+\frac{g'^2+g_2^2}{4}(v_2^2-v_1^2) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.
\end{aligned} \tag{4.7}$$

For the charged Higgses, we left the fields H_1^- and H_2^+ in their complex form. We now impose that those fields behave under complex conjugation as

$$H_1^{-*} = H_1^+, \qquad H_2^{+*} = H_2^- . \tag{4.8}$$

Then we can put up the following mass matrix for the charged Higgses

$$(H_1^-, H_2^-) \begin{pmatrix} m_1^2 + |\mu|^2 + \frac{g'^2+g_2^2}{4}(v_1^2 - v_2^2) + \frac{g_2^2}{2}v_2^2 & m_{12}^2 + \frac{g_2^2}{2}v_1 v_2 \\ m_{12}^2 + \frac{g_2^2}{2}v_1 v_2 & m_2^2 + |\mu|^2 + \frac{g'^2+g_2^2}{4}(v_2^2 - v_1^2) + \frac{g_2^2}{2}v_1^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix}. \tag{4.9}$$

But this is not the whole story, since there are also Goldstone mass terms.

4.3 Goldstone Mass terms

Now we collect all the ξ-dependent mass terms from the kinetic part of the Lagrangian. These are the mass terms of the Goldstone bosons[1].

$$\begin{aligned}
\mathcal{L}_{\mathrm{G}}^{\mathrm{mass}} &= +\frac{g'^2\xi}{8}[-H_1^\dagger\langle H_1\rangle + \langle H_1\rangle^\dagger H_1 + H_2^\dagger\langle H_2\rangle - \langle H_2\rangle H_2]^2 \\
&\quad +\frac{g_2^2\xi}{8}[-\langle H_1\rangle^\dagger\tau^a H_1 + H_1^\dagger\tau^a\langle H_1\rangle + H_2^\dagger\tau^a\langle H_2\rangle - \langle H_2\rangle\tau^a H_2] \\
&= +\frac{g'^2\xi}{8}\Big\{v_1^2[(H_1^0)^2 - 2|H_1^0|^2 + (H_1^{0\dagger})^2] + v_2^2[(H_2^0)^2 - 2|H_2^0|^2 + (H_2^{0\dagger})^2] \\
&\quad +2v_1v_2[H_1^0H_2^{0\dagger} - H_1^0H_2^0 - H_1^{0\dagger}H_2^{0\dagger} + H_1^{0\dagger}H_2^0]\Big\} \\
&\quad +\frac{g_2^2\xi}{8}\Big\{v_1^2[(H_1^0)^2 - 2|H_1^0|^2 + (H_1^{0\dagger})^2] - 4v_1^2|H_1^-|^2 \\
&\quad +v_2^2[(H_2^0)^2 - 2|H_2^0|^2 + (H_2^{0\dagger})^2] - 4v_2^2|H_2^+|^2 \\
&\quad +2v_1v_2[H_1^0H_2^{0\dagger} - H_1^0H_2^0 - H_1^{0\dagger}H_2^{0\dagger} + H_1^{0\dagger}H_2^0] \\
&\quad +2v_1v_2[2H_2^+H_1^- + 2(H_1^-)^\dagger(H_2^+)^\dagger]\Big\}.
\end{aligned} \tag{4.10}$$

Applying for H_1^0 and H_2^0 the complex notation from above, we see that the x-field drops out completely, while what remains is

$$\begin{aligned}
\mathcal{L}_{\mathrm{G}}^{\mathrm{mass}} = &-\frac{\xi(g'^2+g_2^2)}{2}\ (y_1, y_2) \begin{pmatrix} v_1^2 & -v_1v_2 \\ -v_1v_2 & v_2^2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \\
&-\frac{\xi\, g_2^2}{2}\ (H_1^-, H_2^-) \begin{pmatrix} v_1^2 & -v_1v_2 \\ -v_1v_2 & v_2^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix}.
\end{aligned} \tag{4.11}$$

[1]Note that one can actually only speak in this way *after the diagonalization* of the mass matrices where it can be seen that the Goldstone state receives all the ξ-dependent terms, while the other state does not.

4.4 Five physical Higgses in the MSSM

4.4.1 The CP-even Higgses H and h

The mass matrix for the real part of the neutral Higgses was given by

$$(x_1, x_2)\begin{pmatrix} m_1^2 + \frac{g'^2+g_2^2}{4}(3v_1^2 - v_2^2) & -m_{12}^2 - \frac{g'^2+g_2^2}{2}v_1v_2 \\ -m_{12}^2 - \frac{g'^2+g_2^2}{2}v_1v_2 & m_2^2 + \frac{g'^2+g_2^2}{4}(3v_2^2 - v_1^2) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{4.12}$$

where the addends of $|\mu|^2$ have been absorbed in m_1^2 and m_2^2 (as will always be implicit in the following). Using the minimization conditions the matrix simplifies to

$$(x_1, x_2)\begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \frac{g'^2+g_2^2}{2}v_1^2 & -m_{12}^2 - \frac{g'^2+g_2^2}{2}v_1v_2 \\ -m_{12}^2 - \frac{g'^2+g_2^2}{2}v_1v_2 & m_{12}^2\frac{v_1}{v_2} + \frac{g'^2+g_2^2}{2}v_2^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{4.13}$$

Using the identities [2]

$$\begin{aligned} m_A^2 &= m_{12}^2\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right), & (4.14)\\ m_Z^2 &= \frac{g'^2 + g_2^2}{2}v^2, & (4.15)\\ \cos 2\beta &= \cos^2\beta - \sin^2\beta & (4.16) \end{aligned}$$

and our definition for $\tan\beta$ we arrive at the matrix

$$\frac{1}{v^2}(x_1, x_2)\begin{pmatrix} m_A^2 v_2^2 + m_Z^2 v_1^2 & -(m_A^2 + m_z^2)v_1v_2 \\ -(m_A^2 + m_Z^2)v_1v_2 & m_A^2 v_1^2 + m_Z^2 v_2^2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{4.17}$$

from which the eigenvalues can best be read off:

$$\begin{aligned} m_H^2 &= \frac{m_A^2 + m_Z^2}{2} + \sqrt{\frac{(m_A^2 + m_Z^2)^2}{4} - m_Z^2 m_A^2 \cos^2 2\beta}, & (4.18)\\ m_h^2 &= \frac{m_A^2 + m_Z^2}{2} - \sqrt{\frac{(m_A^2 + m_Z^2)^2}{4} - m_Z^2 m_A^2 \cos^2 2\beta}. & (4.19) \end{aligned}$$

The mixing angle can be determined with the help of appendix A

$$\tan 2\alpha = -2\frac{(m_A^2 + m_Z^2)v_1v_2}{(m_A^2 - m_Z^2)(v_2^2 - v_1^2)}, \tag{4.20}$$

[2] The first equation for m_A^2 has to be taken as a definition until it will be shown in the next section that this is really the mass of the CP-odd Higgs A

so that the new fields are

$$\begin{aligned} H &:= \frac{1}{\sqrt{2}}(\cos\alpha x_1 + \sin\alpha x_2), & (4.21)\\ h &:= \frac{1}{\sqrt{2}}(-\sin\alpha x_1 + \cos\alpha x_2). & (4.22) \end{aligned}$$

As the orthogonal transformation of the mass matrix does not change the trace of the matrix, we find the *mass sum rule for the neutral Higgs bosons* at tree level

$$m_A^2 + m_Z^2 = m_H^2 + m_h^2. \tag{4.23}$$

It should be noted that the following relations between the masses hold

$$\begin{aligned} m_h &\leq m_A, & (4.24)\\ m_h &\leq m|\cos\beta| \leq m_Z, \ \text{with}\, m := \min(m_Z, m_A), & (4.25)\\ m_H &\geq m_Z. & (4.26) \end{aligned}$$

Thus, *the mass of the lightest Higgs is smaller than the mass of the Z-boson.* If this was true, the lightest Higgs should already have been obeserved at LEP and (as this was not the case) the MSSM would be ruled out. But note that this is only a *tree level* result. We will see in the following that large radiative corrections push the lightest Higgs mass to much higher values and thereby rescue the MSSM.

4.4.2 The CP-odd Higgs A and the neutral Goldstone boson G

The mass matrix for the imaginary part of the neutral Higgs contains also the Goldstone terms found in (4.11)

$$(y_1, y_2) \begin{pmatrix} m_1^2 + \frac{g'^2+g_2^2}{4}[(1-2\xi)v_1^2 - v_2^2] & m_{12}^2 + \xi\frac{g'^2+g_2^2}{2}v_1 v_2 \\ m_{12}^2 + \xi\frac{g'^2+g_2^2}{2}v_1 v_2 & m_2^2 + \frac{g'^2+g_2^2}{2}[(1-2\xi)v_2^2 - v_1^2] \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \tag{4.27}$$

Also after the use of the minimization conditions we find the nicer form

$$(y_1, y_2) \begin{pmatrix} m_{12}^2\frac{v_2}{v_1} - \frac{g'^2+g_2^2}{2}\xi v_1^2 & m_{12}^2 + \frac{g'^2+g_2^2}{2}\xi v_1 v_2 \\ m_{12}^2 + \frac{g'^2+g_2^2}{2}\xi v_1 v_2 & m_{12}^2\frac{v_1}{v_2} - \frac{g'^2+g_2^2}{2}\xi v_2^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{4.28}$$

with the eigenvalues

$$\begin{aligned} m_A^2 &= m_{12}^2\left(\frac{v_1}{v_2} + \frac{v_2}{v_1}\right), & (4.29)\\ m_G^2 &= -\xi m_Z^2. & (4.30) \end{aligned}$$

The corresponding eigenvectors are

$$m_A^2 \quad : \quad \begin{pmatrix} v_2 \\ v_1 \end{pmatrix} = \begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix}, \tag{4.31}$$

$$m_G^2 \quad : \quad \begin{pmatrix} -v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -\cos\beta \\ \sin\beta \end{pmatrix}, \tag{4.32}$$

so that with the appropriate normalization of the new fields

$$A \quad := \quad \frac{1}{\sqrt{2}}(\sin\beta y_1 + \cos\beta y_2), \tag{4.33}$$

$$G \quad := \quad \frac{1}{\sqrt{2}}(-\cos\beta y_1 + \sin\beta y_2). \tag{4.34}$$

Please note two things: First, we see that the mixing angle is the same β which we defined in the beginning of chapter 2. So the definition of the quantity $\frac{v_2}{v_1}$ as the tangent of an *angle* really makes sense. Second, we see that while the mass of the Goldstone boson depends on the gauge (i.e. on ξ), the eigenstates do not.

4.4.3 The charged Higgses $\mathbf{H}^{\pm}$ and the charged Goldstone Bosons $\mathbf{G}^{\pm}$

The mass matrix for the charged Higgses contains also the Goldstone terms found in (4.11)

$$\left(H_1^-, H_2^-\right) \begin{pmatrix} m_1^2 + \frac{g'^2+g_2^2}{4}(v_1^2 - v_2^2) + \frac{g_2^2}{2}(v_2^2 - \xi v_1^2) & m_{12}^2 + \frac{g_2^2}{2}(1+\xi)v_1 v_2 \\ m_{12}^2 + \frac{g_2^2}{2}(1+\xi)v_1 v_2 & m_2^2 + \frac{g'^2+g_2^2}{4}(v_2^2 - v_1^2) + \frac{g_2^2}{2}(v_1^2 - \xi v_2^2) \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix} \tag{4.35}$$

This is simplified after using the minimization conditions to

$$\left(H_1^-, H_2^-\right) \begin{pmatrix} m_{12}^2 \frac{v_2}{v_1} + \frac{g_2^2}{2}(v_2^2 - \xi v_1^2) & m_{12}^2 + \frac{g_2^2}{2}(1+\xi)v_1 v_2 \\ m_{12}^2 + \frac{g_2^2}{2}(1+\xi)v_1 v_2 & m_{12}^2 \frac{v_1}{v_2} + \frac{g_2^2}{2}(v_1^2 - \xi v_2^2) \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix} \tag{4.36}$$

with eigenvalues

$$m_{H^\pm}^2 \quad = \quad m_A^2 + m_W^2, \tag{4.37}$$

$$m_{G^\pm}^2 \quad = \quad -\xi m_W^2 \tag{4.38}$$

and eigenvectors

$$m_{H^\pm}^2 \quad : \quad \begin{pmatrix} v_2 \\ v_1 \end{pmatrix} = \begin{pmatrix} \sin\beta \\ \cos\beta \end{pmatrix}, \tag{4.39}$$

$$m_{G^\pm}^2 \quad : \quad \begin{pmatrix} -v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} -\cos\beta \\ \sin\beta \end{pmatrix}. \tag{4.40}$$

So here again, β is the mixing angle for these fields. The new eigenstates are

$$\begin{aligned} H^+ &:= \frac{1}{\sqrt{2}}(\sin\beta H_1^+ + \cos\beta H_2^+), \qquad (4.41) \\ G^+ &:= \frac{1}{\sqrt{2}}(-\cos\beta H_1^+ + \sin\beta H_2^+) \qquad (4.42) \end{aligned}$$

and the hermitean conjugate of these equations is valid for H^- and G^-. We note in passing that there is also a mass sum rule for the charged Higgses

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \qquad (4.43)$$

which in turn means that

$$m_{H^\pm} \geq m_W. \qquad (4.44)$$

This finnishes the discussion of the Higgs sector.

Chapter 5

Squarks, Sleptons, Gauginos and Higgsinos

5.1 Squarks

5.1.1 Top-Type Squarks (Stops)

The mass terms for top-type squarks appear in the D-terms, the F-terms and the soft breaking-terms:

$$
\begin{aligned}
\mathcal{L}_{\tilde{t}}^{\text{mass}} &= +\frac{g_2^2}{8}\left(4|\tilde{u}_L|^2 v_1^2 - 2|\tilde{u}_L|^2 v_1^2 - 2|\tilde{u}_L|^2 v_2^2\right) \\
&\quad +\frac{g'^2}{8}\left(-\frac{2}{3}|\tilde{u}_L|^2 v_1^2 + \frac{2}{3}|\tilde{u}_L|^2 v_2^2 + \frac{8}{3}|\tilde{u}_R|^2 v_1^2 - \frac{8}{3}|\tilde{u}_R|^2 v_2^2\right) \\
&\quad -h_t\mu^*\tilde{u}_L\tilde{u}_R v_1 - \mu h_t^*\tilde{u}_L^*\tilde{u}_R^* v_1 + |h_t|^2|\tilde{u}_R|^2 v_2^2 + |h_t|^2|\tilde{u}_L|^2 v_2^2 \\
&\quad +M_{\tilde{Q}}^2|\tilde{t}_L|^2 + M_{\tilde{U}}^2|\tilde{t}_R|^2 + \frac{g}{\sqrt{2}}\frac{m_t A_t}{m_W \sin\beta}(v_2\tilde{t}_L\tilde{t}_R^* + \text{h.c.})
\end{aligned}
\tag{5.1}
$$

so that the mass matrix is

$$
(\tilde{t}_L^*, \tilde{t}_R^*)\begin{pmatrix} M_{\tilde{Q}}^2 + \frac{g_2^2+g'^2}{4}(v_1^2 - v_2^2) - \frac{g'^2}{3}(v_1^2 - v_2^2) + |h_t|^2 v_2^2 & \frac{g_2}{\sqrt{2}}\frac{m_t A_t}{m_W \sin\beta}v_2 - h_t\mu v_1 \\ \frac{g_2}{\sqrt{2}}\frac{m_t A_t}{m_W \sin\beta}v_2 - h_t\mu v_1 & M_{\tilde{U}}^2 + \frac{g'^2}{3}(v_1^2 - v_2^2) + |h_t|^2 v_2^2 \end{pmatrix}\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}.
\tag{5.2}
$$

Using the usual identities this can be brought into the nicer form

$$
(\tilde{t}_L^*, \tilde{t}_R^*)\begin{pmatrix} M_{\tilde{Q}}^2 + m_Z^2\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)\cos 2\beta + m_t^2 & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & M_{\tilde{U}}^2 + \frac{2}{3}m_Z^2\sin^2\theta_W\cos 2\beta + m_t^2 \end{pmatrix}\begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}.
\tag{5.3}
$$

The eigenvalues of this matrix are found with the help of appendix A

$$
\begin{aligned}
m_{\tilde{t}}^2 &= \frac{1}{2}(2m_t^2 + M_{\tilde{Q}}^2 + M_{\tilde{U}}^2 + \frac{1}{2}m_Z^2\cos 2\beta) \\
&\pm\frac{1}{2}\sqrt{[M_{\tilde{Q}}^2 - M_{\tilde{U}}^2 + (\frac{1}{2} - \frac{4}{3}\sin^2\theta_W)m_Z^2\cos 2\beta]^2 + 4m_t^2(A_t - \mu\cot\beta)^2}
\end{aligned} \tag{5.4}
$$

Note that we have *two* squarks, and they are in general not degenerate in mass.

5.1.2 Bottom-Type Squarks (Sbottoms)

The mass terms for bottom-type squarks appear in the D-terms, the F-terms and the soft breaking-terms:

$$
\begin{aligned}
\mathcal{L}_{\tilde{b}}^{\text{mass}} &= +\frac{g_2^2}{8}\left(-2|\tilde{d}_L|^2v_1^2 + 4|\tilde{d}_L|^2v_2^2 - 2|\tilde{d}_L|^2v_2^2\right) \\
&+\frac{g'^2}{8}\left(-\frac{2}{3}|\tilde{d}_L|^2v_1^2 + \frac{2}{3}|\tilde{d}_L|^2v_2^2 - \frac{4}{3}|\tilde{d}_R|^2v_1^2 + \frac{4}{3}|\tilde{d}_R|^2v_2^2\right) \\
&-h_b\mu^*\tilde{d}_L\tilde{d}_R^*v_2 - \mu h_b^*\tilde{d}_L^*\tilde{d}_Rv_2 + |h_b|^2|\tilde{d}_R|^2v_1^2 + |h_b|^2|\tilde{d}_L|^2v_1^2 \\
&+M_{\tilde{Q}}^2|\tilde{b}_L|^2 + M_{\tilde{D}}^2|\tilde{b}_R|^2 + \frac{g_2}{\sqrt{2}}\frac{m_bA_b}{m_W\cos\beta}(v_1\tilde{b}_L\tilde{b}_R^* + \text{h.c.}),
\end{aligned} \tag{5.5}
$$

so that the mass matrix is

$$
\left(\tilde{b}_L^*, \tilde{b}_R^*\right)\begin{pmatrix} M_{\tilde{Q}}^2 + \frac{g_2^2+g'^2}{4}(v_1^2+v_2^2) + \frac{g'^2}{6}(v_1^2-v_2^2) + |h_b|^2v_1^2 & \frac{g_2}{\sqrt{2}}\frac{m_bA_b}{m_W\sin\beta}v_1 - h_b\mu v_2 \\ \frac{g_2}{\sqrt{2}}\frac{m_bA_b}{m_W\sin\beta}v_1 - h_b\mu v_2 & M_{\tilde{D}}^2 + \frac{g'^2}{6}(-v_1^2+v_2^2) + |h_b|^2v_1^2 \end{pmatrix}\begin{pmatrix}\tilde{b}_L \\ \tilde{b}_R\end{pmatrix}. \tag{5.6}
$$

The matrix entries can be rewritten in the nicer form

$$
\left(\tilde{b}_L^*, \tilde{b}_R^*\right)\begin{pmatrix} M_{\tilde{Q}}^2 - m_Z^2\left(\frac{1}{2}-\frac{1}{3}\sin^2\theta_W\right)\cos 2\beta + m_b^2 & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & M_{\tilde{D}}^2 - \frac{1}{3}m_Z^2\sin^2\theta_W\cos 2\beta + m_b^2 \end{pmatrix}\begin{pmatrix}\tilde{b}_L \\ \tilde{b}_R\end{pmatrix}. \tag{5.7}
$$

Diagonalizing this matrix gives as mass-squareds the two eigenvalues

$$
\begin{aligned}
m_{\tilde{b}}^2 &= \frac{1}{2}(2m_b^2 + M_{\tilde{Q}}^2 + M_{\tilde{D}}^2 - \frac{1}{2}m_Z^2\cos 2\beta) \\
&\pm\frac{1}{2}\sqrt{[M_{\tilde{Q}}^2 - M_{\tilde{D}}^2 - (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)m_Z^2\cos 2\beta]^2 + 4m_b^2(A_b - \mu\tan\beta)^2}.
\end{aligned} \tag{5.8}
$$

5.2 Sleptons

5.2.1 Sneutrinos

The sneutrinos receive contributions from the D-terms and the soft breaking-terms

$$\begin{aligned}
\mathcal{L}^{\text{mass}}_{\tilde{\nu}} &= +\frac{g_2^2}{8}\left(4|\tilde{\nu}|^2 v_1^2 - 2|\tilde{\nu}|^2 v_1^2 - 2|\tilde{\nu}|^2 v_2^2\right) \\
&\quad +\frac{g'^2}{8}(2|\tilde{\nu}|^2 v_1^2 - 2|\tilde{\nu}|^2 v_2^2) \\
&\quad +M_{\tilde{L}}^2|\tilde{\nu}|^2. \qquad (5.9)
\end{aligned}$$

