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Off-Shell Supergravity in Five Dimensions and Supersymmetric Brane World Scenarios

by
Maximilian Zucker

Abstract

In order to supersymmetrize the Randall-Sundrum setup it proves useful to start with an off-shell formulation of five dimensional supergravity. So far, this theory has only been known on-shell. The first part of this thesis is concerned with the construction of five dimensional $\mathcal{N} = 2$ off-shell supergravity. We develop the complete local tensor calculus. The basic building block is the minimal multiplet which contains the propagating fields of supergravity. The minimal multiplet is completed by several auxiliary fields, extending the field content to $(40 + 40)$ components. The supersymmetry transformation laws for all matter multiplets with $(8 + 8)$ components are given. Using these results, action formulas for all multiplets are presented. In particular, we give three different off-shell formulations of supergravity and gauged supergravity. Next, gauged supergravity on the orbifold $S^1/\mathbb{Z}_2$ is constructed. Consistent implementation of the $\mathbb{Z}_2$ transformation shows that the minimal multiplet projected to the fixpoints of the orbifold forms a non-minimal $\mathcal{N} = 1$ supergravity multiplet in four dimensions. We use this observation to supersymmetrize the Randall-Sundrum scenario. This result is extended to matter located at the fixpoints which we couple in a manifestly supersymmetric way to the supergravity in the bulk. We consider chiral and super Yang-Mills multiplets at the fixpoints.

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To my parents

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Contents

List of Tables

Chapter 1

Introduction

The standard model of elementary particle physics is the most successful physical theory known. Theoretically, it describes in an elegant manner the electromagnetic, weak and strong forces in a single gauge field theory. The standard model is a unitary, renormalizable and anomaly free relativistic quantum field theory. The experimental evidence for the correctness of this model up to energies like $\sim 100\ GeV$ is overwhelming. The measured data are in excellent agreement with the theoretical predictions. The only missing ingredient is the Higgs particle but it is general belief that it will be found at LHC.

Nevertheless there are good reasons to believe that this theory is not the final theory of the world and we shall now discuss some of these problems.

- The gauge group of the standard model is $SU(3) \times SU(2)_L \times U(1)_Y$. The coupling constants are different for each factor, and thus there is no true unification of forces.

- There is no explanation for charge quantization, i.e. the fact that the electric charges of particles are integer multiples of some basic unit.

- The standard model contains some twenty free parameters, masses, Yukawa couplings, etc., which have to be fixed by experimental measurement. From a fundamental theory one would not expect such a proliferation of free parameters. Ideally, such a theory should contain no free parameter at all.

- One more ambiguity is the number of families. The standard model contains three families, which are essentially copies of each other and there is no compelling reason for this number.

- The standard model cannot be extended to include gravity. This is due to the fact that Einstein's theory of general relativity is nonrenormalizable as a quantum field theory. The seemingly only resolution to this problem requires far reaching conceptual changes like the introduction of superstrings.

- The last problem of the standard model to be mentioned is the hierarchy problem (for reviews see e.g. [59, 62]), which we shall discuss in more detail since it is the main motivation for our further development.

> An important feature of the standard model is its renormalizability. That means, that a priori there are no restrictions on the range of its validity. The basic mass scale which appears in the model is the mass of the massive vector bosons, $M_W \sim 100\,GeV$. Due to the renormalizability of the standard model, this mass scale is completely arbitrary. It is an experimental input parameter and one may ask why it is so small compared to the more fundamental Planck scale $M_{Pl} \sim 10^{19}\ GeV$. This is the so-called hierarchy problem. It results in an unnatural fine tuning, sometimes called technical hierarchy problem, which appears in perturbation theory. The mass parameter of the Higgs field receives quadratically divergent corrections which are regularized by the introduction of an *arbitrary new* mass scale in the theory. The basic mass scale in nature is the Planck scale M_{Pl} so that one would naturally be tempted to identify the new scale which appears in perturbation theory with the Planck scale. However, to keep the masses as small as they are observed, the counterterm which has to be included requires an unnatural fine tuning of one part in 10^{17}. This problem persists order by order in perturbation theory and is one motivation for low energy supersymmetry, which provides an elegant solution to this problem.