As there is no right-handed sneutrino with which a mixing could occur, we can directly read off the mass:

$$\begin{aligned}
m_{\tilde{\nu}}^2 &= M_{\tilde{L}}^2 + \frac{g_2^2}{4}(v_1^2 - v_2^2) + \frac{g'^2}{4}(v_1^2 - v_2^2) \\
&= M_{\tilde{L}}^2 + \frac{1}{2}m_Z^2 \cos 2\beta. \qquad (5.10)
\end{aligned}$$

5.2.2 Tau-Type Sleptons (Staus)

Mass terms are found in the the D-terms, F-terms and soft breaking-terms.

$$\begin{aligned}
\mathcal{L}^{\text{mass}}_{\tilde{\tau}} &= +\frac{g_2^2}{8}\left(-2|\tilde{e}_L|^2 v_1^2 + 4|\tilde{e}_L|^2 v_2^2 - 2|\tilde{e}_L|^2 v_2^2\right) \\
&\quad +\frac{g'^2}{8}\left(2|\tilde{e}_L|^2 v_1^2 - 2|\tilde{e}_L|^2 v_2^2 - 4|\tilde{e}_R|^2 v_1^2 + 4|\tilde{e}_R|^2 v_2^2\right) \\
&\quad -h_\tau \mu^* \tilde{e}_L \tilde{e}_R^* v_2 - \mu h_\tau^* \tilde{e}_L^* \tilde{e}_R v_2 + |h_\tau|^2|\tilde{e}_R|^2 v_1^2 + |h_\tau|^2|\tilde{e}_L|^2 v_1^2 \\
&\quad +M_{\tilde{L}}^2|\tilde{\tau}_L|^2 + M_{\tilde{E}}^2|\tilde{\tau}_R|^2 + \frac{g}{\sqrt{2}}\frac{m_\tau A_\tau}{m_W \cos\beta}(v_1 \tilde{\tau}_L \tilde{\tau}_R^* + \text{h.c.}). \qquad (5.11)
\end{aligned}$$

Then the mass matrix is

$$(\tilde{\tau}_L^*, \tilde{\tau}_R^*)\begin{pmatrix} M_{\tilde{L}}^2 + \frac{g_2^2+g'^2}{4}(-v_1^2+v_2^2) + \frac{g'^2}{4}(v_1^2 - v_2^2) + |h_\tau|^2 v_1^2 & \frac{g_2}{\sqrt{2}}\frac{m_\tau A_\tau}{m_W \cos\beta}v_1 - h_\tau \mu v_2 \\ \frac{g_2}{\sqrt{2}}\frac{m_\tau A_\tau}{m_W \cos\beta}v_1 - h_\tau \mu v_2 & M_{\tilde{E}}^2 + \frac{g'^2}{2}(-v_1^2 + v_2^2) + |h_\tau|^2 v_1^2 \end{pmatrix}\begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}. \qquad (5.12)$$

In a nicer form this is

$$(\tilde{\tau}_L^*, \tilde{\tau}_R^*)\begin{pmatrix} M_{\tilde{L}}^2 + m_\tau^2 - m_Z^2\left(\frac{1}{2} - \sin^2\theta_W\right)\cos 2\beta & m_\tau(A_\tau - \mu\tan\beta) \\ m_\tau(A_\tau - \mu\tan\beta) & M_{\tilde{E}}^2 + m_\tau^2 - m_Z^2 \sin^2\theta_W \cos(2\beta) \end{pmatrix}\begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}. \qquad (5.13)$$

The eigenvalues are:

$$\begin{aligned}
m_{\tilde{\tau}}^2 &= \frac{1}{2}(2m_\tau^2 + M_{\tilde{L}}^2 + M_{\tilde{E}}^2 - \frac{1}{2}m_Z^2\cos 2\beta) \qquad (5.14) \\
&\quad \pm\frac{1}{2}\sqrt{[M_{\tilde{L}}^2 - M_{\tilde{E}}^2 - (\frac{1}{2} - 2\sin^2\theta_W)m_Z^2\cos 2\beta]^2 + 4m_\tau^2(A_\tau - \mu\tan\beta)^2}.
\end{aligned}$$

5.3 Neutralinos

The neutral gauginos ($\tilde{W}^3$ and $\tilde{B}$) and the neutral higgsinos ($\tilde{H}_1^-$ and $\tilde{H}_2^0$) have the same quantum numbers and mix to new mass eigenstates, which we then call *neutralinos*. The neutralino mass terms appear in the SUSY-part, the Yukawa-part and the soft breaking-terms.

$$\begin{aligned}\mathcal{L}^{\text{mass}}_{\text{neutralinos}} &= -\frac{g'}{\sqrt{2}}v_1\bar{\tilde{B}}P_L\tilde{H}_1^0 + \frac{g_2}{\sqrt{2}}v_1\bar{\tilde{W}}^3P_L\tilde{H}_1^0 + \text{h.c.}\\ &\quad +\frac{g'}{\sqrt{2}}v_2\bar{\tilde{B}}P_L\tilde{H}_2^0 - \frac{g_2}{\sqrt{2}}v_2\bar{\tilde{W}}^3P_L\tilde{H}_2^0 + \text{h.c.}\\ &\quad -\mu\overline{\tilde{H}_1^{0c}}P_L\tilde{H}_2^0 + \text{h.c.}\\ &\quad +\frac{1}{2}M_2\bar{\tilde{W}}^3\tilde{W}^3 + \frac{1}{2}M_1\bar{\tilde{B}}\tilde{B}\end{aligned} \tag{5.15}$$

Note that there is *no* +h.c. missing in the last line. This comes in when we apply the identities (A.10) to write this part of the Lagrangian in two-component WEYL-spinors:

$$\begin{aligned}\mathcal{L}^{\text{mass}}_{\text{neutralinos}} &= -\frac{g'}{\sqrt{2}}v_1\tilde{b}\tilde{h}_1^0 + \frac{g_2}{\sqrt{2}}v_1\tilde{w}^3\tilde{h}_1^0 + \text{h.c.}\\ &\quad +\frac{g'}{\sqrt{2}}v_2\tilde{b}\tilde{h}_2^0 - \frac{g_2}{\sqrt{2}}v_2\bar{\tilde{w}}^3\tilde{h}_2^0 + \text{h.c.}\\ &\quad -\mu\tilde{h}_1^{0c}\tilde{h}_2^0 + \text{h.c.}\\ &\quad +\frac{1}{2}M_2\tilde{w}^3\tilde{w}^3 + \frac{1}{2}M_1\tilde{b}\tilde{b} + \text{h.c.}\end{aligned} \tag{5.16}$$

$$= \left(\tilde{b}, \tilde{w}^3, \tilde{h}_1^0, \tilde{h}_2^0\right)\begin{pmatrix} \frac{M_1}{2} & 0 & -\frac{g'}{\sqrt{2}}v_1 & \frac{g'}{\sqrt{2}}v_2\\ 0 & \frac{M_2}{2} & \frac{g_2}{\sqrt{2}}v_1 & -\frac{g_2}{\sqrt{2}}v_2\\ -\frac{g'}{\sqrt{2}}v_1 & \frac{g_2}{\sqrt{2}}v_1 & 0 & -\mu\\ \frac{g'}{\sqrt{2}}v_2 & -\frac{g_2}{\sqrt{2}}v_2 & -\mu & 0\end{pmatrix}\begin{pmatrix}\tilde{b}\\ \tilde{w}^3\\ \tilde{h}_1^0\\ \tilde{h}_2^0\end{pmatrix} + \text{h.c.} \quad . \tag{5.17}$$

The matrix can also be written as

$$\frac{1}{2}\begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_W\cos\beta & m_Z\sin\theta_W\sin\beta\\ 0 & M_2 & m_Z\cos\theta_W\cos\beta & -m_Z\cos\theta_W\sin\beta\\ -m_Z\sin\theta_W\cos\beta & m_Z\cos\theta_W\cos\beta & 0 & -\mu\\ m_Z\sin\theta_W\sin\beta & -m_Z\cos\theta_W\sin\beta & -\mu & 0\end{pmatrix}. \tag{5.18}$$

5.4 Charginos

The charged gauginos ($\tilde{W}^{\pm}$, which are linear combinations of $\tilde{W}^1$ and $\tilde{W}^2$) and the charged higgsinos ($\tilde{H}^{\pm}$) have the same quantum numbers and mix to new mass eigenstates, which are then called *charginos*. Their fermionic mass terms can be found in the SUSY-part, the Yukawa-part and the soft breaking-terms.

$$\begin{aligned}\mathcal{L}^{\text{mass}}_{\text{charginos}} &= +\frac{g_2}{\sqrt{2}}v_1\overline{\tilde{W}^-}P_L\tilde{H}_1^- + \text{h.c.}\\ &\quad +\frac{g_2}{\sqrt{2}}v_2\overline{\tilde{W}^+}P_L\tilde{H}_2^+ + \text{h.c.}\\ &\quad +\mu\overline{\tilde{H}_1^{-c}}P_L\tilde{H}_2^+ + \text{h.c.}\\ &\quad +\frac{1}{2}M_2(\bar{\tilde{W}}^1\tilde{W}^1 + \bar{\tilde{W}}^2\tilde{W}^2).\end{aligned} \tag{5.19}$$

Note that there is *no* +h.c. missing in the last line. This comes in when we write the Lagrangian in two component WEYL-spinors[1]:

$$\begin{aligned}\mathcal{L}_{\text{charginos}} &= \frac{g_2}{\sqrt{2}}v_1\tilde{w}^+\tilde{h}_1^- + \text{h.c.}\\ &\quad +\frac{g_2}{\sqrt{2}}v_2\tilde{w}^-\tilde{h}_2^+ + \text{h.c}\\ &\quad +\mu\tilde{h}_1^-\tilde{h}_2^+ + \text{h.c.}\\ &\quad +\frac{1}{2}M_2\left(\tilde{w}^1\tilde{w}^1 + \tilde{w}^2\tilde{w}^2\right) + \text{h.c.},\end{aligned} \tag{5.20}$$

where the last line can be rewritten into

$$+M_2\tilde{w}^+\tilde{w}^- + \text{h.c.} \quad . \tag{5.21}$$

So the chargino mass matrix is

$$\left(\tilde{w}^-, \tilde{h}_1^-\right)\begin{pmatrix} M_2 & \frac{g_2}{\sqrt{2}}v_2 \\ \frac{g_2}{\sqrt{2}}v_1 & \mu \end{pmatrix}\begin{pmatrix} \tilde{w}^+ \\ \tilde{h}_2^+ \end{pmatrix} + \text{h.c.} \tag{5.22}$$

and the matrix can also be written as

$$\begin{pmatrix} M_2 & \sqrt{2}m_W\sin\beta \\ \sqrt{2}m_W\cos\beta & \mu \end{pmatrix}. \tag{5.23}$$

If we denote this matrix by X, the mass (-squared) matrix is given by

[1] For the notation, please see (A.13) and (A.14) in appendix A.

$$\begin{aligned} m^2_{\text{chargino}} &= X^\dagger X \qquad (5.24) \\ &= \begin{pmatrix} M_2^2 + 2m_W^2 \cos^2\beta & M_2\sqrt{2}m_W \sin\beta + \mu\sqrt{2}m_W \cos\beta \\ M_2\sqrt{2}m_W \sin\beta + \mu\sqrt{2}m_W \cos\beta & 2m_W^2 \sin^2\beta \end{pmatrix} \end{aligned}$$

and the eigenvalues are

$$\begin{aligned} m^2_{1,2} &= \frac{1}{2}(M_2^2 + \mu^2 + 2m_W^2) \qquad (5.25) \\ &\pm\sqrt{(M_2^2 - \mu^2)^2 + 4m_W^4 \cos^2 2\beta + 4m_W^2(M^2 + \mu^2 + 2M_2\mu \sin 2\beta)}. \end{aligned}$$

Chapter 6

Radiative Corrections To The Charge Neutral Higgs Boson Mass Sum Rule

We have seen that the *tree level* mass of the light CP-even Higgs boson h in the MSSM is below the mass of the Z boson. If the tree level mass was the whole mass, then the h boson should have been seen at LEP. As this was not the case, the MSSM would have been ruled out. But in the 1990's, physicists [14, 15, 16] found out that there are huge radiative corrections to the mass of the lightest Higgs, which push the numerical value over the critical limit into the yet unobserved region, thereby restoring the hope in the validity of the MSSM.

In the preceding chapters, we have displayed the tree level mass spectrum of the MSSM. Especially we found with formula (4.23) the tree level mass sum rule for the charge neutral Higgses. Following the argumentation of [10] we show how a formalism for the calculation of *radiative corrections to the mass sum rule* is established. In chapter 7 we are going to discuss that a convenient approximation for the correction lies in the evaluation of one-loop top/stop and bottom/sbottom diagrams and use the correction to the mass sum rule to estimate the shift in the mass of the lightest Higgs.

6.1 Renormalization in the Presence Of Mixing

Due to the existence of particles with the same quantum numbers it comes to a *mixing* which could already be seen in the derivation of the mass spectrum. Thus, the quantities that appear in the renormalization programme take on a matrix form. An excellent overview on these matters can be found in [11].

CP even sector

The *bare* Lagrangian for the real part of the charge neutral Higgses is given by

$$\frac{1}{2}\partial_\mu\,(x_1,x_2)_b\,\partial^\mu \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_b - \frac{1}{2}\,(x_1,x_2)_b \begin{pmatrix} A & B \\ B & C \end{pmatrix}_b \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_b, \tag{6.1}$$

where

$$\begin{aligned} A_b &= (m_1^2)_b + \tfrac{1}{2}(m_Z^2)_b \left(\frac{3v_1^2 - v_2^2}{v_1^2+v_2^2}\right)_b, \\ B_b &= -(m_{12}^2)_b - (m_Z^2)_b \left(\frac{v_1^2 v_2^2}{v_1^2+v_2^2}\right)_b, \\ C_b &= (m_2^2)b + \tfrac{1}{2}(m_Z^2)_b \left(\frac{3v_2^2 - v_1^2}{v_1^2+v_2^2}.\right)_b \end{aligned} \tag{6.2}$$

Now we start the renormalization procedure by splitting up the bare terms into renormalized quantities and counterterms.

$$\begin{aligned} (m_1^2)_b &= (m_1^2)_r + \delta m_1^2, \\ (m_2^2)_b &= (m_2^2)_r + \delta m_2^2, \\ (m_{12}^2)_b &= (m_{12}^2)_r + \delta m_1^2, \\ (m_Z^2)_b &= (m_Z^2)_r + \delta m_Z^2, \\ (v_1)_b &= (v_1)_r + \delta v_1, \\ (v_2)_b &= (v_2)_r + \delta v_1. \end{aligned} \tag{6.3}$$

This generates also a shift in coefficients of the mass matrix

$$\begin{aligned} A_b &= A_r + \delta A, \\ B_b &= B_r + \delta B, \\ C_b &= C_r + \delta C. \end{aligned} \tag{6.4}$$

Then we have

$$\frac{1}{2}\partial_\mu\,(x_1,x_2)_b\,\partial^\mu \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_b - \frac{1}{2}\,(x_1,x_2)_b \begin{pmatrix} A & B \\ B & C \end{pmatrix}_r \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_b - \frac{1}{2}\,(x_1,x_2)_b \begin{pmatrix} \delta A & \delta B \\ \delta B & \delta C \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_b. \tag{6.5}$$

Next we rotate into the mass eigenstate basis

$$
\begin{aligned}
&\tfrac{1}{2}\partial_\mu\,(H,h)_b\,\partial^\mu \begin{pmatrix} H \\ h \end{pmatrix}_b - \tfrac{1}{2}\,(H,h)_b \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}_r \begin{pmatrix} H \\ h \end{pmatrix}_b \\
&\qquad -\tfrac{1}{2}\,(H,h)_b \begin{pmatrix} \delta m_H^2 & \delta m_{Hh}^2 \\ \delta m_{hH}^2 & \delta m_h^2 \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}_b ,
\end{aligned}
\tag{6.6}
$$

where the entries of the diagonalized renormalized mass matrix are

$$
\begin{aligned}
(m_H^2)_r &= \tfrac{1}{2}\left[A_r + C_r + \sqrt{(A_r - C_r)^2 + 4B_r^2}\right], \\
(m_h^2)_r &= \tfrac{1}{2}\left[A_r + C_r + \sqrt{(A_r - C_r)^2 + 4B_r^2}\right]
\end{aligned}
\tag{6.7}
$$

and the counterterm matrix is then calculated to be

$$
\begin{aligned}
\begin{pmatrix} \delta m_H^2 & \delta m_{Hh}^2 \\ \delta m_{hH}^2 & \delta m_h^2 \end{pmatrix} &= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \delta A & \delta B \\ \delta B & \delta C \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \\
&= \begin{pmatrix} \delta A\cos^2\alpha + \delta B\sin 2\alpha + \delta C\sin^2\alpha & (\delta C - \delta A)\sin\alpha\cos\alpha + \delta B\cos 2\alpha \\ (\delta C - \delta A)\sin\alpha\cos\alpha + \delta B\cos 2\alpha & \delta A\sin^2\alpha - \delta B\sin 2\alpha + \delta C\cos^2\alpha \end{pmatrix}.
\end{aligned}
\tag{6.8}
$$

We pause the renormalization programme shortly to derive a useful expression for later use: From the above expressions it can be seen that

$$
\begin{aligned}
\delta m_H^2 + \delta m_h^2 &= \delta A + \delta C \\
&= (A_b - A_r) + (C_b - C_r) \\
&= \delta m_1^2 + \delta m_2^2 + \delta m_Z^2.
\end{aligned}
\tag{6.9}
$$

We have not yet carried out the wavefunction renormalization which also takes a matrix form

$$
\begin{aligned}
&\tfrac{1}{2}\partial_\mu\,(H,h)_r\,[Z_s^{1/2}]^T[Z_s^{1/2}]\partial^\mu \begin{pmatrix} H \\ h \end{pmatrix}_r - \tfrac{1}{2}\,(H,h)_r\,[Z_s^{1/2}]^T \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}_r [Z_s^{1/2}] \begin{pmatrix} H \\ h \end{pmatrix}_r \\
&\qquad -\tfrac{1}{2}\,(H,h)_r\,[Z_s^{1/2}]^T \begin{pmatrix} \delta m_H^2 & \delta m_{Hh}^2 \\ \delta m_{hH}^2 & \delta m_h^2 \end{pmatrix} [Z_s^{1/2}] \begin{pmatrix} H \\ h \end{pmatrix}_r
\end{aligned}
\tag{6.10}
$$

with

$$
Z_s^{1/2} = \begin{pmatrix} Z_{HH}^{1/2} & Z_{Hh}^{1/2} \\ Z_{hH}^{1/2} & Z_{hh}^{1/2} \end{pmatrix}.
\tag{6.11}
$$

Note that the diagonal elements $Z_{HH}^{1/2}$ and $Z_{hh}^{1/2}$ are of the order $1+\mathcal{O}(\alpha)$ and the off-diagonal elements $Z_{Hh}^{1/2}$ and $Z_{hH}^{1/2}$ are newly generated due to the mixing, so that they are of $\mathcal{O}(\alpha)$. When we now drop terms of second order in perturbation theory (i.e. $\mathcal{O}(\alpha^2)$), the last term simplifies a bit, so that we get

$$\frac{1}{2}\partial_\mu\,(H,h)_r\,[Z_s^{1/2}]^T[Z_s^{1/2}]\partial^\mu\begin{pmatrix}H\\h\end{pmatrix}_r-\frac{1}{2}\,(H,h)_r\,[Z_s^{1/2}]^T\begin{pmatrix}m_H^2&0\\0&m_h^2\end{pmatrix}_r[Z_s^{1/2}]\begin{pmatrix}H\\h\end{pmatrix}_r$$
$$-\frac{1}{2}\,(H,h)_r\begin{pmatrix}\delta m_H^2&\delta m_{Hh}^2\\\delta m_{hH}^2&\delta m_h^2\end{pmatrix}\begin{pmatrix}H\\h\end{pmatrix}_r. \qquad (6.12)$$

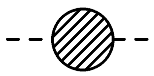
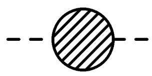
The propagator can be defined as the inverse of the operator in the quadratic terms in the Lagrangian. Thus, we can simply read off the *inverse propagator*. We replace $\partial_\mu\to -ip_\mu$ and gain

$$\begin{aligned}i\Gamma_s(p^2)&=\begin{pmatrix}i\Gamma_{HH}(p^2)&i\Gamma_{Hh}(p^2)\\i\Gamma_{hH}(p^2)&i\Gamma_{hh}(p^2)\end{pmatrix} \qquad (6.13)\\&=[Z_s^{1/2}]^T[Z_s^{1/2}]p^2-[Z_s^{1/2}]^T\begin{pmatrix}(m_H^2)_r&0\\0&(m_h^2)_r\end{pmatrix}[Z_s^{1/2}]-\begin{pmatrix}\delta m_H^2&\delta m_{Hh}^2\\\delta m_{hH}^2&\delta m_h^2\end{pmatrix},\end{aligned}$$