A geometric way of looking at the hierarchy problem has been suggested by Randall and Sundrum [66]. They considered five dimensional gravity with a cosmological constant on the orbifold $S^1/\mathbb{Z}_2$. One may visualize this orbifold as an interval, the endpoints being the fixpoints of the $\mathbb{Z}_2$, sometimes also called boundaries or branes. Randall and Sundrum localize at these fixpoints additional cosmological constants. The resulting Einstein equations are solved using a warped product ansatz. The authors go on and evaluate the four dimensional effective action of their theory and find that due to the warp factor, the hierarchy problem can be solved. Namely, the TeV scale is generated by an exponential suppression, resulting from the warp factor in the metric. In their model, all appearing scales can be traced back to one fundamental scale, the Planck scale. Thus, the model of Randall-Sundrum provides an elegant geometrization of the hierarchy problem.

However, the described brane world scenario introduces a new problem. The four dimensional effective action contains a massless scalar, the so-called radion $T(x^m)$, which is experimentally excluded. But also theoretically the radion is a weak point. It is a modulus, i.e. a field with vanishing potential and since the vacuum expectation value of the radion measures the distance r_c between the branes, it follows that r_c may be chosen arbitrarily. This modulus problem is well known from standard Kaluza-Klein compactification where the magnitude of the compact dimensions is described by moduli, i.e. the magnitude is unfixed.

The arbitrariness of the radion vacuum expectation value causes a more serious problem, the radius stabilization problem. In general, quantum effects will generate an effective potential for the radion. This potential leads to forces between the branes which can be either attractive or repulsive and ultimately cause instability of the configuration [31].

One solution to this problem has been suggested by Goldberger and Wise [40]. By including an additional massive scalar in the bulk with Higgs-like potentials on the boundaries, they find that it is possible to stabilize the radius r_c. They claim that no additional fine tuning is required, which is true as long as no backreactions of the bulk scalar with gravity is included [28]. An open question is the stability of the bulk scalar under quantum corrections.

A more promising option is the supersymmetrization of the Randall-Sundrum scenario. Besides dealing with the radius stabilization problem, supersymmetry has the virtue of solving the technical aspect of the hierarchy problem. Further, the motivation for supersymmetry from a more microscopic theory is quite easy, since theories like string theories seem to require supersymmetry for consistency.

For reasons to be described below, we choose minimal supersymmetry, $\mathcal{N} = 2$ in the bulk. The effective four dimensional theory is then $\mathcal{N} = 1$ supersymmetric and the radion should belong to a chiral multiplet. Again, we have to analyze the potential for $T(x)$ in order to understand the stability of the system. The potential for a chiral multiplet is generally derivable from a superpotential, for which a powerful non-renormalization theorem exists: the superpotential receives no perturbative quantum corrections at all [42]. Thus the potential for the radion is stabilized by supersymmetry and therefore, a supersymmetrization of the brane world scenario would solve the radius stabilization problem.

Before continuing our development we should shortly digress and present another, independent motivation for the study of supergravity on $S^1/\mathbb{Z}_2$. For phenomenologically relevant supersymmetric theories like the minimal supersymmetric standard model it turns out that the breaking of rigid supersymmetry represents a serious problem. For such theories, the supertrace of the squared mass matrix has to vanish (on tree level), which results in superpartners, lighter than the original particles, which is in conflict with the experimental observations. To solve this problem, one has invented the so-called hidden sector supergravity models [59]. These models contain two sectors, the visible sector, which contains the minimal supersymmetric standard model or some grand unified generalization thereof and the hidden sector, which is responsible for the breaking of supersymmetry. These two sectors are connected by supergravity which transmits the supersymmetry breaking from the hidden to the visible sector. Models of this type get a nice interpretation within the contex of supergravity on $S^1/\mathbb{Z}_2$, as explored for the first time in the context of Hořava-Witten theory [51, 52]. These authors considered eleven dimensional supergravity on $S^1/\mathbb{Z}_2$ coupled to super Yang-Mills multiplets at the boundaries. Supersymmetry breaking is effected by placing a gaugino condensate on one boundary. The supersymmetry breaking is transmitted to the other wall by the supergravity in the bulk [50, 60, 61], thus leading to a geometric interpretation of hidden sector supergravity models in the context of supergravity on $S^1/\mathbb{Z}_2$: we simply identify the two sectors with the two branes and the higher dimensional supergravity theory in the bulk with the mediator of supersymmetry breaking.