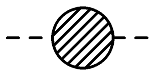
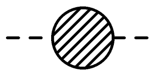
Note that although renormalized quantities appear in the inverse propagator, this is *still* the *bare* inverse propagator from the beginning, just rewritten into renormalized quantities and counterterms. To get from the inverse propagator to the *renormalized* inverse propagator, we have to add the self-energies to all matrix entries:

$$i\Gamma_s(p^2)=\begin{pmatrix}i\Gamma_{HH}&i\Gamma_{Hh}\\i\Gamma_{hH}&i\Gamma_{hh}\end{pmatrix}\longrightarrow i\tilde{\Gamma}_s(p^2)=\begin{pmatrix}i\Gamma_{HH}+\Pi_{HH}&i\Gamma_{Hh}+\Pi_{Hh}\\i\Gamma_{hH}+\Pi_{hH}&i\Gamma_{hh}+\Pi_{hh}\end{pmatrix} \qquad (6.14)$$

where

$i\Pi_{HH}:=$ H  H , $i\Pi_{Hh}:=$ H  h,

(6.15)

$i\Pi_{hH}:=$ h  H , $i\Pi_{hh}:=$ h  h

and the blobs denote the usual one particle irreducible (1PI) self-energies. The matrix entries of the renormalized inverse propagator are

$$
\begin{aligned}
i\tilde{\Gamma}_{HH}(p^2) &= (Z_{HH}+Z_{hH})p^2-(m_H^2)_r Z_{HH}-(m_h^2)_r Z_{hH}-\delta m_H^2+\Pi_{HH}(p^2),\\
i\tilde{\Gamma}_{hh}(p^2) &= (Z_{hh}+Z_{Hh})p^2-(m_h^2)_r Z_{hh}-(m_H^2)_r Z_{Hh}-\delta m_h^2+\Pi_{hh}(p^2),\\
i\tilde{\Gamma}_{Hh}(p^2) &= (Z_{HH}^{1/2}Z_{Hh}^{1/2}+Z_{hh}^{1/2}Z_{hH}^{1/2})p^2-(m_H^2)_r Z_{HH}^{1/2}Z_{Hh}^{1/2}-(m_h^2)_r Z_{hh}^{1/2}Z_{hH}^{1/2},\\
&\quad -\delta m_{Hh}^2+\Pi_{Hh}(p^2).\\
i\tilde{\Gamma}_{hH}(p^2) &= i\tilde{\Gamma}_{Hh}(p^2)
\end{aligned}
\tag{6.16}
$$

CP odd sector

The same analysis is carried out in the CP odd sector. The noteworthy differences are that the matrix entries are now given by

$$
\begin{aligned}
A'_b &= (m_1^2)_b+\tfrac{1}{2}(m_Z^2)_b\left(\frac{[1-\xi]v_1^2-v_2^2}{v_1^2+v_2^2}\right)_b,\\
B'_b &= -(m_{12}^2)_b-(m_Z^2)_b\left(\frac{v_1^2v_2^2}{v_1^2+v_2^2}\right)_b,\\
C'_b &= (m_2^2)b+\tfrac{1}{2}(m_Z^2)_b\left(\frac{[1-\xi]v_2^2-v_1^2}{v_1^2+v_2^2}\right)_b,
\end{aligned}
\tag{6.17}
$$

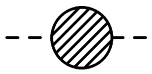
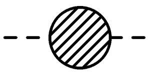
so that after the rotation into the mass eigenstate basis and in Landau gauge (i.e. $\xi=0$) the corresponding calculation to the one which led to (6.9) yields

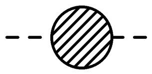
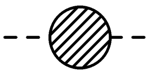
$$\delta m_A^2+\delta m_G^2=\delta m_1^2+\delta m_2^2. \tag{6.18}$$

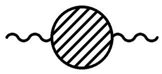
We conclude from (6.9) and (6.18) that

$$\delta m_H^2+\delta m_h^2=\delta m_G^2+\delta m_A^2+\delta m_Z^2. \tag{6.19}$$

In order to build the renormalized inverse propagator in the CP-odd sector, we define the 1PI-self-energies

$i\Pi_{AA} :=$ A  A , $i\Pi_{AG} :=$ A  G,

$i\Pi_{GA} :=$ G  A , $i\Pi_{GG} :=$ G  G, (6.20)

$i\Pi_{hH} :=$ Z  Z .

The self-energy for the Z boson has the general structure

$$\Pi_{ZZ}^{\mu\nu}=\mathcal{A}_{ZZ}g^{\mu\nu}+\mathcal{B}_{ZZ}p^{\mu}p^{\nu}. \tag{6.21}$$

We define

$$A_{ZZ} := Re(\mathcal{A}_{ZZ}). \tag{6.22}$$

6.2 On-Shell Renormalization

The connection to the *physical quantities* is established by the *renormalization conditions.* We use the socalled *on-shell renormalization scheme.* The renormalization conditions read:

1. The renormalized mass is the real part of the pole of the propagator.

2. The real part of the residue of the pole is equal to one.

Mathematically speaking, the conditions in the CP even sector are

$$\begin{aligned} i\tilde{\Gamma}_{HH}(m_H^2) = 0, &\qquad i\tilde{\Gamma}_{Hh}(m_H^2) = 0, \\ i\tilde{\Gamma}_{Hh}(m_h^2) = 0, &\qquad i\tilde{\Gamma}_{hh}(m_h^2) = 0, \\ i\frac{d\tilde{\Gamma}_{HH}}{d(p^2)}(m_H^2) = 1, &\qquad i\frac{d\tilde{\Gamma}_{hh}}{d(p^2)}(m_h^2) = 1. \end{aligned} \tag{6.23}$$

In addition to these conditions we impose that the renormalized $(m_H)_r$ is equal to the physical mass. Then the evaluation of $i\tilde{\Gamma}_{HH}(p^2)$ at $p^2 = m_H^2$ gives with the help of (6.23)

$$i\tilde{\Gamma}_{HH}(m_H^2) \quad = \quad Z_{hH}[m_H^2 - (m_h^2)_r] - \delta m_H^2 + \Pi_{HH}(m_H^2) = 0. \tag{6.24}$$

As Z_{hH} is of $\mathcal{O}(\alpha^2)$, we can drop it and have

$$\delta m_H^2 = \Pi_{HH}(m_H^2). \tag{6.25}$$

In the CP odd sector we also impose that $(m_A)_r$ and $(m_G)_r$ are equal to their physical masses and get in the same way as in the CP even sector

$$\delta m_A^2 \quad = \quad \Pi_{AA}(m_A^2) \tag{6.26}$$

$$\delta m_A^2 \quad = \quad \Pi_{GG}(0) \tag{6.27}$$

The renormalized quantities again fulfill the tree level mass sum rule, because they have the same structure as the bare masses as can be seen from (6.7), while the physical quantities satisfy the radiatively corrected mass sum rule

$$(m_h^2)_r - (m_H^2)_r \quad = \quad (m_A^2)_r + (m_Z^2)_r, \tag{6.28}$$

$$m_h^2 + m_H^2 \quad = \quad m_A^2 + m_Z^2 + \Delta. \tag{6.29}$$

Thus,

$$m_h^2 \quad = \quad (m_h^2)_r + \Delta \tag{6.30}$$

is valid. In the same way

$$\delta m_Z^2 = -A_{ZZ}(m_Z^2) \tag{6.31}$$

holds. Therefore, the evaluation of $i\tilde{\Gamma}_{hh}(p^2)$ at $p^2 = m_h^2$, which must be zero according to (6.23) leads to

$$\delta m_h^2 \quad = \quad \Pi_{hh}(m_h^2) + \Delta, \tag{6.32}$$

which is again true to $\mathcal{O}(\alpha)$. Finally we substitute (6.25), (6.26), (6.27), (6.31) and (6.32) into (6.19) and receive the result for the correction to the mass sum rule

$$\Delta = -\Pi_{HH}(m_H^2) - \Pi_{hh}(m_h^2) + \Pi_{AA}(m_A^2) + \Pi_{GG}(0) - A_{ZZ}(m_Z^2). \tag{6.33}$$

Chapter 7

Evaluation of the Correction to the Mass Sum Rule

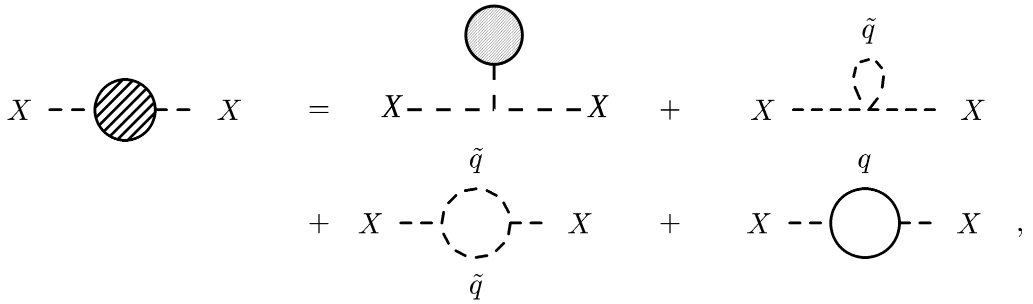
In this chapter we will derive an approximation for the correction to the mass sum rule at one loop. To obtain the leading correction, loop diagrams with tops and stops seem to be most promising, because their couplings to the Higgses are proportional to the large mass of the top quark of about 175 GeV. A second big contribution, but lower than the first one, could come from bottom/sbottom loops. Their couplings are proportional to the bottom mass of 4.25 GeV.

7.1 Calculation of the Correction due to Top/Stop Loops

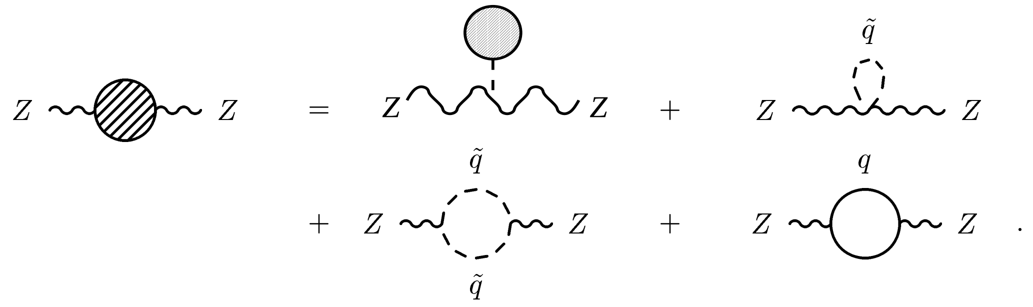
The formula for the correction Δ was given by equation (6.33) of chapter 6

$$\Delta = -\Pi_{HH}(m_H^2) - \Pi_{hh}(m_h^2) + \Pi_{AA}(m_A^2) + \Pi_{GG}(0) - A_{ZZ}(m_Z^2). \tag{7.1}$$

The idea is now to find all self-energy diagrams which contain one loop with quarks or squarks. A general self-energy diagram of one of the Higgses (including the Goldstone boson) can be decomposed as follows:



where X stands for the H, h, A or G. With q we denote an arbitrary quark and $\tilde{q}$ its superpartner. For the Z-boson the decomposition looks like



Tadpole Cancellation

Because the vacuum expectation values v_1 and v_2 have been renormalized, we cannot ensure any longer that there are no tadpole contributions. However, it is shown in appendix C that all tadpole contributions to the correction of the mass sum rule add up to zero.

Calculation of Loop Diagrams

Then only the three last general loop diagrams contribute to the correction to the mass sum rule. We decide to include first only diagrams where q is the top quark and $\tilde{q}$ its superpartner, the stop. The explicit calculation of Δ for top/stop contributions can be found in the appendix and is structured as follows: In appendix D we display the diagram table (table D.1). All relevant self-energy diagrams can be found there including their sign in formula (6.33). From the diagram table it can be seen, that the diagrams fall into six classes, depending firstly on whether the particle on the external lines is either one of the Higgses or the Z boson and secondly on the three types of loops that appear: We have loops with one squark, two squarks or one quark. Next, we calculate six general diagrams as representatives for their class in appendix E. In this calculation we leave the couplings unspecified, but they can be found in appendix F. Note that we have rotated the couplings from the interaction basis into the mass eigenstate basis. Finally, everything is put together in appendix G: The couplings are inserted into the corresponding diagrams and they are summed according to the formula (6.33). The result consists of two parts: The *divergent* part is a sum of the singularities of the diagrams and is shown to vanish. This must be the case, since Δ is after all a physical quantity, so it must not not be divergent. The *finite* part contains the terms which we call in *logarithmic* and *nonlogarithmic* terms. This is the endresult for Δ which is evaluated numerically in the following sections.

7.2 The mass of the lightest Higgs

We have now derived the corrected mass sum rule and explicitly written down the expression for Δ from top/stop loops in appendix G. Taking the limit of a large mass of the CP-odd Higgs A, we already noted that A and H are almost degenerate. In this limit, the two mass terms m_A^2 and m_H^2 cancel from the sum rule, and what remains is

$$m_h^2 = m_Z^2 + \Delta. \tag{7.2}$$

From this equation we see that we can use the correction to the mass sum rule to calculate the corrections to the mass of the lightest Higgs in the large m_A limit.[1] We will first evaluate the contributions coming from the top/stop sector and then those from the bottom/sbottom sector.

7.3 Evaluation of the Top/Stop Contributions

Parameter Space for the Evaluation

The formula for the one-loop correction of the Higgs mass sum rule depends on seven parameters: Two of them are the value of $\tan\beta$ and one Higgs mass, say m_A which then characterize the Higgs Sector completely. (They fix m_H, m_h at tree level and also the mixing angle α.) In the squark sector, we have the A_t-parameter from the trilinear soft breaking terms, the two stop masses $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, the stop mixing angle θ_t and the μ-parameter from the superpotential. But there exists the relation

$$A_t = \frac{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)\sin 2\theta_t}{2m_t} + \mu\cot\beta, \tag{7.3}$$

which can be directly read off from the stop mass matrix (5.3) of chapter 5. Thus, we can choose A_t to be fixed. This makes six parameters. In addition to that we will also display the dependence of the correction to the h mass on the mass of the top quark m_t, which is in principle known, but with a large uncertainty and therefore regarded as a seventh parameter. We choose the mass parameters to be around the expected SUSY scale of 1 Tev: $m_A = 1$ TeV, $m_{\tilde{t}_1} = 900$ GeV, $m_{\tilde{t}_2} = 1100$ GeV. We distinguish two scenarios for the stop mixing angle: $\theta_t = 0$ (*minimal mixing*) and $\theta_t = \frac{\pi}{4}$ (*maximal mixing*). The mass of the top quark is taken to be 175 GeV. The μ-parameter receives the conservative value of 300 Gev. We first display the dependence of the Higgs mass of $\tan\beta$ in figure 7.1.

[1] We will take a mass of m_A around the expected SUSY scale of 1 TeV.

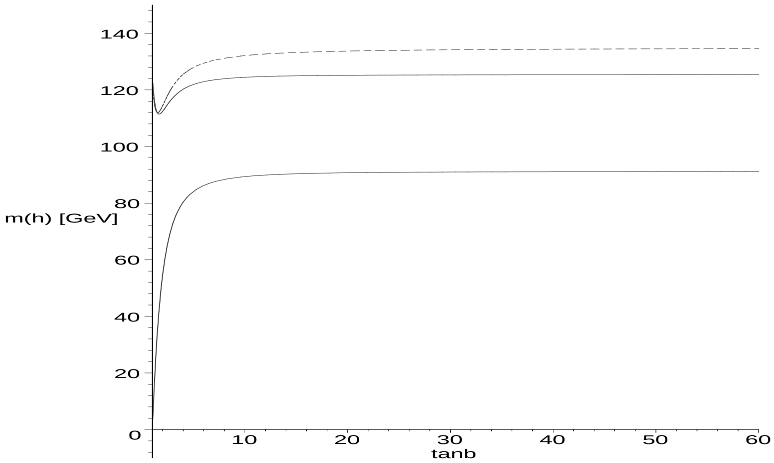


Figure 7.1: The mass of h, depending on the ratio of the vevs, $\tan\beta$. The lowest line shows the tree level mass of the lightest Higgs which approaches the mass of the Z as an upper bound. The upper lines display the one-loop corrected masses in the minimal mixing scenario (full line, $\theta_t = 0$) and the maximal mixing scenario (dotted line, $\theta_t = \frac{\pi}{4}$).

The figure tells us that for reasonably high $\tan\beta \geq 15$ we reach a plateau. We decide to fix $\tan\beta = 20$ as conventional value for the next plots. In figure 7.2 we display the dependence on m_h on the two stop masses. The first observation is that the graph grows with growing stop masses. In fact, this is the logarithmic growth with the SUSY scale which was to be expected from the leading log result in from equation (G.2). As we are looking for a maximum of m_h, only the maximal mixing scenario is interesting to us. In order to graphically obtain the maximum value, we rotate the graph so that the highest edge points towards us (figure 7.3). We learn that the maximum has shifted from 91 GeV to around 150 GeV. The maximum is obtained for stop masses around $m_{\tilde{t}_1} = 900$ GeV and $m_{\tilde{t}_2} = 1100$ GeV. Next we display the dependence of m_h on the mass of the top-quark in figure 7.4. We see that the uncertainty in the top mass has an effect of approximately 6 GeV over the whole range, irrespective of the mixing angle. Variations of the correction with respect to the mass of the CP-odd Higgs and the μ-parameter from the superpotential do not show much effect.

7.4 Evaluation of the Bottom/Sbottom Contributions

Our result for the correction to the mass sum rule (and thus the correction to the mass of the lightest Higgs) due to top/stop loops in appendix G can directly be translated into a correction due to bottom/sbottom loops [10]. Because the FEYNMAN rules are

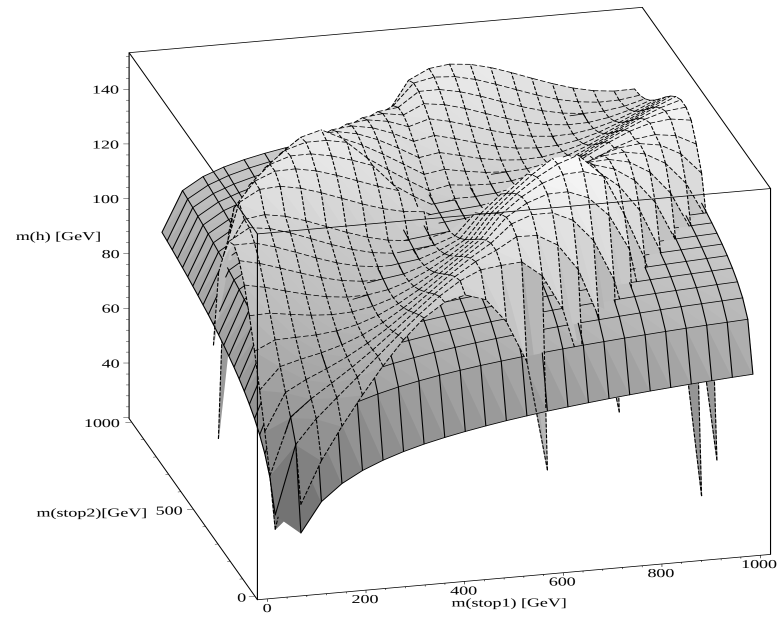


Figure 7.2: The mass of h, depending on the two stop-masses, $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$. The lower/upper planes show the one-loop corrected mass in the minimal/maximal mixing scenarios, respectively.

very similar, it is only neccessary to make the following replacements in the formula:

$$\theta_u \longrightarrow \theta_d, \tag{7.4}$$

$$m_u \longrightarrow m_d, \qquad m_{\tilde{u}_{1,2}} \longrightarrow m_{\tilde{d}_{1,2}}, \tag{7.5}$$

$$e_u \longrightarrow e_d, \qquad T_3 = \tfrac{1}{2} \longrightarrow -\tfrac{1}{2}, \tag{7.6}$$

$$\sin\beta \longrightarrow \cos\beta, \qquad \cos\beta \longrightarrow \sin\beta, \tag{7.7}$$

$$\cos\alpha \longrightarrow \sin\alpha, \qquad \sin\alpha \longrightarrow \cos\alpha, \tag{7.8}$$

$$\sin(\alpha+\beta) \longrightarrow -\sin(\alpha+\beta), \qquad \cos(\alpha+\beta) \longrightarrow \cos(\alpha+\beta). \tag{7.9}$$

Let us discuss the parameter space: Again we have the two parameters that characterize the Higgs sector at tree level, m_A and $\tan\beta$. In the sbottom sector, we have the A_b parameter from the soft breaking terms, the two sbottom masses $m_{\tilde{b}_1}$ and $m_{\tilde{b}_2}$ and the sbottom mixing angle θ_b. A relation that combines them all is

$$A_b = \frac{(m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}) \sin 2\theta_b}{2m_b} + \mu \tan\beta. \tag{7.10}$$

Now there is a crucial difference to the discussion in the top/stop sector. Let us look at the two addends which contribute to A_b and compare them with (7.3). The first addend has the bottom mass in the denominator whereas in case of the A_t parameter

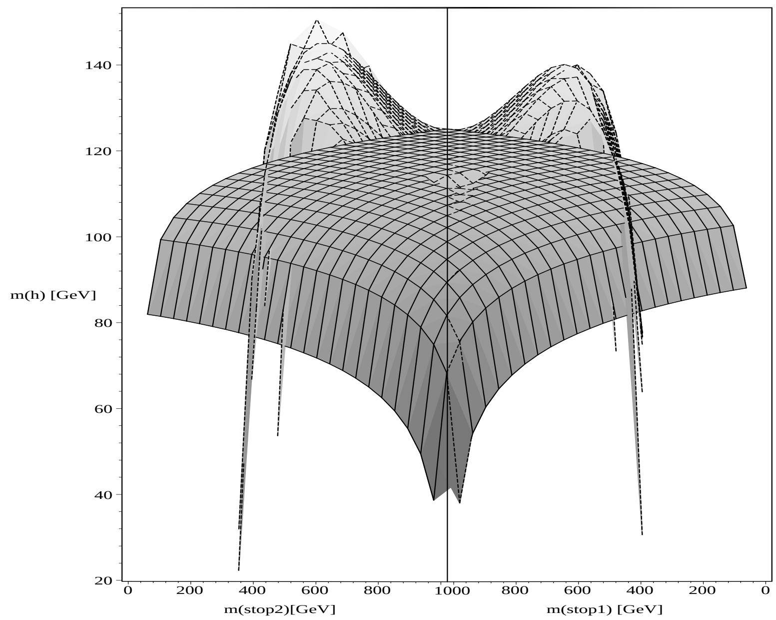


Figure 7.3: The mass of h, depending on the two stop-masses, front view. The lower/upper cuts show the one-loop corrected mass in the minimal/maximal mixing scenarios, respectively.

we had the top mass. Thus, a large mass splitting $m^2_{\tilde{b}_1} - m^2_{\tilde{b}_2}$ and a strong mixing $\theta_b \to \frac{\pi}{4}$ give large contributions to A_b. The second addend contains the product of the μ-parameter with $\tan\beta$, so that this term can also become very large. In the case of (7.3) μ was multiplied with $\cot\beta$. The problem comes about that for seemingly harmless parameters, A_b will blow up and destroy the stability of the potential. But we can overcome this problem by constraining the A_b parameter (which has dimensions of a mass) by the usual bound that masses should not exceed the TeV scale so that the potential is stable and also we do not introduce a hierarchy problem again. In fact, we will now regard A_b as an input parameter and choose instead the mixing angle θ_b to be fixed by (7.10). A conventional value for A_b will be 1 TeV.

One more remark is in order to avoid confusion: Of course this effect that the A parameter gets large does in principle also occur in the top/stop sector. But the effect is first mildened by the large mass of the top quark, and second, it comes about for very *small* values of $\tan\beta$ due to the cotangent in (7.3). It is only the fact that small values of $\tan\beta$ are excluded that made the effect seemingly disappear in the previous discussion.

With A_b kept under control in this way, the dependence of the Higgs mass corrected only by bottom/sbottom loops on $\tan\beta$ is presented in figure 7.5. The remaining

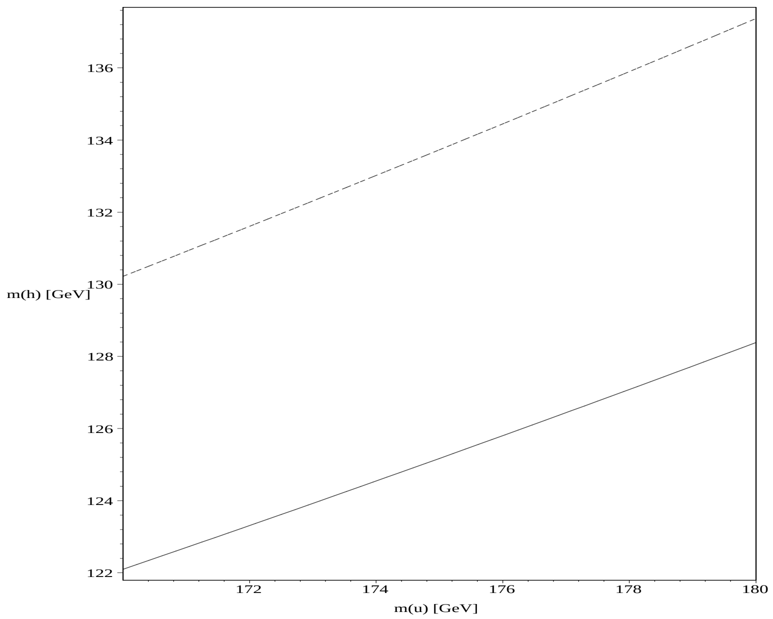


Figure 7.4: The mass of h, depending on the mass of the top-quark. The lower/upper line displays the behaviour in the minimal/maximal mixing scenario, respectively.

values that were chosen for this figure were the same as in the top/stop sector (m_A=1 TeV, $m_{\tilde{b}_1} = 900$ GeV, $m_{\tilde{b}_2} = 1100$ GeV, μ= 300 GeV. The mass of the bottom quark is 4.25 GeV. As we can see the variation of $\tan\beta$ has almost no effect on the corrected Higgs mass, so we conclude that we can stay at $\tan\beta$=20, which we also used in the top/stop sector.

Plot 7.6 shows the dependence of the corrected Higgs mass on the choice of the sbottom masses. We first notice that the effect is smaller than the effect from the top/stop loops and then take care of the two small hills on the plateau of the surface. The hills lie on a raised line which fulfills $m_{\tilde{b}_2} = -m_{\tilde{b}_1} + 1000$ GeV and one of them corresponds approximately to values of $m_{\tilde{b}_1} = 470$ GeV for the mass of the first bottom squark and $m_{\tilde{b}_2} = 530$ GeV of the second bottom squark. The overall effect of the bottom contribution is to raise the Higgs mass to approximately 97 GeV.

Now we are going to put the results together. We know that the best values to find a maximum in the Higgs mass which is consistent with the mass parameters to be around the TeV scale (m_a= 1 TeV, A_b= 1 TeV) is to look at maximal mixing in the top/stop sector ($\theta_u = \frac{\pi}{4}$) with stop masses $m_{\tilde{t}_1}$ around 900 GeV and $m_{\tilde{t}_2}$ around 1100 GeV and $\tan\beta = 20$, where in the bottom/sbottom sector the values $m_{\tilde{b}_1} = 470$ GeV and $m_{\tilde{b}_2} = 530$ GeV for the sbottom masses were chosen. We display the figure for the combined top/stop and bottom/sbottom corrected result for a top quark mass of 175

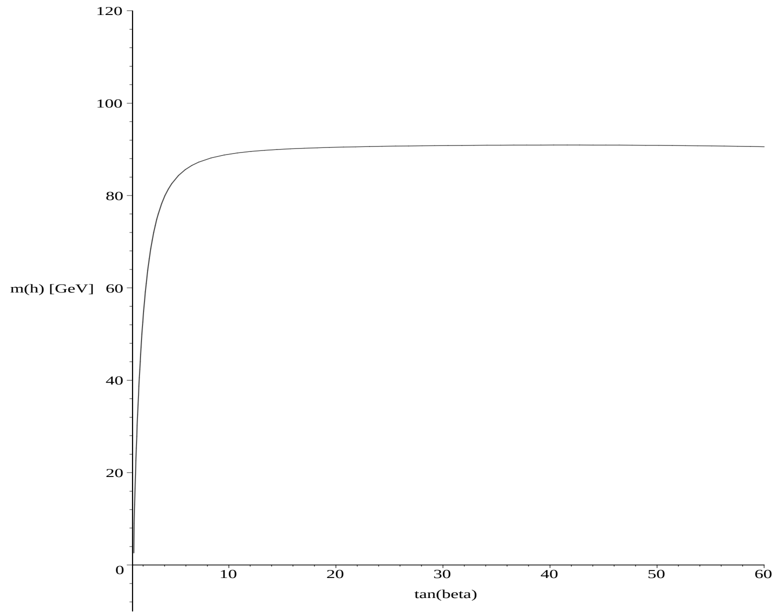


Figure 7.5: The mass of h, corrected by bottom/sbottom loops, depending on $\tan\beta$.

GeV in figure 7.7 . The maximal Higgs mass is around 156 GeV. A top quark mass higher than 175 GeV would shift this result upwards.

7.5 The SUSY Scale

What can we learn from the corrected mass sum rule except for the estimation of the Higgs mass? One could for example try to deduce statements about the SUSY scale ($m^2_{\mathrm{SUSY}} := m_{\tilde{t}_1} m_{\tilde{t}_2}$) as soon as masses of Higgs particles have been determined. Let us suppose that LEP saw the Higgs last year at 115 GeV and that this was the light Higgs of the MSSM. Then we can vary all parameters and see for which values of the SUSY scale we are able to produce a Higgs mass of 115 GeV. This was done in figure 7.8: We let m_{SUSY} run from 0 to 2 TeV, and vary the other parameters $\tan\beta$ (from 2 to 60), $m_{\tilde{t}_1}$, $m_{\tilde{t}_1}$, $m_{\tilde{b}_1}$, $m_{\tilde{b}_1}$ (which we expect around the SUSY scale and therefore take values from $m_{\mathrm{SUSY}} - 200$ GeV to $m_{\mathrm{SUSY}} + 200$ GeV), θ_t (for values 0, $\frac{\pi}{8}$, $\frac{\pi}{4}$), A_b (smaller or equal 1 TeV) and m_t (from 170 GeV to 180 GeV). We fix $m_A = 1$ TeV and $\mu = 300$ GeV. Whenever for a given value of m_{SUSY} there is a combination of the other parameters, such that our one-loop corrected formula gives a Higgs mass of 115 GeV (we allow for an uncertainty of ± 2 GeV), we have a point in figure 7.8. We learn from this figure that there is a triangle structure with the variation of m_t. What is more remarkable, is the absence of points. There are no entries below 200 GeV and

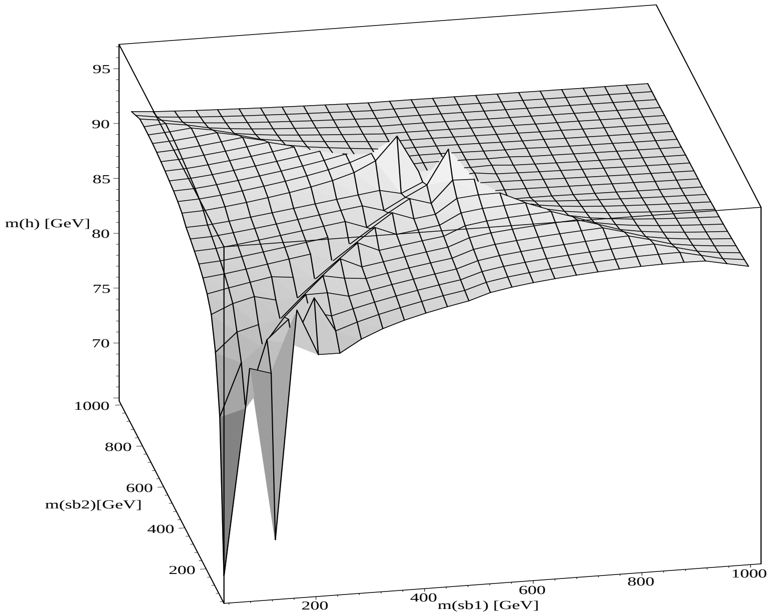


Figure 7.6: The mass of h, depending on the two sbottom-masses.

no entries above 950 GeV (for $m_t = 170$ GeV). The value goes down with increasing top mass. So if we knew the Higgs mass, we could deduce from our one-loop corrected result an upper bound on the SUSY breaking scale.

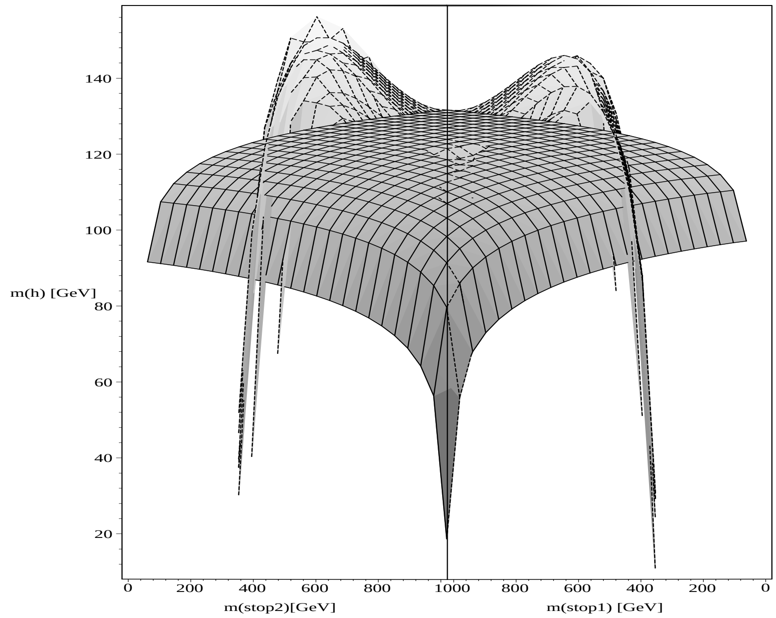


Figure 7.7: The top/stop and bottom/sbottom one loop corrected mass of h, depending on the two stop masses, for the choice of parameter see text.

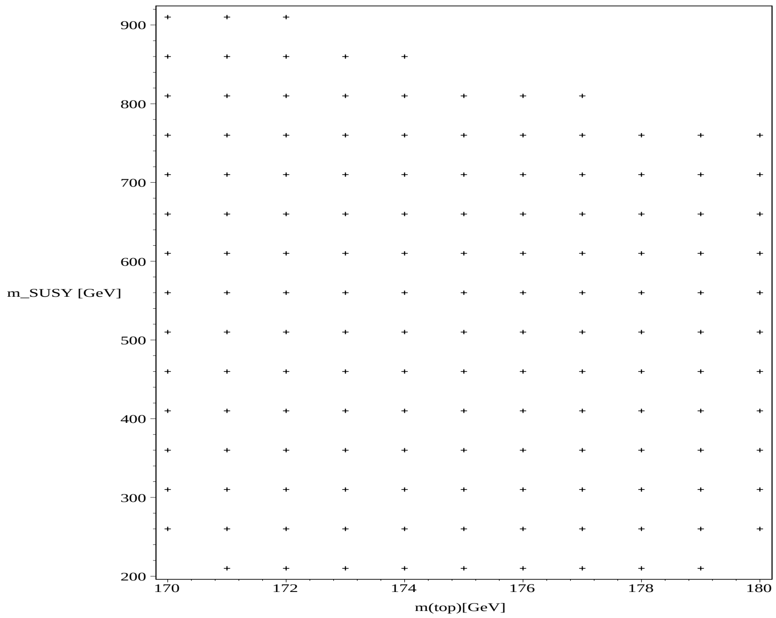


Figure 7.8: The SUSY scale, depending on the mass of the top quark. Note that there are no points below 200 GeV and above 950 GeV.

Chapter 8

Conclusions

In this thesis we derived the MSSM mass spectrum at the tree level and showed that the mass of the lightest Higgs h is bounded from above by the Z mass. We calculated explicitly the one-loop correction to the mass sum rule for the neutral Higgs bosons due to top/stop and bottom/sbottom loops. We demonstrated that this correction can be used in the limit of a large A mass to estimate the shift in the mass of the h Higgs. The new maximum is found to be around 156 GeV. We demonstrated how one can use our formula to give an approximate upper bound on the SUSY scale once the h mass is known. For an h mass of 115 ± 2 GeV we find an upper bound on the SUSY scale of 950 GeV.

Appendix A

Conventions and Useful Formulae

A.1 Conventions

Our conventions follow [2, 3, 4]. We use natural units $\hbar = c = 1$ and the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.1}$$

The momentum four vector is

$$p^\mu = \begin{pmatrix} E \\ \mathbf{p} \end{pmatrix}. \tag{A.2}$$

The Pauli matrices are

$$\sigma^\mu = (1, \sigma)\,, \qquad \bar{\sigma}^\mu = (1, \bar{\sigma})\,. \tag{A.3}$$

The two-component Weyl-spinor ξ_α transforms under a matrix M of $SL(2,C)$. The spinors $\bar{\xi}_{\dot{\alpha}}$, ξ^α and $\bar{\xi}^{\dot{\alpha}}$ transform under matrices M^*, M^{-1} and $(M^{-1})^*$, respectively. The two-component Dirac equations for these spinors are

$$(\bar{\sigma}_\mu p^\mu)^{\dot{\alpha}\beta}\xi_\beta = m\bar{\eta}^{\dot{\alpha}}, \qquad (\sigma_\mu p^\mu)_{\alpha\dot{\beta}}\bar{\eta}^{\dot{\beta}} = m\xi_\alpha. \tag{A.4}$$

Then we can construct a Dirac four spinor such that it fulfils the four component equation

$$(\gamma_\mu p^\mu - m)\Psi = 0. \tag{A.5}$$

We identify

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix} \quad \text{and} \quad \gamma_\mu = \begin{pmatrix} 0 & (\sigma_\mu)_{\alpha\dot{\beta}} \\ (\bar{\sigma}_\mu)^{\dot{\alpha}\beta} & 0 \end{pmatrix} \tag{A.6}$$

and define

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A.7}$$

As γ^5 is seen to be diagonal, we have obviously obtained the *chiral* representation. We also define left- and right-handed projection operators

$$P_L := \tfrac{1}{2}(1-\gamma^5) \quad \text{and} \quad P_R := \tfrac{1}{2}(1+\gamma^5). \tag{A.8}$$

The charge-conjugated spinor is given by

$$\Psi^c := C\bar{\Psi}^T \quad \text{with} \quad C = i\gamma^2\gamma^0. \tag{A.9}$$

Then the following equalities, which mediate between the two-component and four-component notation for the fermions, hold

$$\begin{aligned} \overline{\Psi}_1 P_L \Psi_2 &= \eta_1\xi_2, & \overline{\Psi}{}_1^c P_L \Psi_2^c &= \xi_1\eta_2, \\ \overline{\Psi}_1 P_R \Psi_2 &= \bar{\eta}_2\bar{\xi}_1, & \overline{\Psi}{}_1^c P_R \Psi_2^c &= \bar{\xi}_2\bar{\eta}_1, \\ \overline{\Psi}{}_1^c P_L \Psi_2 &= \xi_1\xi_2, & \overline{\Psi}_1 P_L \Psi_2^c &= \eta_1\eta_2, \\ \overline{\Psi}_1 P_R \Psi_2^c &= \bar{\eta}_1\bar{\eta}_2, & \overline{\Psi}{}_1^c P_R \Psi_2 &= \bar{\eta}_1\eta_2, \end{aligned} \tag{A.10}$$

where we have dropped summation indices. A MAJORANA spinor fulfills the condition

$$\Psi^c = \Psi. \tag{A.11}$$

Gauginos as Majorana Spinors

The supersymmetry breaking terms of chapter 1 contain mass terms for the gauginos. This passage wants to clarify the notation that can be used when one wants to decompose these spinors into their WEYL-substructure: In the review of HABER [2] the gaugino spinors $\tilde{B}$ and $\tilde{W}^a$ are supposed to be MAJORANA-spinors. That means that we can write them as follows

$$\tilde{B} = \begin{pmatrix} \tilde{b} \\ \bar{\tilde{b}} \end{pmatrix}, \qquad \tilde{W}^a = \begin{pmatrix} \tilde{w}^a \\ \bar{\tilde{w}}^a \end{pmatrix}, \tag{A.12}$$

where $\tilde{B}$ and $\tilde{W}^a$ are 4-component spinors and $\tilde{b}$ and $\tilde{w}^a$ are two-component WEYL-spinors. Next we define the charged linear combinations

$$\tilde{w}^+ = \frac{1}{\sqrt{2}}\left(\tilde{w}^1 - i\tilde{w}^2\right), \qquad \tilde{w}^- = \frac{1}{\sqrt{2}}\left(\tilde{w}^1 + i\tilde{w}^2\right). \tag{A.13}$$

Therefore, for the charged DIRAC-spinors we have the entries

$$\begin{aligned}\tilde{W}^+ &= \frac{1}{\sqrt{2}}(\tilde{W}^1 - i\tilde{W}^2) = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{w}^1 - i\tilde{w}^2) \\ \frac{1}{\sqrt{2}}(\bar{\tilde{w}}^1 - i\bar{\tilde{w}}^2) \end{pmatrix} = \begin{pmatrix} \tilde{w}^+ \\ \bar{\tilde{w}}^- \end{pmatrix}, \\ \tilde{W}^- &= \frac{1}{\sqrt{2}}(\tilde{W}^1 + i\tilde{W}^2) = \begin{pmatrix} \frac{1}{\sqrt{2}}(\tilde{w}^1 + i\tilde{w}^2) \\ \frac{1}{\sqrt{2}}(\bar{\tilde{w}}^1 + i\bar{\tilde{w}}^2) \end{pmatrix} = \begin{pmatrix} \tilde{w}^- \\ \bar{\tilde{w}}^+ \end{pmatrix}. \end{aligned} \tag{A.14}$$

A.2 Mass Matrix Diagonalization

The typical mass (squared) matrix is a symmetric 2×2 matrix

$$\bar{\chi} M \chi = (\bar{\chi_1}, \bar{\chi_2}) \begin{pmatrix} A & \frac{1}{2}C \\ \frac{1}{2}C & B \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{A.15}$$

The eigenvalues are

$$m_{1,2}^2 = \tfrac{1}{2}(A+B) \pm \tfrac{1}{2}\sqrt{(A-B)^2 + C^2}. \tag{A.16}$$

As a convention we choose to rotate the eigenstates in mathematically positive direction

$$\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{A.17}$$

Then the diagonal matrix D with the eigenvalues m_1^2 and m_2^2 satisfies

$$\bar{\eta} D \eta = \bar{\chi} M \chi \tag{A.18}$$

and from (A.18) we can read off the relations for the matrix entries

$$\begin{aligned} \cos^2\theta m_1^2 + \sin^2\theta m_2^2 &= A, \\ \sin^2\theta m_1^2 + \cos^2\theta m_2^2 &= B, \\ \cos\theta\sin\theta(m_2^2 - m_1^2) &= \tfrac{1}{2}C. \end{aligned} \tag{A.19}$$

So we have as valid relations for the mixing angle for example

$$\begin{aligned} \sin 2\theta &= \frac{C}{m_2^2 - m_1^2}, \\ \tan 2\theta &= \frac{C}{A-B}. \end{aligned} \tag{A.20}$$

A.3 Helpful Formulae for Regularization

The formulae from this section can be found in [13].