Let us now discuss the supersymmetric Randall-Sundrum scenario in more detail. Which supergravity theory should be taken as bulk theory? Due to the requirement of chiral fermions in the four dimensional effective theory, this theory has to have $\mathcal{N} = 1$ supersymmetry. Since the $\mathbb{Z}_2$ projections eliminate half of the supersymmetry, we have to start with an $\mathcal{N} = 2$ supergravity in the bulk. The bulk theory has to contain a cosmological constant, which is only consistent with supersymmetry if the vacuum is an anti de Sitter space. There are various ways of getting such an adS supergravity, all of them require a gauging of the $\mathcal{R}$-symmetry group or a subgroup thereof. A minimal choice is the gauging of a $U(1)$ subgroup by the graviphoton, since no additional fields are introduced. Thus we take the bulk theory to be $\mathcal{N}=2$ gauged supergravity, where a $U(1)$ subgroup of the $\mathcal{R}$-symmetry is gauged by the graviphoton, but let us stress that larger gaugings could be interesting, too.

To construct the supersymmetrization of the boundary cosmological constants various methods have been developed. The authors of [32] start from ungauged supergravity to which they add a cosmological constant on the boundary. They subsequently render the action supersymmetric by using some sort of generalized Noether procedure, first developed by Hořava and Witten in order to couple eleven dimensional supergravity on $S^1/\mathbb{Z}_2$ to super Yang-Mills multiplets at the fixpoints. They find a gravitino mass term in the bulk which is multiplied by a step function, which results effectively in projecting out this term on the boundaries. In some sense a generalization is the approach of [4], which has also shed some light on the connection between this and the approach to be discussed next.

An alternative method is the one of Altendorfer et al. [2] who leave the bulk theory as it stands and find the Killing spinors in the bulk. These spinors solve the Killing spinor equations everywhere except for the fixpoints. By modifying the supersymmetry transformation laws, the Killing spinors are turned to solutions of the Killing spinor equations everywhere. Due to the modified supersymmetry transformation laws, the bulk action is no longer invariant. This is finally cured by adding a cosmological constant and a mass term for the gravitino to the boundary lagrangian. A disadvantage of this approach is that it seems to be quite complicated to extend this method to more complicated configurations including matter located at the branes [4, 30].

We tackle the task of supersymmetrizing the Randall-Sundrum scenario differently. In contradistinction to the former authors who work on-shell, our approach is off-shell, i.e. the gauge algebra closes without reference to the equations of motion. In principle there are two approaches to off-shell supersymmetry: either one uses superspace techniques or develops a tensor calculus. Both techniques are equivalent and it is possible to derive them from each other. The first method is more useful in performing quantum calculations, where the machinery of supergraphs may be used. Superspace is a very elegant concept and also in practice very useful for theories with less than four real supercharges[1]. For theories with more than four supercharges superspace becomes more complicated, the basic reason being the complexity of the constraints on the superspace geometry which have to be solved. Couplings to matter multiplets transforming nontrivially under a central charge can only be described reasonably using harmonic superspace which requires more involved techniques. Since we are concerned with $\mathcal{N}=2$ supersymmetry and since our goal is not to perform quantum calculations, we restrict ourselves in what follows to the tensor calculus. Let us compile the general benefits of using off-shell supersymmetry.

From the fact that the gauge algebra closes of shell, a practical advantage arises. Namely, we can construct first the transformation laws and then move on to the construc-

[1] This corresponds to $\mathcal{N}=1$ supersymmetry in four dimensions.

tion of lagrangians. When treating on-shell theories one has to construct transformation laws and lagrangians simultaneously, since the gauge algebra closes only up to equations of motion. In particular, if an on-shell theory is changed, the equations of motion change and the transformation laws have to be changed, too. This results in tedious calculations which can be circumvented by using off-shell formulations.