We only need the simplest formula for FEYNMAN-parameterization:

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[xA + (1-x)B]^2}. \tag{A.21}$$

(We will usually take A to be the more complicated denominator and B the easier one.) For dimensional regularization we make use of the following formulae. Starting from d-dimensional MINKOWSKI space, the following identities hold

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^n} = \frac{(-1)^n i}{(4\pi)^{d/2}} \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}}, \tag{A.22}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}, \tag{A.23}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1}. \tag{A.24}$$

For the DIRAC-algebra in four dimensions, we need that the trace of an odd number of γ matrices vanishes and the following identities

$$\begin{aligned}
\mathrm{tr}(\mathbf{1}) &= 4, \\
\mathrm{tr}(\gamma^\mu\gamma^\nu) &= 4g^{\mu\nu}, \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}), \\
\mathrm{tr}(\gamma^5) &= 0, \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) &= 0, \\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma}
\end{aligned} \tag{A.25}$$

A.4 Passarino-Veltman Functions

The F, G and H-integrals of appendix G can be evaluated properly with the help of a paper by PASSARINO and VELTMAN [17]. We define the following functions[1]:

$$\begin{aligned} B_0'(m_1,m_2,p) &= -\left[\ln(-p^2-i\epsilon)+\sum_j\{\ln(1-x_j)+\mathcal{F}(1,x_j)\}\right], \\ B_1'(m_1,m_2,p) &= \tfrac{1}{2}\left[\ln(-p^2-i\epsilon)+\sum_j\{\ln(1-x_j)+\mathcal{F}(2,x_j)\}\right], \\ B_{21}'(m_1,m_2,p) &= -\tfrac{1}{3}\left[\ln(-p^2-i\epsilon)+\sum_j\{\ln(1-x_j)-\mathcal{F}(3,x_j)\}\right], \quad \text{(A.26)} \end{aligned}$$

where x_j are the roots of the equation

$$-p^2x^2+(p^2+m_2^2-m_1^2)x+m_1^2=0 \tag{A.27}$$

and $\mathcal{F}(n,x)$ is given by

$$\mathcal{F}(n,x)=-x^n\ln\frac{x-1}{x}-x^{n-1}-\tfrac{1}{2}x^{n-2}-\tfrac{1}{3}x^{n-3}\ldots-\frac{1}{n}. \tag{A.28}$$

Then we can identify

$$\begin{aligned} F(m_1,m_2,m_3) &= -B_0(m_1,m_2,p), & \text{(A.29)} \\ G(m_1,m_2,m_3) &= B_1(m_1,m_2,p)+B_{21}(m_1,m_2,p), & \text{(A.30)} \\ H(m_1,m_2,m_3) &= B_1(m_2,m_1,p). & \text{(A.31)} \end{aligned}$$

[1] The original functions B_0, B_1, B_{21} from [17] also contain a divergent term, which we have subtracted. Therefore, we denote our functions in A.26 with a prime.

Appendix B

The MSSM Lagrangian

Here we display the complete MSSM Lagrangian before symmetry breaking, reduced to the field notation.

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{D}} + \mathcal{L}_{\mathrm{F}} + \mathcal{L}_{\mathrm{SUSY}} + \mathcal{L}_{\mathrm{Y}} + \mathcal{L}_{\mathrm{soft}} \tag{B.1}$$

The Kinetic Part

$$\begin{aligned}
\mathcal{L}_{\mathrm{kin}} \;=\;& -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu)^2 \\
& -\frac{1}{4}(\partial_\mu g^i_\nu - \partial_\nu g^i_\mu + g_3 f^{ijk} g^j_\mu g^k_\nu)^2 \\
& + i\bar{\tilde{B}}\gamma^\mu \partial_\mu \bar{B} + i\bar{\tilde{W}}^a \gamma^\mu (\partial_\mu + g_2 \epsilon^{abc} W^b_\mu \tilde{W}^c)\bar{W}^a + i\bar{\tilde{g}}^i \gamma^\mu (\partial_\mu + g_3 f^{ijk} g^j_\mu \tilde{g}^k)\bar{g}^k \\
& + |\partial_\mu \tilde{L} + ig' B_\mu \tilde{L} - \frac{ig_2}{2} W^a_\mu \tau^a \tilde{L}|^2 + |\partial_\mu \tilde{Q} - \frac{ig'}{3} B_\mu \tilde{Q} - \frac{ig_2}{2} W^a_\mu \tau^a \tilde{Q} - \frac{ig_3}{2} g^a \lambda^a \tilde{Q}|^2 \\
& + |\partial_\mu \tilde{E} - 2ig' B_\mu \tilde{E}|^2 + |\partial_\mu \tilde{U} - \frac{4ig'}{3} B_\mu \tilde{U} - \frac{ig_3}{2} g^a (\lambda^a)^* \tilde{U}|^2 \\
& + |\partial_\mu \tilde{D} - \frac{2ig'}{3} B_\mu \tilde{D} - \frac{ig_3}{2} g^a (\lambda^a)^* \tilde{D}|^2 + |\partial_\mu H_1 + ig' B_\mu H_1 - \frac{ig_2}{2} W^a_\mu \tau^a H_1|^2 \\
& + |\partial_\mu H_2 - ig' B_\mu H_2 - \frac{ig_2}{2} W^a_\mu \tau^a H_2|^2 \\
& + (i\bar{L}\gamma^\mu(\partial_\mu L + ig' B_\mu L - \frac{ig_2}{2} W^a_\mu \tau^a L) + \mathrm{h.c.}) \\
& + (i\bar{E}\gamma^\mu(\partial_\mu E - 2ig' B_\mu E) + \mathrm{h.c.}) \\
& + (i\bar{Q}\gamma^\mu(\partial_\mu Q - \frac{ig'}{3} B_\mu Q - \frac{ig_2}{2} W^a_\mu \tau^a Q - \frac{ig_3}{2} g^a \lambda^a Q) + \mathrm{h.c.}) \\
& + (i\bar{U}\gamma^\mu(\partial_\mu U + \frac{4ig'}{3} B_\mu U - \frac{ig_3}{2} g^a (\lambda^a)^* U) + \mathrm{h.c.}) \\
& + (i\bar{D}\gamma^\mu(\partial_\mu U - \frac{2ig'}{3} B_\mu D - \frac{ig_3}{2} g^a (\lambda^a)^* D) + \mathrm{h.c.})
\end{aligned}$$

$$
\begin{aligned}
&+(i\bar{\tilde{H}}_1\gamma^\mu(\partial_\mu\tilde{H}_1 + ig'B_\mu\tilde{H}_1 - \frac{ig_2}{2}W^a_\mu\tau^a\tilde{H}_1) + \text{h.c.})\\
&+(i\bar{\tilde{H}}_2\gamma^\mu(\partial_\mu\tilde{H}_1 - ig'B_\mu\tilde{H}_2 - \frac{ig_2}{2}W^a_\mu\tau^a\tilde{H}_1) + \text{h.c.}) \qquad \text{(B.2)}
\end{aligned}
$$

The D-Terms

$$
\begin{aligned}
-\mathcal{L}_\mathrm{D} \;=\; & \frac{g_2^2}{8}\{[|\tilde{\nu}|^4 + 2|\tilde{\nu}|^2|\tilde{e}_L|^2 + |\tilde{e}_L|^4]\\
&+4\left[|\tilde{\nu}|^2|\tilde{u}_L|^2 + \tilde{\nu}^*\tilde{e}_L\tilde{d}_L^*\tilde{u}_L + \tilde{e}_L^*\tilde{\nu}\tilde{u}_L^*\tilde{d}_L + |\tilde{e}_L|^2|\tilde{d}_L|^2\right]\\
&-2\left[|\tilde{\nu}|^2|\tilde{u}_L|^2 + |\tilde{\nu}^2||\tilde{d}_L|^2 + |\tilde{e}_L|^2|\tilde{u}_L|^2 + |\tilde{e}_L|^2|\tilde{d}_L|^2\right]\\
&+4\left[|\tilde{\nu}|^2|H_1^0|^2 + \tilde{\nu}^*\tilde{e}_LH_1^{-*}H_1^0 + \tilde{e}_L^*\tilde{\nu}H_1^{0*}H_1^- + |\tilde{e}_L|^2|H_1^-|^2\right]\\
&-2\left[|\tilde{\nu}|^2|H_1^0|^2 + |\tilde{\nu}^2||H_1^-|^2 + |\tilde{e}_L|^2|H_1^0|^2 + |\tilde{e}_L|^2|H_1^-|^2\right]\\
&+4\left[|\tilde{\nu}|^2|H_2^+|^2 + \tilde{\nu}^*\tilde{e}_LH_2^{0*}H_2^+ + \tilde{e}_L^*\tilde{\nu}H_2^{+*}H_2^0 + |\tilde{e}_L|^2|H_2^0|^2\right]\\
&-2\left[|\tilde{\nu}|^2|H_2^+|^2 + |\tilde{\nu}^2||H_2^0|^2 + |\tilde{e}_L|^2|H_2^+|^2 + |\tilde{e}_L|^2|H_2^0|^2\right]\\
&+\left[|\tilde{u}_L|^4 + 2|\tilde{u}_L|^2|\tilde{d}_L|^2 + |\tilde{d}_L|^4\right]\\
&+4\left[|\tilde{u}_L|^2|H_1^0|^2 + \tilde{u}_L^*\tilde{d}_LH_1^{-*}H_1^0 + \tilde{d}_L^*\tilde{u}_LH_1^{0*}H_1^- + |\tilde{d}_L|^2|H_1^-|^2\right]\\
&-2\left[|\tilde{u}_L|^2|H_1^0|^2 + |\tilde{u}_L|^2||H_1^-|^2 + |\tilde{d}_L|^2|H_1^0|^2 + |\tilde{d}_L|^2|H_1^-|^2\right]\\
&+4\left[|\tilde{u}_L|^2|H_2^+|^2 + \tilde{u}_L^*\tilde{d}_LH_2^{0*}H_2^+ + \tilde{d}_L^*\tilde{u}_LH_2^{+*}H_2^0 + |\tilde{d}_L|^2|H_2^-|^2\right]\\
&-2\left[|\tilde{u}_L|^2|H_2^+|^2 + |\tilde{u}_L|^2||H_2^0|^2 + |\tilde{d}_L|^2|H_2^+|^2 + |\tilde{d}_L|^2|H_2^0|^2\right]\\
&+\left[|H_1^0|^4 + 2|H_1^0|^2|H_1^-|^2 + |H_1^-|^4\right]\\
&+4\left[|H_1^0|^2|H_2^+|^2 + H_1^{0*}H_1^-H_2^{0*}H_2^+ + H_1^{-*}H_1^0H_2^{+*}H_2^0 + |H_1^-|^2|H_2^0|^2\right]\\
&-2\left[|H_1^0|^2|H_2^+|^2 + |H_1^0|^2||H_2^0|^2 + |H_1^-|^2|H_2^+|^2 + |H_1^-|^2|H_2^0|^2\right]\\
&+\left[|H_2^+|^4 + 2|H_2^+|^2|H_2^0|^2 + |H_2^0|^4\right]\}\\
&+\frac{g_3^2}{6}\Big\{2|\tilde{Q}^\alpha\tilde{Q}^\beta|^2 + 3\tilde{Q}^{\alpha*}\tilde{Q}^{\beta*}\tilde{U}^\beta\tilde{U}^\alpha - |\tilde{Q}^\alpha|^2|\tilde{U}^\beta|^2 + 3\tilde{Q}^{\alpha*}\tilde{Q}^\beta\tilde{D}^{\beta*}\tilde{D}^\alpha - |\tilde{Q}^\alpha|^2|\tilde{D}^\beta|^2\\
&+\tilde{U}^{\alpha*}\tilde{U}^\beta\tilde{U}^{\beta*}\tilde{U}^\alpha + 3\tilde{U}^{\alpha*}\tilde{U}^\beta\tilde{D}^{\beta*}\tilde{D}^\alpha - \tilde{U}^{\alpha*}\tilde{U}^\alpha\tilde{D}^{\beta*}\tilde{D}^\beta + \tilde{D}^{\alpha*}\tilde{D}^\beta\tilde{D}^{\beta*}\tilde{D}^\alpha\Big\}\\
&+\frac{g'^2}{8}\Big\{[|\tilde{\nu}|^2|\tilde{u}_L|^2 + \tilde{\nu}^*\tilde{e}_L\tilde{d}_L^*\tilde{u}_L + \tilde{e}_L^*\tilde{\nu}\tilde{u}_L^*\tilde{d}_L + |\tilde{e}_L|^2|\tilde{d}_L|^2]\\
&-4[|\tilde{\nu}|^2|\tilde{e}_R|^2 + |\tilde{e}_L|^2|\tilde{e}_R|^2]\\
&-\frac{2}{3}[|\tilde{\nu}|^2|\tilde{u}_L|^2 + |\tilde{\nu}|^2|\tilde{d}_L|^2 + |\tilde{e}_L|^2|\tilde{u}_L|^2 + |\tilde{e}_L|^2|\tilde{d}_L|^2]\\
&+\frac{8}{3}[|\tilde{\nu}|^2|\tilde{u}_R|^2 + |\tilde{e}_L|^2|\tilde{u}_R|^2] - \frac{4}{3}[\tilde{\nu}|^2|\tilde{d}_R|^2 + |\tilde{\nu}|^2|\tilde{d}_R|^2\\
&+2[\tilde{\nu}|^2|H_1^0|^2 + |\tilde{\nu}|^2|H_1^-|^2 + |\tilde{e}_L|^2|H_1^0|^2 + |\tilde{e}_L|^2|H_1^-|^2]\\
&-2[\tilde{\nu}|^2|H_2^+|^2 + |\tilde{\nu}|^2|H_2^0|^2 + |\tilde{e}_L|^2|H_2^+|^2 + |\tilde{e}_L|^2|H_2^0|^2]\\
&+4|e_R|^4 + \frac{4}{3}[\tilde{e}_R|^2|\tilde{u}_L|^2 + |\tilde{e}_R|^2|\tilde{d}_L|^2] - \frac{16}{3}|\tilde{e}_R|^2|\tilde{u}_R|^2 + \frac{8}{3}|\tilde{e}_R|^2|\tilde{d}_R|^2
\end{aligned}
$$

$$
\begin{aligned}
&-4[|\tilde{e}_R|^2|H_1^0|^2+|\tilde{e}_R|^2|H_1^-|^2]+4[|\tilde{e}_R|^2|H_2^+|^2+|\tilde{e}_R|^2|H_2^0|^2]\\
&+\frac{1}{9}[|\tilde{u}_L|^4+2|\tilde{u}|^2|\tilde{d}_L|^2+|\tilde{d}_L|^4]-\frac{8}{9}[|\tilde{u}_L|^2|\tilde{u}_R|^2+|\tilde{d}_L|^2|\tilde{u}_R|^2]\\
&+\frac{4}{9}[|\tilde{u}_L|^2|\tilde{d}_R|^2+|\tilde{d}_L|^2|\tilde{d}_R|^2]\\
&-\frac{2}{3}[|\tilde{u}_L|^2|H_1^0|^2+|\tilde{u}_L|^2|H_1^-|^2+|\tilde{d}_L|^2|H_1^0|^2+|\tilde{d}_L|^2|H_1^-|^2]\\
&+\frac{2}{3}[|\tilde{u}_L|^2|H_2^+|^2+|\tilde{u}_L|^2|H_2^0|^2+|\tilde{d}_L|^2|H_2^+|^2+|\tilde{d}_L|^2|H_2^0|^2]\\
&+\frac{16}{9}|\tilde{u}_r|^4-\frac{16}{9}|\tilde{u}_R|^2|\tilde{d}_R|^2+\frac{8}{3}[|\tilde{u}_R|^2|H_1^0|^2+|\tilde{u}_R|^2|H_1^-|^2]\\
&-\frac{8}{3}[|\tilde{u}_R|^2|H_2^+|^2+|\tilde{u}_R|^2|H_2^0|^2]+\frac{8}{9}|\tilde{d}_R|^4-\frac{4}{3}[|\tilde{d}_R|^2|H_1^0|^2+|\tilde{d}_R|^2|H_1^-|^2]\\
&+\frac{4}{3}[|\tilde{d}_R|^2|H_2^+|^2+|\tilde{d}_R|^2|H_2^0|^2]\\
&+[|H_1^0|^4+2|H_1^0|^2|H_1^-|^2+|H_1^-|^4]-2[|H_1^0|^2|H_2^+|^2+|H_1^0|^2|H_2^0|^2]\\
&-2[|H_1^-|^2|H_2^+|^2+|H_1^-|^2|H_2^0|^2]+[|H_2^+|^4+2|H_2^+|^2|H_2^0|^2+|H_2^0|^4]\}
\end{aligned}
\tag{B.3}
$$

The F-Terms

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{F}} \;=\; &+|h_\tau|^2(\tilde{\nu}\tilde{e}_R^*\tilde{\nu}^*\tilde{e}_R+\tilde{e}_L\tilde{e}_R^*\tilde{e}_L^*\tilde{e}_R)+h_\tau h_b^*(\tilde{\nu}\tilde{e}_R^*\tilde{u}_L^*\tilde{d}_R+\tilde{e}_L\tilde{e}_R^*\tilde{d}_L^*\tilde{d}_R)\\
&-h_\tau\mu^*(\tilde{\nu}\tilde{e}_R^*H_2^{+*}+\tilde{e}_L\tilde{e}_R^*H_2^{0*})+h_b h_\tau^*(\tilde{u}_L\tilde{d}_R^*\tilde{\nu}^*\tilde{e}_R+\tilde{d}_L\tilde{d}_R^*\tilde{e}_L^*\tilde{e}_R)\\
&+|h_b|^2(\tilde{u}_L\tilde{d}_R^*\tilde{u}_L^*\tilde{d}_R+\tilde{d}_L\tilde{d}_R^*\tilde{d}_L^*\tilde{d}_R)-h_b\mu^*(\tilde{u}_L\tilde{d}_R^*H_2^{+*}+\tilde{d}_L\tilde{d}_R^*H_2^{0*})\\
&-\mu h_\tau^*(H_2^+\tilde{\nu}^*\tilde{e}_R+H_2^0\tilde{e}_L^*\tilde{e}_R)-\mu h_b^*(H_2^+\tilde{u}_L^*\tilde{d}_R+H_2^0\tilde{d}_L^*\tilde{d}_R)\\
&+|\mu|^2(|H_2^+|^2+|H_2^0|^2)+|h_t|^2(|\tilde{u}_L|^2|\tilde{u}_R|^2+|\tilde{d}_L|^2|\tilde{d}_R|^2)\\
&-h_t\mu^*(\tilde{u}_L\tilde{u}_R H_1^{0*}+\tilde{d}_L\tilde{u}_R H_1^{-*})+h_t^*\mu(\tilde{u}_L^*\tilde{u}_R^*H_1^0+\tilde{u}_L^*\tilde{u}_R^*H_1^-)\\
&+|\mu|^2(|H_1^0|^2+|H_1^-|^2)+|h_\tau|^2(|H_1^0|^2|\tilde{e}_R|^2+|H_1^-|^2|\tilde{e}_R|^2)\\
&+|h_\tau|^2(|H_1^0|^2|\tilde{e}_L|^2-H_1^0\tilde{e}_L H_1^{-*}\tilde{\nu}-H_1^-\tilde{\nu}H_1^{0*}\tilde{e}_L+|H_1^-|^2|\nu|^2)\\
&+|h_b|^2(|H_1^0|^2|\tilde{d}_R|^2+|H_1^-|^2|\tilde{d}_R|^2)-h_b h_t^*(H_1^0\tilde{d}_R^*H_2^+\tilde{u}_R+H_1^-\tilde{d}_R^*H_2^0\tilde{u}_R)\\
&-h_b^*h_t(H_2^+\tilde{u}_R^*H_1^0\tilde{u}_R+H_2^0\tilde{u}_R^*H_1^-\tilde{u}_R)+|h_t|^2(|H_2^+|^2|\tilde{u}_R|^2+|H_2^0|^2|\tilde{u}_R|^2)\\
&+|h_t|^2(|H_2^+|^2|\tilde{d}_L|^2-H_2^+\tilde{d}_L H_2^{0*}\tilde{u}_L^*-H_2^0\tilde{u}_L H_2^{+*}\tilde{d}_L+|H_2^0|^2|\tilde{u}_L|^2)\\
&+|h_b|^2(|H_1^0|^2|\tilde{d}_L|^2-H_1^0\tilde{d}_L H_1^{-*}\tilde{u}_L^*-H_1^-\tilde{u}_L H_1^{0*}\tilde{d}_L^*+|H_1^-|^2|\tilde{u}_L|^2)
\end{aligned}
\tag{B.4}
$$

The SUSY Part

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{SUSY}} \;=\; & -\sqrt{2}g'\{\tilde{e}_R\bar{\tilde{B}}P_L e_L^c\} + \frac{g'}{\sqrt{2}}\{\tilde{\nu}^*\bar{\tilde{B}}P_L\nu + \tilde{e}_L^*\bar{\tilde{B}}P_L e_L\} \\
& -\frac{g_2}{\sqrt{2}}\{\tilde{\nu}^*\bar{\tilde{W}}_3 P_L\nu + \tilde{e}_L^*\bar{\tilde{W}}^+ P_L\nu + \tilde{\nu}^*\bar{\tilde{W}}^- P_L e_L - \tilde{e}_L^*\bar{\tilde{W}}_3 P_L e_L\} \\
& -\frac{g'}{3\sqrt{2}}\{\tilde{u}_L^*\bar{\tilde{B}}P_L u_L + \tilde{d}_L^*\bar{\tilde{B}}P_L d_L\} \\
& -\frac{g_2}{\sqrt{2}}\{\tilde{u}_L^*\bar{\tilde{W}}_3 P_L u_L + \tilde{d}_L^*\bar{\tilde{W}}^+ P_L u_L + \tilde{u}_L^*\bar{\tilde{W}}^- P_L d_L - \tilde{d}_L^*\bar{\tilde{W}}_3 P_L d_L\} + SU(3) \\
& +\frac{4g'}{3\sqrt{2}}\{\tilde{u}_R\bar{\tilde{B}}P_L u_L^c\} - \frac{\sqrt{2}g'}{3}\{\tilde{d}_R\bar{\tilde{B}}P_L d_L^c\} \\
& +\frac{g'}{\sqrt{2}}\{H_1^{0*}\bar{\tilde{B}}P_L\tilde{H}_1^0 + H_1^{-*}\bar{\tilde{B}}P_L\tilde{H}_1^-\} \\
& -\frac{g_2}{\sqrt{2}}\{H_1^{0*}\bar{\tilde{W}}_3 P_L\tilde{H}_1^0 + H_1^{-*}\bar{\tilde{W}}^+ P_L\tilde{H}_1^0 + H_1^{0*}\bar{\tilde{W}}^- P_L\tilde{H}_1^- - H_1^{-*}\bar{\tilde{W}}_3 P_L\tilde{H}_1^-\} \\
& -\frac{g'}{\sqrt{2}}\{H_2^{+*}\bar{\tilde{B}}P_L\tilde{H}_2^+ + H_2^{0*}\bar{\tilde{B}}P_L\tilde{H}_2^0\} \\
& -\frac{g_2}{\sqrt{2}}\{H_2^{+*}\bar{\tilde{W}}_3 P_L\tilde{H}_2^+ + H_2^{0*}\bar{\tilde{W}}^+ P_L\tilde{H}_2^+ + H_2^{+*}\bar{\tilde{W}}^- P_L\tilde{H}_2^0 - H_2^{0*}\bar{\tilde{W}}_3 P_L\tilde{H}_2^0\} \\
& -\frac{g_3}{\sqrt{2}}(\lambda^a)_{\alpha\beta}\tilde{Q}^{\alpha*}\bar{\tilde{g}}^a P_L Q^\beta - \frac{g_3}{\sqrt{2}}(\lambda^{*a})_{\alpha\beta}\tilde{U}^{\alpha*}\bar{\tilde{g}}^a P_L U^\beta \\
& -\frac{g_3}{\sqrt{2}}(\lambda^{*a})_{\alpha\beta}\tilde{D}^{\alpha*}\bar{\tilde{g}}^a P_L D^\beta \\
& +\text{h.c.}
\end{aligned} \tag{B.5}
$$

The Yukawa Part

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} \;=\; & +\mu\overline{\tilde{H}_1^{0c}}P_L\tilde{H}_2^0 - \mu\overline{\tilde{H}_1^{-c}}P_L\tilde{H}_2^+ - h_\tau\tilde{E}\overline{\tilde{H}_1^{0c}}P_L e_L + h_\tau\tilde{E}\overline{\tilde{H}_1^{-c}}P_L\nu \\
& +h_\tau\tilde{\nu}\overline{\tilde{H}_1^{-c}}P_L e_L^c - h_\tau\tilde{e}_L\overline{\tilde{H}_1^{0c}}P_L e_L^c - h_b\tilde{D}\overline{\tilde{H}_1^{0c}}P_L d_L + h_b\tilde{D}\overline{\tilde{H}_1^{-c}}P_L u_L \\
& +h_b\tilde{u}_L\overline{\tilde{H}_1^{-c}}P_L d_L^c - h_b\tilde{d}_L\overline{\tilde{H}_1^{0c}}P_L d_L^c + h_t\tilde{U}\overline{\tilde{H}_2^{+c}}P_L d_L - h_t\tilde{U}\overline{\tilde{H}_2^{0c}}P_L u_L \\
& -h_t\tilde{u}_L\overline{\tilde{H}_2^{0c}}P_L u_L^c + h_t\tilde{d}_L\overline{\tilde{H}_2^{+c}}P_L u_L^c - h_\tau H_1^0\overline{e_L^c}P_L e_L^c + h_\tau H_1^-\overline{\nu^c}P_L e_L^c \\
& -h_b H_1^0\overline{d_L^c}P_L d_L^c + h_b H_1^-\overline{u_L^c}P_L d_L^c + h_t H_2^+\overline{d_L^c}P_L u_L^c - h_t H_2^0\overline{u_L^c}P_L u_L^c \\
& +\text{h.c.}
\end{aligned} \tag{B.6}
$$

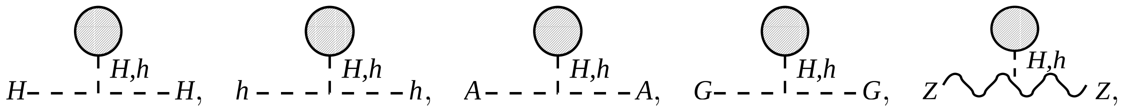
The Soft Breaking Terms

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{soft}} = \; & m1^2(|H_1^0|^2 + |H_1^-|^2) + m_2^2(|H_2^+|^2 + |H_2^0|^2) \\
& -m_{12}^2(H_1^0 H_2^0 - H_1^- H_2^+ + \mathrm{h.c.}) \\
& +M_{\tilde{Q}}^2 \tilde{t}_L^* \tilde{t}_L + M_{\tilde{Q}}^2 \tilde{b}_L^* \tilde{b}_L + M_{\tilde{U}}^2 \tilde{t}_R^* \tilde{t}_R + +M_{\tilde{D}}^2 \tilde{b}_R^* \tilde{b}_R \\
& M_{\tilde{L}}^2 \tilde{\nu}^* \tilde{\nu} + M_{\tilde{L}}^2 \tilde{\tau}_L^* \tilde{\tau}_L + M_{\tilde{E}}^2 \tilde{\tau}_R^* \tilde{\tau}_R \\
& +\frac{g}{\sqrt{2} m_W} \frac{m_\tau A_\tau}{\cos\beta}(H_1^0 \tilde{\tau}_L \tilde{\tau}_R^* - H_1^- \tilde{\nu} \tilde{\tau}_R^* + \mathrm{h.c.}) \\
& +\frac{g}{\sqrt{2} m_W} \frac{m_b A_b}{\cos\beta}(H_1^0 \tilde{b}_L \tilde{b}_R^* - H_1^- \tilde{t}_L \tilde{b}_R^* + \mathrm{h.c.}) \\
& -\frac{g}{\sqrt{2} m_W} \frac{m_t A_t}{\sin\beta}(H_2^+ \tilde{b}_L \tilde{t}_R^* - H_2^0 \tilde{t} \tilde{t}_R^* + \mathrm{h.c.}) \\
& +\frac{1}{2} M_3 \bar{\tilde{g}}^\alpha \tilde{g}^\alpha + \frac{1}{2} M_2 \bar{\tilde{W}}^a \tilde{W}^a + \frac{1}{2} M_1 \bar{\tilde{B}} \tilde{B}
\end{aligned} \tag{B.7}
$$

Appendix C

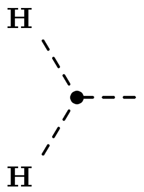
Tadpole Cancellation

The tadpole diagrams that appear in the correction to the mass sum rule Δ are the following

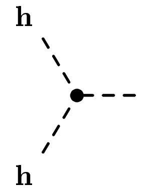


where the line running upwards is either the h or the H-Higgs and the blob denotes any process with one incoming h or H-Higgs. Irrespective of what happens in the blob, we can see that the tadpole contribution to Δ is zero, because the couplings for these diagrams add up to zero in the mass sum rule. (They vanish separately for the h and H diagrams.) The couplings are given in in [10] and agree with those of [4].

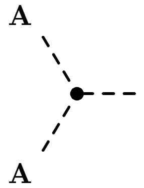
Rules for the H-Tadpoles



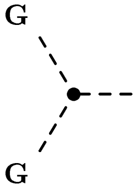
$$\mathbf{H} \quad = \quad -\frac{3igm_Z}{2\cos\theta_W}\cos 2\alpha\cos(\alpha+\beta) \tag{C.1}$$



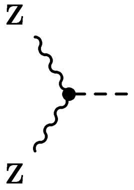
$$\mathbf{H} \quad = \quad -\frac{igm_Z}{2\cos\theta_W}\left[2\sin 2\alpha\sin(\alpha+\beta) - \cos 2\alpha\cos(\alpha+\beta)\right] \tag{C.2}$$



$$\mathbf{H} \quad = \quad \frac{igm_Z}{2\cos\theta_W}\cos 2\beta\cos(\alpha+\beta) \tag{C.3}$$

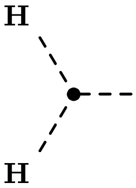


$$= -\frac{igm_Z}{2\cos\theta_W}\cos 2\beta\cos(\alpha+\beta) \tag{C.4}$$

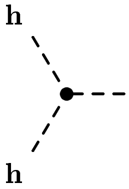


$$= \frac{igm_Z}{\cos\theta_W}\cos(\beta-\alpha)g^{\mu\nu} \tag{C.5}$$

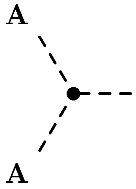
Rules for the h-Tadpoles



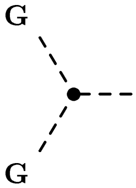
$$= -\frac{igm_Z}{2\cos\theta_W}\left[2\sin 2\alpha\cos(\alpha+\beta)+\cos 2\alpha\sin(\alpha+\beta)\right] \tag{C.6}$$



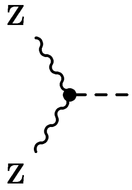
$$= -\frac{3igm_Z}{2\cos\theta_W}\cos 2\alpha\sin(\beta+\alpha) \tag{C.7}$$



$$= -\frac{igm_Z}{2\cos\theta_W}\cos 2\beta\sin(\alpha+\beta) \tag{C.8}$$



$$= \frac{igm_Z}{2\cos\theta_W}\cos 2\beta\sin(\alpha+\beta) \tag{C.9}$$



$$= \frac{igm_Z}{\cos\theta_W}\sin(\beta-\alpha)g^{\mu\nu} \tag{C.10}$$

Appendix D

The Diagram Table

In table D.1 we display all diagrams that are calculated to obtain the explicit formula for Δ due to one-loop top/stop corrections. The table also includes the signs for the different contributions according to the formula

$$\Delta = -\Pi_{HH}(m_H^2) - \Pi_{hh}(m_h^2) + \Pi_{AA}(m_A^2) - \Pi_{GG}(m_G^2) - A_{ZZ}(m_Z^2).$$

Please note that the diagrams are self-energy diagrams and therefore meant to be calculated without the external legs (which are however important in order to know which coupling has to be inserted). Also, for the diagrams with the Z-boson in the last line we do of course mean that part of the diagram which is proportional to $g^{\mu\nu}$ and will thus contribute to A_{ZZ}. One can see that the diagrams fall nicely into 6 distinct fields which correspond to the 6 general diagrams calculated in appendices E.1 - E.6. When we discuss the summation of the contributions, we will refer to the fields with the following numbers

No.	**Field**
1	Higgs with one scalar in the loop
2	Higgs with two scalars in the loop
3	Higgs with fermion in the loop
4	Z with one scalar in the loop
5	Z with two scalars in the loop
6	Z with fermion in the loop

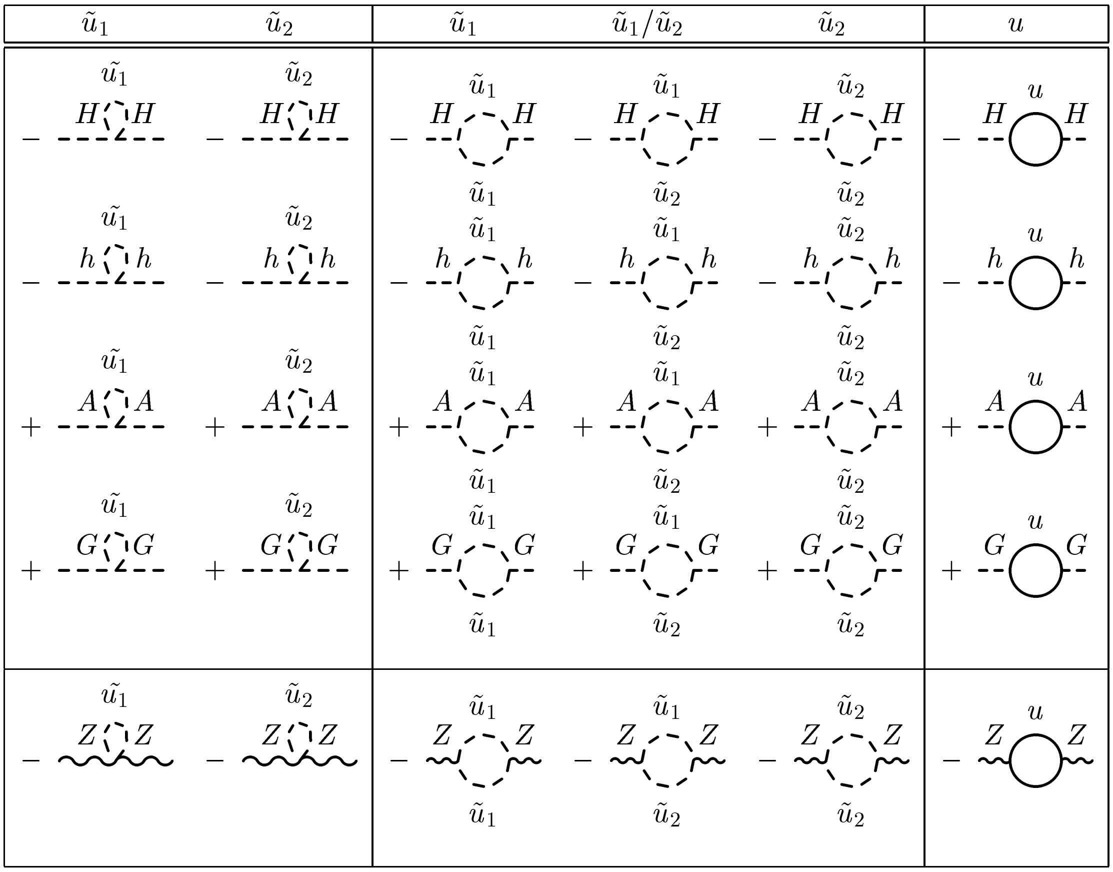


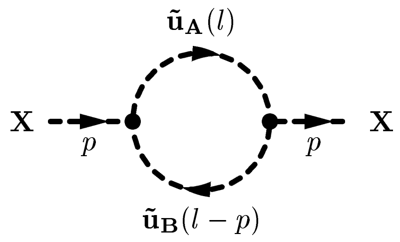
Table D.1: The diagram table

Appendix E

One-Loop Calculations

The one-loop diagrams are calculated in the $\overline{DR}$ scheme (*dimensional reduction*) which respects supersymmetry. The scheme says that first the γ-algebra is performed in 4 dimensions, then the integral is evaluated with the help of dimensional regularization.[1] The *bar* means that the renormalization point is chosen in the same way as in the $\overline{MS}$ scheme. We will see later in the explicit calculation what that means.

E.1 Higgs with Two Squarks in the Loop



We denote the couplings on the left and right with g_1 and g_2, respectively. Then the diagram gives

$$D_1(\tilde{u}_A, \tilde{u}_B) = g_1 g_2 \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - m_A^2 + i\epsilon} \frac{i}{(l-p)^2 - m_B^2 + i\epsilon}. \tag{E.1}$$

We use FEYNMAN-parameters to combine the denominators and analytically continue the integral to d dimensions. In order to preserve the correct mass dimension, a factor

[1] The difference to the ordinary *dimensional regularization* scheme is that there also the γ-algebra is performed in d dimensions.

of μ^{4-d} (the *renormalization scale*) is introduced.

$$D_1(\tilde{u}_A, \tilde{u}_B) = -g_1 g_2 \mu^{4-d} \int_0^1 dx \int \frac{d^d l}{(2\pi)^4} \frac{1}{[(l-xp)^2 - [m_B^2 x + m_A^2(1-x) - x(1-x)p^2]^2 + i\epsilon]^2}. \tag{E.2}$$

Now the loop momentum l is shifted $l \to l' = l - xp$ and renamed to l

$$D_1(\tilde{u}_A, \tilde{u}_B) = -g_1 g_2 \mu^{4-d} \int_0^1 dx \int \frac{d^d l}{(2\pi)^4} \frac{1}{[l^2 - [m_B^2 x + m_A^2(1-x) - x(1-x)p^2]^2 + i\epsilon]^2}. \tag{E.3}$$

Using formula (A.22), we arrive at

$$D_1(\tilde{u}_A, \tilde{u}_B) = -g_1 g_2 \mu^{4-d} \int_0^1 dx \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \tfrac{d}{2}) \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}}, \tag{E.4}$$

where

$$\Delta = m_B^2 x + m_A^2(1-x) - x(1-x)p^2. \tag{E.5}$$

Now let $d = 4 - \epsilon$ in order to approach $d = 4$ from below. Then it follows to $\mathcal{O}(\epsilon)$

$$\mu^{4-d} = 1 + \frac{\epsilon}{2} \log \mu^2, \tag{E.6}$$

$$\frac{1}{(4\pi)^{d/2}} = \frac{1}{(4\pi)^2}(1 + \frac{\epsilon}{2} \log 4\pi), \tag{E.7}$$

$$\Gamma(2 - \tfrac{d}{2}) = \frac{2}{\epsilon} - \gamma, \tag{E.8}$$

$$\left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} = 1 - \frac{\epsilon}{2} \log \Delta. \tag{E.9}$$

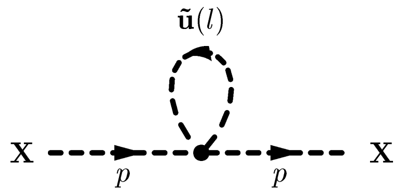
Multiplying the expression out and keeping only terms up to $\mathcal{O}(1)$, we have

$$D_1(\tilde{u}_A, \tilde{u}_B) = \frac{i g_1 g_2}{(4\pi)^2} \left[\frac{2}{\epsilon} - \int_0^1 dx \log \Delta - \gamma + \log 4\pi - \log \mu^2 \right]. \tag{E.10}$$

Here we can see now what the *bar* in $\overline{DR}$ means. We define $\frac{2}{\bar{\epsilon}} := \frac{2}{\epsilon} - \gamma + \log 4\pi$ and draw the two finite terms in the same way into the singularity as in the common $\overline{MS}$ scheme. Therefore, the final result is

$$D_1(\tilde{u}_A, \tilde{u}_B) = \frac{i g_1 g_2}{(4\pi)^2} \left[\frac{2}{\bar{\epsilon}} - \int_0^1 dx \log \frac{m_B^2 x + m_A^2(1-x) - x(1-x)p^2}{\mu^2} \right]. \tag{E.11}$$

E.2 Higgs with One Squark in the Loop



The coupling here is denoted by g, so that the diagram is given by

$$D_2(\tilde{u}) = g \int \frac{d^d l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\epsilon}. \tag{E.12}$$

Going to d dimensions and applying (A.22), we directly come to

$$D_2(\tilde{u}) = \mu^{4-d} \frac{g}{(4\pi)^{d/2}} \Gamma(1 - \tfrac{d}{2}) \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}}. \tag{E.13}$$