Another advantage is the possibility of a tensor calculus. This has been developed for rigid supersymmetry by Wess and Zumino [78] and has been extended to local supersymmetry by various authors [36, 76, 35, 75]. It rests upon the fact that off-shell supersymmetry variations fulfill the Leibniz rule, i.e. it is possible to define multiplication of multiplets. Using this tensor calculus, the most general matter couplings to supergravity have been found for $\mathcal{N} = 1$ [12, 13] and for $\mathcal{N} = 2$ [20, 19] supersymmetry in four dimensions.

Although it is of no importance to us, we shall mention one more advantage of off-shell formulations. The quantization of a theory with closed gauge algebra is much simpler than the quantization of a theory with open gauge algebra. The quantization of off-shell supergravity can be performed using standard BRST techniques (this is reviewed in e.g. [58]), whereas on-shell supergravity would require more involved techniques, like the method of de Wit and van Holten [23] or even the field-antifield formalism of Batalin and Vilkovisky (for a review, see [41]).

Let us now proceed by describing the method we use. It is based on the work of Mirabelli and Peskin [57]. One starts by implementing the $\mathbb{Z}_2$ transformation in a supersymmetric way on the fields. The observation is then, that the projections of the supermultiplets on the boundary form $\mathcal{N} = 1$ supermultiplets in four dimensions. This is in particular true for the minimal multiplet, a submultiplet of the supergravity multiplet, which contains the propagating fields of five dimensional supergravity. Its projection to the boundaries forms an $\mathcal{N} = 1$ off-shell supergravity multiplet in four dimensions. Since this is an off-shell statement, we can simply couple matter located on the brane to this supergravity. After elimination of the auxiliary fields we obtain lagrangians of the desired type.

Of primary interest is the supersymmetric Randall-Sundrum scenario which can be constructed straightforwardly and it seems that the result is a well defined theory. An extension of great importance is the inclusion of chiral multiplets on the boundary, which is on the off-shell level a well defined theory. The elimination of the auxiliary fields turns out to be a complicated task which we have not completed. Finally, we locate super Yang-Mills multiplets at the boundaries. In this case, the elimination of auxiliary fields is an easy exercise, leading to a well defined field theory.

The structure of this work is as follows.
In chapter 2 the minimal multiplet is constructed. It is the basic building block of local $\mathcal{N} = 2$ supersymmetry in five dimensions, containing the graviton, the gravitino and the graviphoton. We find its transformation laws starting from the rigidly supersymmetric Maxwell multiplet for which we compute the Noether currents. These currents are completed to the multiplet of currents which we dualize in order to obtain the supersymmetry transformation laws of the linearized minimal multiplet. We then complete these transformation laws to the full nonlinear ones.

Chapter 3 is devoted to the construction of all matter multiplets which contain $(8+8)$ components. Besides standard multiplets like the super Yang-Mills and the hypermultiplet we also construct the linear multiplet which is very important for our tensor calculus. It turns out that almost all matter multiplets may be embedded in the linear multiplet. Thus, knowing an action for the linear multiplet, one can find straightforwardly an action for the embedded multiplet.

The results from the foregoing chapter are used in chapter 4 to construct supersymmetric actions. We use an action formula for the linear multiplet to construct lagrangians for various matter multiplets and the minimal multiplet. By manipulating these lagrangians we are able to deduce three different off-shell supergravity actions. These actions can be extended to gauged supergravity, which we need for the preceding chapter.

Chapter 5 is dealing with supersymmetric brane world scenarios. Our basic observation is that the minimal multiplet, subjected to $S^1/\mathbb{Z}_2$ forms at the fixpoints a non-minimal supergravity multiplet of four dimensional $\mathcal{N}=1$ supersymmetry. We develop a tensor calculus, using this non-minimal supergravity multiplet, which is located at the fixpoints. This tensor calculus allows us to find various supersymmetric actions with gauged supergravity in the bulk, coupled to matter and cosmological constants on the boundaries.

Finally, we give some conclusions and an outlook in chapter 6.