For $d = 4 - \epsilon$ this means

$$\begin{aligned} \Gamma(1 - \tfrac{d}{2}) &= -\frac{2}{\epsilon} + \gamma - 1, & \text{(E.14)} \\ \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} &= \Delta(1 - \frac{\epsilon}{2} \log \Delta), & \text{(E.15)} \end{aligned}$$

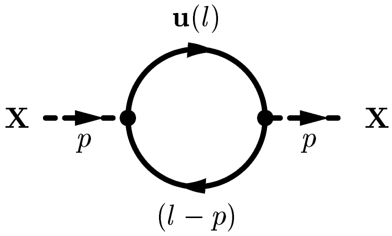
which leads to the $\mathcal{O}(1)$ result

$$D_2(\tilde{u}) = -\frac{gm^2}{(4\pi)^2} \left[\frac{2}{\epsilon} - \gamma + \log 4\pi - \log m^2 + 1 + \log \mu^2\right], \tag{E.16}$$

so that finally

$$D_2(\tilde{u}) = -\frac{gm^2}{(4\pi)^2} \left[\frac{2}{\bar{\epsilon}} - \log \frac{m^2}{\mu^2} + 1\right]. \tag{E.17}$$

E.3 Higgs with Fermion Loop



The couplings have a somewhat more complicated structure and are denoted by $g_0 + g_1\gamma_5$ for the left vertex and $h_0 + h_1\gamma_5$ for the right vertex. Then the diagram gives

$$\begin{aligned} D_3(u) &= -\int \frac{d^4 l}{(2\pi)^4} \mathrm{tr}\left[(g_0 + g_1\gamma_5)\frac{i}{\not{l} - m}(h_0 + h_1\gamma_5)\frac{i}{\not{l} - \not{p} - m}\right] \\ &= \int \frac{d^4 l}{(2\pi)^4} \frac{\mathrm{tr}\left[(g_0 + g_1\gamma_5)(\not{l} + m)(h_0 + h_1\gamma_5)(\not{l} - \not{p} + m)\right]}{[l^2 - m^2]\,[(l-p)^2 - m^2]} \end{aligned} \tag{E.18}$$

In the numerator, we use that the trace of an odd number of γ-matrices vanishes and then some identities for the γ-matrices in four dimensions from A.25. *Then* we analytically continue the integral to d dimensions. To combine the factors in the denominator, we use FEYNMAN-parameters. We get

$$D_3(u) = 4\mu^{4-d} \int \frac{d^d l}{(2\pi)^4} \int_0^1 dx \frac{(g_0 h_0 - g_1 h_1) l^\mu (l-p)_\mu + (g_0 h_0 + g_1 h_1) m^2}{\left[(l - xp)^2 - [m^2 - x(1-x)p^2] + i\epsilon\right]^2}. \tag{E.19}$$

After the shift in the loop momentum $l \to l' = l - xp$ and relabelling l' to l we drop the terms linear in l, because they vanish in the symmetric l-integration and get

$$\begin{aligned} D_3(u) &= 4\mu^{4-d} \int \frac{d^d l}{(2\pi)^4} \int_0^1 dx \left\{ \frac{(g_0 h_0 - g_1 h_1) l^\mu l_\mu}{\left[(l - xp)^2 - [m^2 - x(1-x)p^2] + i\epsilon\right]^2} \right. \\ &\quad \left. + \frac{-(g_0 h_0 - g_1 h_1) x(1-x) p_\mu p^\mu + (g_0 h_0 + g_1 h_1) m^2}{\left[(l - xp)^2 - [m^2 - x(1-x)p^2] + i\epsilon\right]^2} \right\}. \end{aligned} \tag{E.20}$$

For the first part of the sum, we use formula (A.23) and formula (A.22) for the second part. This yields

$$\begin{aligned} D_3(u) &= 4\int_0^1 dx \left\{ (g_0 h_0 - g_1 h_1) \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \Gamma(1 - \tfrac{d}{2}) \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} \right. \qquad \text{(E.21)} \\ &\quad \left. + \left[-(g_0 h_0 - g_1 h_1) x(1-x) p_\mu p^\mu + (g_0 h_0 + g_1 h_1) m^2\right] \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \tfrac{d}{2}) \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \right\}. \end{aligned}$$

Now we let again $d = 4 - \epsilon$ and collect all terms of $\mathcal{O}(1)$. We already know the result for the second part of the sum, which is the same integral as in E.1. For evaluating the first part, note that it is trivial but important not to forget to use

$$\frac{d}{2} = 2 - \frac{\epsilon}{2}. \tag{E.22}$$

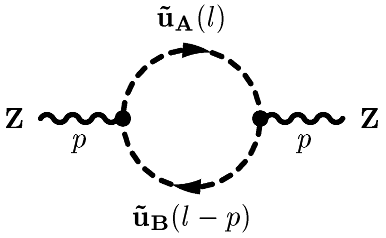
Then we arrive to $\mathcal{O}(1)$ at the result

$$\begin{aligned} D_3(u) &= \frac{4i}{(4\pi)^2} \Bigg\{ (g_0 h_0 - g_1 h_1) \int_0^1 dx \Delta \left[\frac{4}{\epsilon} + 1 - 2\gamma - 2\log\frac{\Delta}{\mu^2} + 2\log 4\pi \right] \\ &\quad + m^2 (g_0 h_0 + g_1 h_1) \left[\frac{2}{\epsilon} - \int_0^1 dx \log\frac{\Delta}{\mu^2} - \gamma + \log 4\pi \right] \\ &\quad + p^2 (g_0 h_0 - g_1 h_1) \int_0^1 dx \left(-x(1-x) \left[\frac{2}{\epsilon} - \log\frac{\Delta}{\mu^2} - \gamma + \log 4\pi \right] \right) \Bigg\}, \end{aligned} \tag{E.23}$$

which in the $\overline{DR}$-scheme is

$$\begin{aligned} D_3(u) &= \frac{4i}{(4\pi)^2} \Bigg\{ (g_0 h_0 - g_1 h_1) \int_0^1 dx \Delta \left[\frac{4}{\bar{\epsilon}} - 2\log\frac{\Delta}{\mu^2} + 1 \right] \\ &\quad + m^2 (g_0 h_0 + g_1 h_1) \left[\frac{2}{\bar{\epsilon}} - \int_0^1 dx \log\frac{\Delta}{\mu^2} \right] \\ &\quad + p^2 (g_0 h_0 - g_1 h_1) \int_0^1 dx \left(-x(1-x) \left[\frac{2}{\bar{\epsilon}} - \log\frac{\Delta}{\mu^2} \right] \right) \Bigg\}. \end{aligned} \tag{E.24}$$

E.4 Z with Two Squarks in the Loop



The left vertex carries the coupling $g_1(p+p')$ and the right vertex the coupling $g_2(p+p')$. Then the diagram stands for

$$D_4(\tilde{u}_A, \tilde{u}_B) = g_1 g_2 \int \frac{d^4l}{(2\pi)^4}(l^\mu + (l-p)^\mu)\frac{i}{l^2 - m_A^2}((l-p)^\nu - l^\nu)\frac{i}{(l-p)^2 - m_B^2}. \quad \text{(E.25)}$$

Next we apply FEYNMAN-parameterization, shift the loop variable and drop terms linear in l. After going to d dimensions, we have

$$D_4(\tilde{u}_A, \tilde{u}_B) = -4\mu^{4-d} g_1 g_2 \int \frac{d^dl}{(2\pi)^4}\int_0^1 dx \left\{\frac{l^\mu l^\nu + (x(x-1)-1)p^\mu p^\nu}{[l^2-\Delta]^2}\right\}, \quad \text{(E.26)}$$

where

$$\Delta = m_B^2 x + m_A^2(1-x) - x(1-x)p^2. \quad \text{(E.27)}$$

Obviously, we have a part proportional to $p^\mu p^\nu$, which does not contribute and is therefore dropped. The rest is evaluated with the help of (A.24).

$$D_4(\tilde{u}_A, \tilde{u}_B) = 4\mu^{4-d} g_1 g_2 \int_0^1 dx \frac{i}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\Gamma(1-\tfrac{d}{2})\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}}. \quad \text{(E.28)}$$

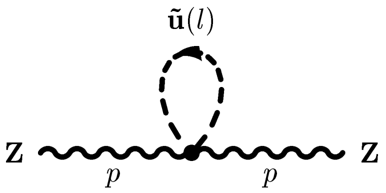
For $d = 4 - \epsilon$ we get the $\mathcal{O}(1)$ result

$$D_4(\tilde{u}_A, \tilde{u}_B) = -\frac{4ig_1g_2}{(4\pi)^2}\frac{g^{\mu\nu}}{2}\int_0^1 dx\Delta\left[\frac{2}{\epsilon} - \gamma + 1 + \log 4\pi - \log\Delta + \log\mu^2\right] \quad \text{(E.29)}$$

and therefore, in the $\overline{DR}$ scheme

$$D_4(\tilde{u}_A, \tilde{u}_B) = -\frac{2ig_1g_2}{(4\pi)^2}g^{\mu\nu}\int_0^1 dx\Delta\left[\frac{2}{\bar{\epsilon}} - \log\frac{\Delta}{\mu^2} + 1\right]. \quad \text{(E.30)}$$

E.5 Z with One Squark in the Loop



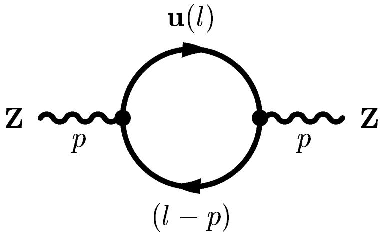
The coupling has the structure $g \cdot g^{\mu\nu}$. The diagram is given by

$$D_5(\tilde{u}) = g g^{\mu\nu} \int \frac{d^d l}{(2\pi)^4} \frac{i}{l^2 - m^2 + i\epsilon}. \tag{E.31}$$

The calculation is exactly the same as in E.2 so that we can directly give the result

$$D_5(\tilde{u}) = -\frac{g}{(4\pi)^2} g^{\mu\nu} m^2 \left[\frac{2}{\epsilon} - \log \frac{m^2}{\mu^2} + 1 \right]. \tag{E.32}$$

E.6 Z with Fermion Loop



The coupling of a boson to two fermions has a part proportional to γ_μ and a part proportional to $\gamma_\mu\gamma_5$. The couplings are denoted by $a_1\gamma_\mu + b_1\gamma_\mu\gamma_5$ and $a_2\gamma_\nu + b_2\gamma_\nu\gamma_5$. Then the graph is[2]

$$\begin{aligned} D_6(u) &= -\int \frac{d^4l}{(2\pi)^4} \mathrm{tr}\left[(a_1\gamma_\mu + b_1\gamma_\mu\gamma_5)\frac{i}{\not{l} - m}(a_2\gamma_\nu + b_2\gamma_\nu\gamma_5)\frac{i}{\not{l} - \not{p} - m}\right] \\ &= \int \frac{d^4l}{(2\pi)^4} \frac{\mathrm{tr}\left[(a_1\gamma_\mu + b_1\gamma_\mu\gamma_5)(\not{l} + m)(a_2\gamma_\nu + b_2\gamma_\nu\gamma_5)(\not{l} - \not{p} + m)\right]}{[l^2 - m^2]\,[(l-p)^2 - m^2]} \end{aligned} \tag{E.33}$$

After some γ-algebra gymnastics in 4 dimensions (i.e. dropping products with odd number of γs and using equations (A.25) we arrive at

$$\begin{aligned} D_6(u) &= 4\int \frac{d^4l}{(2\pi)^4} \frac{(a_1a_2 + b_1b_2)(-l^\lambda(l-p)_\lambda) + (a_1a_2 - b_1b_2)m^2}{[l^2 - m^2]\,[(l-p)^2 - m^2]} g^{\mu\nu} \\ &\quad +4\int \frac{d^4l}{(2\pi)^4} \frac{(a_1a_2 + b_1b_2)(l_\mu(l-p)_\nu + l_\nu(l-p)_\mu)}{[l^2 - m^2]\,[(l-p)^2 - m^2]}. \end{aligned} \tag{E.34}$$

The result for the first integral is already known to us, since the structure already appeared in E.3. It is

$$\begin{aligned} &\frac{4i}{(4\pi)^2}\left\{-(a_1a_2 + b_1b_2)\int_0^1 dx \Delta\left[\frac{4}{\bar{\epsilon}} - 2\log\Delta + 1\right]\right. \\ &-p^2(a_1a_2 + b_1b_2)\int_0^1 dx\left[-x(1-x)\left(\frac{2}{\bar{\epsilon}} - \log\Delta\right)\right] \\ &\left.+m^2(a_1a_2 - b_1b_2)\left[\frac{2}{\bar{\epsilon}} - \int_0^1 dx \log\Delta\right]\right\} g^{\mu\nu}. \end{aligned} \tag{E.35}$$

Only the last part has to be evaluated. After the introduction of FEYNMAN-parameters and the shift of the loop momentum it is in d dimensions

$$8\mu^{4-d}\int_0^1 dx \int \frac{d^dl}{(2\pi)^d} \frac{(a_1a_2 + b_1b_2)l_\mu l_\nu}{[l^2 - \Delta]^2}, \tag{E.36}$$

[2] Note that this is a normal Standard Model Graph

where

$$\Delta = m^2 - x(1-x)p^2. \tag{E.37}$$

Then equation (A.24) gives

$$-8\mu^{4-d}(a_1a_2 + b_1b_2)\int_0^1 dx \frac{i}{(4\pi)^{d/2}}\frac{g^{\mu\nu}}{2}\Gamma(1-\tfrac{d}{2})\left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}}, \tag{E.38}$$

so that for $d = 4 - \epsilon$ we get the $\mathcal{O}(1)$ result

$$\frac{4i}{(4\pi)^2}(a_1a_2 + b_1b_2)\int_0^1 dx\Delta\left[\frac{2}{\epsilon} - \gamma + \log 4\pi - \log\Delta + \log\mu^2 + 1\right] \tag{E.39}$$

and in the $\overline{DR}$ scheme

$$\frac{4i}{(4\pi)^2}(a_1a_2 + b_1b_2)\int_0^1 dx\Delta\left[\frac{2}{\bar{\epsilon}} - \log\frac{\Delta}{\mu^2} + 1\right]. \tag{E.40}$$

Adding the two contributions, the final result is

$$\begin{aligned} D_6(u) &= \frac{8i}{(4\pi)^2}\left\{-a_1a_2\int_0^1 dx(-x(1-x)p^2)\left[\frac{2}{\bar{\epsilon}} - \log\frac{\Delta}{\mu^2}\right]\right. \\ &\quad \left. -b_1b_2)\int_0^1 dx(m^2 - x(1-x)p^2)\left[\frac{2}{\bar{\epsilon}} - \log\frac{\Delta}{\mu^2}\right]\right\} g^{\mu\nu}. \end{aligned} \tag{E.41}$$

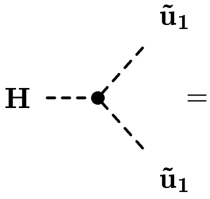
Appendix F

The Feynman Rules

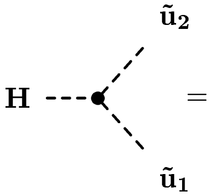
Here we display the FEYNMAN rules needed for the calculation. The rules involving Higgs bosons were taken from [4] while those involving the Z boson can be found in [3]. In these works, the rules are stated in the interaction base. For our purpose they were transformed into the mass eigenstate basis in which we display them here.

F.1 Rules for Diagram 1

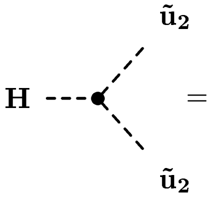
Higgs H and Two Stops



$$= -\frac{igm_Z}{\cos\theta_W}\left[T_3\cos^2\theta_u - e_u\sin^2\theta_W\cos 2\theta_u\right]\cos(\alpha+\beta) - \frac{igm_u^2}{m_W\sin\beta}\sin\alpha$$
$$+\sin 2\theta_u\frac{igm_u}{2m_W\sin\beta}(\mu\cos\alpha - A_u\sin\alpha)$$

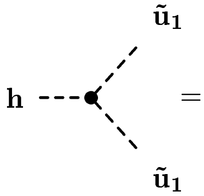


$$= -\frac{igm_Z}{\cos\theta_W}\left[-T_3 + 2e_u\sin^2\theta_W\right]\cos(\alpha+\beta)\frac{\sin 2\theta_u}{2}$$
$$+\cos 2\theta_u\frac{igm_u^2}{2m_W\sin\beta}(\mu\cos\alpha - A_u\sin\alpha)$$

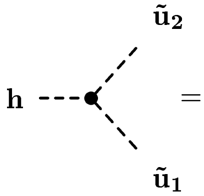


$$= -\frac{igm_Z}{\cos\theta_W}\left[T_3\sin^2\theta_u + e_u\sin^2\theta_W\cos 2\theta_u\right]\cos(\alpha+\beta) - \frac{igm_u^2}{m_W\sin\beta}\sin\alpha$$
$$-\sin 2\theta_u\frac{igm_u}{2m_W\sin\beta}(\mu\cos\alpha - A_u\sin\alpha)$$

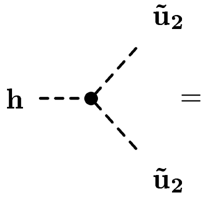
Higgs h and Two Stops



$$= \frac{igm_Z}{\cos\theta_W}\left[T_3\cos^2\theta_u - e_u\sin^2\theta_W\cos 2\theta_u\right]\sin(\alpha+\beta) - \frac{igm_u^2}{m_W\sin\beta}\cos\alpha - \frac{igm_u}{2m_W\sin\beta}(\mu\sin\alpha + A_u\cos\alpha)\sin 2\theta_u$$

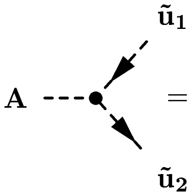


$$= \frac{igm_Z}{2\cos\theta_W}\left[-T_3 + 2e_u\sin^2\theta_W\right]\sin(\alpha+\beta)\sin 2\theta_u - \frac{igm_u}{2m_W\sin\beta}(\mu\sin\alpha + A_u\cos\alpha)\cos 2\theta_u$$

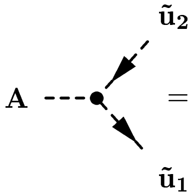


$$= \frac{igm_Z}{\cos\theta_W}\left[T_3\sin^2\theta_u + e_u\sin^2\theta_W\cos 2\theta_u\right]\sin(\alpha+\beta) - \frac{igm_u^2}{m_W\sin\beta}\cos\alpha + \frac{igm_u}{2m_W\sin\beta}(\mu\sin\alpha + A_u\cos\alpha)\sin 2\theta_u$$

Higgs A and Two Stops

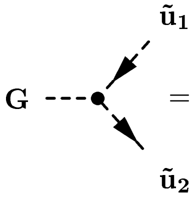


$$= \frac{gm_u}{2m_W}(\mu + A_u\cot\beta)$$

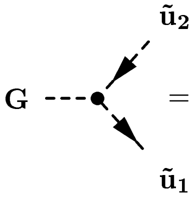


$$= -\frac{gm_u}{2m_W}(\mu + A_u\cot\beta)$$

Goldstone G and Two Stops



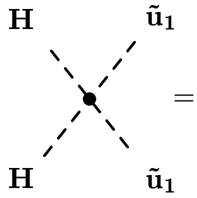
$$= -\frac{gm_u}{2m_W}(\mu\cot\beta - A_u)$$



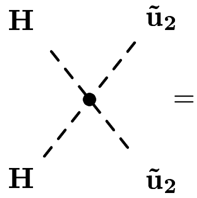
$$= \frac{gm_u}{2m_W}(\mu\cot\beta - A_u)$$

F.2 Rules for Diagram 2

Two Higgses H and Two Stops

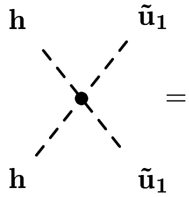


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} - e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(-\cos 2\alpha) - \frac{m_u^2}{m_W^2}\frac{\sin^2\alpha}{\sin^2\beta}\right]$$

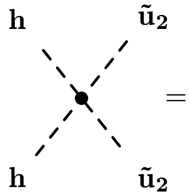


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} + e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(-\cos 2\alpha) - \frac{m_u^2}{m_W^2}\frac{\sin^2\alpha}{\sin^2\beta}\right]$$

Two Higgses h and Two Stops

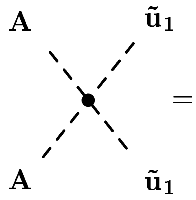


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} - e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(\cos 2\alpha) - \frac{m_u^2}{m_W^2}\frac{\cos^2\alpha}{\sin^2\beta}\right]$$

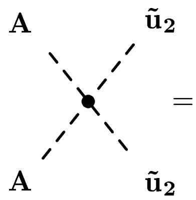


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} + e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(\cos 2\alpha) - \frac{m_u^2}{m_W^2}\frac{\cos^2\alpha}{\sin^2\beta}\right]$$

Two Higgses A and Two Stops

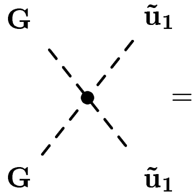


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} - e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(\cos 2\beta) - \frac{m_u^2}{m_W^2}\cot^2\beta\right]$$

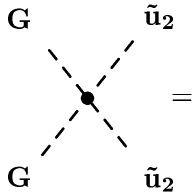


$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} + e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(\cos 2\beta) - \frac{m_u^2}{m_W^2}\cot^2\beta\right]$$

Two Goldstones G and Two Stops



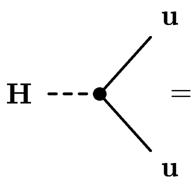
$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} - e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(-\cos 2\beta) - \frac{m_u^2}{m_W^2}\right]$$



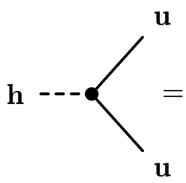
$$= \frac{ig^2}{2}\left[\frac{\frac{1}{2} + e_u \sin^2\theta_W \cos 2\theta_u}{\cos^2\theta_W}(-\cos 2\beta) - \frac{m_u^2}{m_W^2}\right]$$

F.3 Rules for Diagram 3

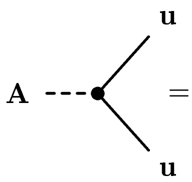
Higgs H,h,A/Goldstone G and Two Tops



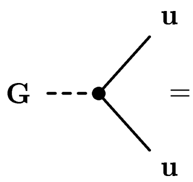
$$= -\frac{igm_u \sin\alpha}{2m_W \sin\beta}$$



$$= -\frac{igm_u \cos\alpha}{2m_W \sin\beta}$$



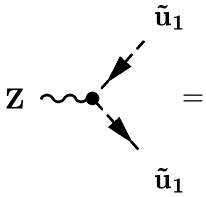
$$= -\frac{gm_u \cot\beta}{2m_W}\gamma_5$$



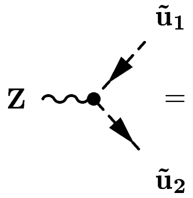
$$= -\frac{gm_u}{2m_W}\gamma_5$$

F.4 Rules for Diagram 4

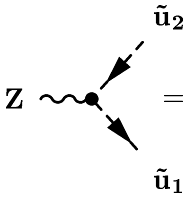
Z-Boson and Two Stops



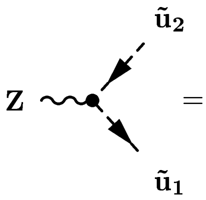
$$= \frac{ig}{\cos\theta_W}\left[(-T_3 + e_u \sin^2\theta_W)\cos^2\theta_u + (e_u \sin^2\theta_W)\sin^2\theta_u\right](p+p')^\mu$$



$$= \frac{ig}{\cos\theta_W}\frac{\sin 2\theta_u}{4}(p+p')^\mu$$



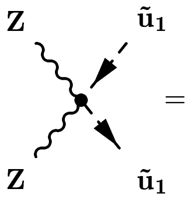
$$= \frac{ig}{\cos\theta_W}\frac{\sin 2\theta_u}{4}(p+p')^\mu$$



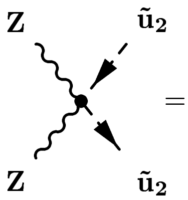
$$= \frac{ig}{\cos\theta_W}\left[(-T_3 + e_u \sin^2\theta_W)\sin^2\theta_u + (e_u \sin^2\theta_W)\cos^2\theta_u\right](p+p')^\mu$$

F.5 Rules for Diagram 5

Two Z-Bosons and Two Stops



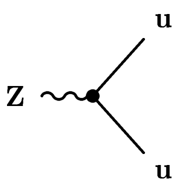
$$= \frac{2ig^2}{\cos^2\theta_W}\left[(T_3 - e_u \sin^2\theta_W)^2\cos^2\theta_u + (-e_u \sin^2\theta_W)^2\sin^2\theta_u\right] g_{\mu\nu}$$



$$= \frac{2ig^2}{\cos^2\theta_W}\left[(T_3 - e_u \sin^2\theta_W)^2\sin^2\theta_u + (-e_u \sin^2\theta_W)^2\cos^2\theta_u\right] g_{\mu\nu}$$

F.6 Rules for Diagram 6

Z-Boson and Two Tops



$$= -\frac{ig}{2\cos\theta_W}\gamma^\mu\left[(T_3 - e_u \sin^2\theta_W)(1-\gamma_5) - e_u \sin^2\theta_W(1+\gamma_5)\right]$$

Appendix G

Calculation of the Correction to the Mass Sum Rule

From the calculations in appendix E it is clear that the six diagrams have a common typical structure

$$\sim \int_0^1 \left[\frac{2}{\epsilon} + A(x)\log\frac{B(m_1^2, m_2^2, m_3^2)}{\mu^2} + C\right]. \tag{G.1}$$

For the calculation of Δ it seems convenient to look at the three addends seperately. We will therefore call the part which goes with $\frac{2}{\epsilon}$ the *divergent part*, the terms which contain the logarithms make up the *logarithmic part* and the rest which we denoted with C is the *nonlogarithmic part*. The divergent terms will have to cancel against each other, while the logarithmic and constant terms form the finite result.

G.1 The Divergent Part

The correction to the mass sum rule is a *physical* quantity which can be measured in collider experiments, so it cannot be a divergent expression. Thus, all the terms from the different diagram which go with $\frac{2}{\epsilon}$ must cancel among themselves. We will see that this is indeed the case. We now display the divergent contributions from all six fields from table D.1 in appendix D. We denote them by D_n for the n-th field. The diagrams in the first field vanish completely, because the couplings add up to zero. Thus,

$$D_1 = 0.$$

In the second field, we have many cancellations within the field and only a small rest, which is

$$
\begin{aligned}
D_2 \quad = \quad & -\frac{2g^2 m_u^4}{(4\pi)^2 m_W^2 \sin^2\beta}\frac{2}{\epsilon} + \frac{g^2 m_u^2}{8\pi^2\cos^2\theta_W}T_3\frac{2}{\epsilon} - \frac{g^2 m_Z^2}{32\pi^2\cos^2\theta_W}\left[-T_3 + 2e_u\sin^2\theta_W\right]^2\sin^2\theta_u\frac{2}{\epsilon} \\
& -\frac{g^2 m_Z^2}{(4\pi)^2\cos^2\theta_W}\left[\sin^2\theta_u(T_3 - e_u\sin^2\theta_W) + e_u\cos^2\theta_u\sin^2\theta_W\right]^2\frac{2}{\epsilon}
\end{aligned}
$$

The other fields do not have so many inner cancellations. Their results are

$$
\begin{aligned}
D_3 \quad = \quad & +\frac{g^2 m_u^4}{8\pi^2 m_W^2\sin^2\beta}\frac{2}{\epsilon} + \frac{g^2 m_u^2}{32\pi^2 m_W^2\sin^2\beta}\left[-m_H^2\sin^2\alpha - m_h^2\cos^2\alpha + m_A^2\cos^2\beta\right]\frac{2}{\epsilon}, \\
D_4 \quad = \quad & -\frac{g^2 m_{\tilde{u}_1}^2}{8\pi^2\cos^2\theta_W}\left\{\left[(-T_3 + e_u\sin^2\theta_W)\cos^2\theta_u + e_u\sin^2\theta_W\sin^2\theta_u\right]^2 + \tfrac{1}{4}\cos^2\theta_u\sin^2\theta_u\right\}\frac{2}{\epsilon} \\
& -\frac{g^2 m_{\tilde{u}_2}^2}{8\pi^2\cos^2\theta_W}\left\{\left[(-T_3 + e_u\sin^2\theta_W)\sin^2\theta_u + e_u\sin^2\theta_W\cos^2\theta_u\right]^2 + \tfrac{1}{4}\cos^2\theta_u\sin^2\theta_u\right\}\frac{2}{\epsilon} \\
& +\frac{g^2 m_Z^2}{48\pi^2\cos^2\theta_W}\left\{\left[(-T_3 + e_u\sin^2\theta_W)\cos^2\theta_u + e_u\sin^2\theta_W\sin^2\theta_u\right]^2 + \tfrac{1}{2}\cos^2\theta_u\sin^2\theta_u\right., \\
& \left. + \left[(-T_3 + e_u\sin^2\theta_W)\sin^2\theta_u + e_u\sin^2\theta_W\cos^2\theta_u\right]^2\right\}\frac{2}{\epsilon} \\
D_5 \quad = \quad & +\frac{g^2 m_{\tilde{u}_1}^2}{8\pi^2\cos^2\theta_W}\left(\left[(T_3 - e_u\sin^2\theta_W)^2\cos^2\theta_u + (-e_u\sin^2\theta_w)^2\sin^2\theta_u\right]\right) \\
& +\frac{g^2 m_{\tilde{u}_1}^2}{8\pi^2\cos^2\theta_W}\left(\left[(T_3 - e_u\sin^2\theta_W)^2\cos^2\theta_u + (-e_u\sin^2\theta_w)^2\sin^2\theta_u\right]\right), \\
D_6 \quad = \quad & +\frac{g^2 m_Z^2}{(4\pi)^2\cos^2\theta_W 3}\left(\left[T_3 - 2e_u\sin^2\theta_W\right]^2 + T_3^2\right)\frac{2}{\epsilon} - \frac{g^2 m_u^2}{8\pi^2\cos^2\theta_W}T_3^2\frac{2}{\epsilon}.
\end{aligned}
$$

The sum of all these terms is zero. The tricky part is to see that the very last term can be rewritten into

$$
\begin{aligned}
-\frac{g^2 m_u^2}{8\pi^2\cos^2\theta_W}T_3^2 \quad & = \quad +\frac{g^2 m_u^2}{8\pi^2\cos^2\theta_W}T_3^2 - 2\frac{g^2 m_u^2}{8\pi^2\cos^2\theta_W}T_3^2 \\
& = \quad \frac{g^2 m_u^2}{32\pi^2 m_W^2\sin^2\beta}m_Z^2\sin^2\beta - \frac{g^2 m_u^2}{16\pi^2\cos^2\theta_W}
\end{aligned}
$$

so that it cancels the corresponding terms from the second and third field by using that

$$
-m_H^2\sin^2\alpha - m_h^2\cos^2\alpha + m_a^2\cos^2\beta + m_z^2\sin^2\beta = 0.
$$

This *Cancellation of Divergences* is a neccessary consistency check for the calculation.

G.2 The Finite Part

G.2.1 Logarithmic Terms

For the logarithmic terms, we adopt the notation of [10] and define

$$\begin{aligned} F(m_1, m_2, m_3) &= \int_0^1 dx \log\left[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu^2}\right], \\ G(m_1, m_2, m_3) &= \int_0^1 dx\, x(1-x) \log\left[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu^2}\right], \\ H(m_1, m_2, m_3) &= \int_0^1 dx\, x \log\left[\frac{xm_1^2 + (1-x)m_2^2 - x(1-x)m_3^2}{\mu^2}\right]. \end{aligned}$$

It is useful to note that F and G are symmetric in m_1 and m_2, while H fulfills

$$H(m_1, m_2, m_3) = F(m_2, m_1, m_3) - H(m_2, m_1, m_3).$$

In order to evaluate the F,G and H functions numerically, we make use of the functions B_0', B_1^prime and B_{21}' defined in (A.26). But let us now discuss the contributions from the various fields of table D.1. The first field vanishes completely, so it does not contribute terms here, either. The second field brings in the largest contribution.

$$\begin{aligned} L_2 = \quad & +\frac{g^2 m_u^4 N}{(4\pi)^2 m_W^2 \sin^2\beta} && \left\{\sin^2\alpha\left[F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H)\right]\right\} \\ & && + \cos^2\alpha\left[F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h) + F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h)\right]\} \\ & +\frac{g^2 m_u^3 \sin 2\theta_u N}{(4\pi)^2 m_W^2 \sin^2\beta} && \{(A_u \sin\alpha - \mu\cos\alpha)\sin\alpha\left[F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H)\right] \\ & && + (A_u\cos\alpha + \mu\sin\alpha)\cos\alpha\left[F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h)\right]\} \\ & -\frac{g^2 m_u^2 \sin^2 2\theta_u N}{4(4\pi)^2 m_W^2 \sin^2\beta} && \left\{(A_u\sin\alpha - \mu\cos\alpha)^2\left[-F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H)\right]\right. \\ & && \left. + (A_u\cos\alpha + \mu\sin\alpha)^2\left[-F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h) - F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h)\right]\right\} \\ & -\frac{g^2 m_u^2 \cos^2 2\theta_u N}{4(4\pi)^2 m_W^2 \sin^2\beta} && \left\{(A_u\sin\alpha - \mu\cos\alpha)^2\left[-2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_H)\right]\right. \\ & && \left. + (A_u\cos\alpha + \mu\sin\alpha)^2\left[-2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_h)\right]\right\} \\ & -\frac{g^2 m_u^2 N}{4(4\pi)^2 m_W^2 \sin^2\beta} && \left\{(A_u\cos\beta + \mu\sin\beta)^2 2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_A)\right. \\ & && \left. + (A_u\sin\beta - \mu\cos\beta)^2 2F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_G)\right\} \\ & +\frac{g^2 m_u^2 N}{8\pi^2\cos^2\theta_W\sin\beta} && \left\{\left[\cos^2\theta_u(T_3 - e_u\sin^2\theta_W) + \sin^2\theta_u(e_u\sin^2\theta_W)\right]\right. \\ & && \times(\sin\alpha\cos(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) - \cos\alpha\sin(\alpha+\beta)F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h)) \\ & && + \left[\sin^2\theta_u(T_3 - e_u\sin^2\theta_W) + \cos^2\theta_u(e_u\sin^2\theta_W)\right] \\ & && \left. \times(\sin\alpha\cos(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H) - \cos\alpha\sin(\alpha+\beta)F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h))\right\} \end{aligned}$$

$$
\begin{aligned}
&+\frac{g^2 m_u \sin 2\theta_u N}{(4\pi)^2 \cos^2\theta_W \sin\beta} && \cos(\alpha+\beta)(A_u \sin\alpha - \mu\cos\alpha) \\
& && \times \Big\{ \left[\cos^2\theta_u (T_3 - e_u \sin^2\theta_W) + \sin^2\theta_u (e_u \sin^2\theta_W)\right] F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) \\
& && + \left[(-T_3 + 2e_u \sin^2\theta_W)\cos 2\theta_u\right] F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_H) \\
& && - \left[\sin^2\theta_u (T_3 - e_u \sin^2\theta_W) + \cos^2\theta_u (e_u \sin^2\theta_W)\right] F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H) \Big\} \\
&-\frac{g^2 m_u \sin 2\theta_u N}{(4\pi)^2 \cos^2\theta_W \sin\beta} && \sin(\alpha+\beta)(A_u \cos\alpha + \mu\sin\alpha) \\
& && \times \Big\{ \left[\cos^2\theta_u (T_3 - e_u \sin^2\theta_W) + \sin^2\theta_u (e_u \sin^2\theta_W)\right] F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h) \\
& && + \left[(-T_3 + 2e_u \sin^2\theta_W)\cos 2\theta_u\right] F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_h) \\
& && - \left[\sin^2\theta_u (T_3 - e_u \sin^2\theta_W) + \cos^2\theta_u (e_u \sin^2\theta_W)\right] F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h) \Big\} \\
&+\frac{g^2 m_Z^2 N}{(4\pi)^2 \cos^2\theta_W} && \Big\{ \left[\cos^2\theta_u (T_3 - e_u \sin^2\theta_W) + \sin^2\theta_u (e_u \sin^2\theta_W)\right]^2 \\
& && \times \left(\cos^2(\alpha+\beta) F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_H) + \sin^2(\alpha+\beta) F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_h)\right) \\
& && + \frac{1}{2}\left[(-T_3 + 2e_u \sin^2\theta_W\right]^2 \sin^2 2\theta_u \\
& && \times \left(\cos^2(\alpha+\beta) F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_H) + \sin^2(\alpha+\beta) F(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_h)\right) \\
& && + \left[\sin^2\theta_u (T_3 - e_u \sin^2\theta_W) + \cos^2\theta_u (e_u \sin^2\theta_W)\right]^2 \\
& && \times \left(\cos^2(\alpha+\beta) F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_H) + \sin^2(\alpha+\beta) F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_h)\right) \Big\} .
\end{aligned}
$$

The terms that come from the third field are

$$
\begin{aligned}
L_3 = \; &+\frac{g^2 m_u^4 N}{(4\pi)^2 m_W^2 \sin^2\beta} && \left\{-3\sin^2\alpha F(m_u, m_u, m_H) - 3\cos^2\alpha F(m_u, m_u, m_h)\right. \\
& && \left. + \cos^2\beta F(m_u, m_u, m_A) + \sin^2\beta F(m_u, m_u, m_G)\right\} \\
&+\frac{3 g^2 m_u^2 N}{(4\pi)^2 m_W^2 \sin^2\beta} && \left\{+\sin^2\alpha m_H^2 G(m_u, m_u, m_H) + \cos^2\alpha m_h^2 G(m_u, m_u, m_h)\right. \\
& && \left. - \cos^2\beta m_A^2 G(m_u, m_u, m_A) - \sin^2\beta m_G^2 G(m_u, m_u, m_G)\right\} .
\end{aligned}
$$

The terms from the fourth field are

$$
\begin{aligned}
L_4 = \; &-\frac{g^2 m_{\tilde{u}_1}^2 N}{8\pi^2 \cos^2\theta_W} && \left[(T_3 - e_u \sin^2\theta_W)^2 \cos^2\theta_u + (e_u \sin^2\theta_W)^2 \sin^2\theta_u\right] F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, 0) \\
&-\frac{g^2 m_{\tilde{u}_2}^2 N}{8\pi^2 \cos^2\theta_W} && \left[(T_3 - e_u \sin^2\theta_W)^2 \sin^2\theta_u + (e_u \sin^2\theta_W)^2 \cos^2\theta_u\right] F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, 0).
\end{aligned}
$$

From the fifth field

$$
\begin{aligned}
L_5 = \; & +\frac{g^2 m_{\tilde{u}_1}^2 N}{8\pi^2 \cos^2\theta_W} && \Big\{ \big[(-T_3 + e_u \sin^2\theta_W)\cos^2\theta_u + e_u \sin^2\theta_W \sin^2\theta_u \big]^2 F(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_Z) \\
& && +\tfrac{1}{2}\sin^2\theta_u \cos^2\theta_u H(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_Z) \Big\} \\
& +\frac{g^2 m_{\tilde{u}_2}^2 N}{8\pi^2 \cos^2\theta_W} && \Big\{ \big[(-T_3 + e_u \sin^2\theta_W)\sin^2\theta_u + e_u \sin^2\theta_W \cos^2\theta_u \big]^2 F(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_Z) \\
& && +\tfrac{1}{2}\sin^2\theta_u \cos^2\theta_u H(m_{\tilde{u}_2}, m_{\tilde{u}_1}, m_Z) \Big\} \\
& -\frac{g^2 m_Z^2 N}{8\pi^2 \cos^2\theta_W} && \Big\{ \big[(-T_3 + e_u \sin^2\theta_W)\cos^2\theta_u + (e_u \sin^2\theta_W) sin^2\theta_u \big]^2 G(m_{\tilde{u}_1}, m_{\tilde{u}_1}, m_Z) \\
& && +\tfrac{1}{2}\sin^2\theta_u \cos^2\theta_u G(m_{\tilde{u}_1}, m_{\tilde{u}_2}, m_Z) \\
& && \big[(-T_3 + e_u \sin^2\theta_W)\sin^2\theta_u + (e_u \sin^2\theta_W)\cos^2\theta_u \big]^2 G(m_{\tilde{u}_2}, m_{\tilde{u}_2}, m_Z) \Big\} .
\end{aligned}
$$

From the sixth field we get

$$
\begin{aligned}
L_6 = \; & +\frac{g^2 m_u^2 N}{8\pi^2 \cos^2\theta_W} && T_3^2 F(m_u, m_u, m_Z) \\
& -\frac{g^2 m_Z^2 N}{4\pi^2 \cos^2\theta_W} && \big[T_3^2 - 2e_u T_3 \sin^2\theta_W + 2e_u^2 \sin^4\theta_W \big] G(m_u, m_u, m_Z).
\end{aligned}
$$

G.2.2 Nonlogarithmic Terms

The first field vanishes completely as already stated above. The second field does not have any nonlogarithmic terms to contribute. The terms from field 4 and 5 which are proportional $m_{\tilde{u}_1}^2$ and $m_{\tilde{u}_2}^2$ cancel against each other. Also the 6th field does not have a nonlogarithmic term. Two terms remain (from fields 3 and 5) which are not cancelled. They can be written as

$$
-\frac{g^2 m_u^2}{96\pi^2 \cos^2\theta_W} + \frac{g^2 m_Z^2}{(4\pi)^2 \cos^2\theta_W 3} \big[T_3^2 - 2e_u T_3 \sin^2\theta_W + 2e_u^2 \sin^4\theta_W \big] .
$$

G.2.3 Leading Log Approximation

In the case of very high top and stop masses, the other mass parameters can be neglected and an approximate formula for the correction to the mass sum rule is

$$
\Delta_{\text{leading log}} = \frac{3g^2 m_t^4}{16\pi^2 m_W^2 \sin^2\beta} \ln\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} \right] . \tag{G.2}
$$

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