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- M. Zucker, *Minimal Off-Shell Supergravity in Five Dimensions*, Nucl. Phys. **B570** (2000), 267, `hep-th/9907082`.
- M. Zucker, *Gauged Off-Shell Supergravity in Five Dimensions*, `hep-th/9909144`.
- M. Zucker, *Supersymmetric Brane World Scenarios from Off-Shell Supergravity*, to appear.

Chapter 2

The Minimal Multiplet

This chapter is devoted to the construction of the local off-shell supersymmetry transformation laws of the minimal multiplet in five dimensional $\mathcal{N} = 2$ supergravity [83].

The minimal multiplet[1] is the main building block of our construction. It contains the propagating fields of supergravity and various auxiliary fields. All in all, it comprises $(40 + 40)$ degrees of freedom, which is not enough to construct a physically meaningful action for supergravity, as one easily deduces by dimensional reduction from five to four dimensions. For this purpose, one considers the on-shell theory [10, 8], which contains the vielbein e_M^A, the gravitino ψ_M and the graviphoton A_M. Schematically, the reduction to four dimensions may be depicted as follows:

$$(e_M^A,\ \psi_M,\ A_M) \overset{\text{Reduction}}{\longrightarrow} (e_m^a,\ e_m^{\dot{5}},\ e_5^{\dot{5}};\ \ \psi_m,\ \psi_5;\ \ A_m,\ A_5)$$

The lower case indices are four dimensional. One finds a $D = 4$, $\mathcal{N} = 2$ supergravity multiplet and a Maxwell multiplet, i.e. an abelian super Yang-Mills multiplet. The four dimensional $\mathcal{N} = 2$ supergravity multiplet contains the vielbein, the gravitino and a graviphoton. The remaining degrees of freedom form an $\mathcal{N} = 2$ Maxwell multiplet, consisting of a gauge field, the gaugino and two real scalars. To extend these multiplets to off-shell configurations, one has to add non-dynamical degrees of freedom, see e.g. [6, 5]. One finds, that the $\mathcal{N} = 2$, $D = 4$ supergravity multiplet requires $(40+40)$ and the Maxwell multiplet $(8+8)$ components. Thus we conclude that the smallest five dimensional off-shell supergravity theory contains at least $(48 + 48)$ components.

As in the four and six dimensional cases, these $(48+48)$ components are not irreducible; they contain a submultiplet with $(40 + 40)$ components, namely the minimal multiplet. In table 2.1 the particle content and some basic properties of this multiplet are collected. The propagating degrees of freedom are the before mentioned fields $(e_M^A,\ \psi_M,\ A_M)$. Besides these and as stated above, the minimal multiplet contains several auxiliary fields which have in five spacetime dimensions the canonical mass dimension $5/2$ (see the fourth column in the table). Note the presence of the vectorfield $\vec{V}_M$ which is a gauge field for the $SU(2)_\mathcal{R}$ $\mathcal{R}$-symmetry. Hence, this symmetry is gauged by an *auxiliary* field.

[1] A precise definition of the minimal multiplet is that it is the smallest multiplet within Poincaré supersymmetry, which contains a spin 2 excitation.

Type	Field	$SU(2)_\mathcal{R}$	Dimension	Components	
				bosonic	fermionic
Vielbein	e_M^A	**1**	0	10	
Graviphoton	A_M	**1**	3/2	4	
Gravitino	ψ_M	**2**	2		32
Isotriplet	$\vec{t}$	**3**	5/2	3	
Antisymmetric tensor	v_{AB}	**1**	5/2	10	
$SU(2)_\mathcal{R}$ gauge field	$\vec{V}_M$	**3**	5/2	12	
Spinor	λ	**2**	3		8
Scalar	C	**1**	7/2	1	
				40 +	40

Table 2.1: Field content of the minimal multiplet.

Further, the multiplet contains an auxiliary field C with dimension 7/2, for which the canonical action is not bilinear as for dimension 5/2 auxiliary fields, but linear,

$$S \sim \frac{1}{\kappa} \int d^5x \; eC. \tag{2.1}$$

Here, κ is the gravitational coupling constant whose mass-dimension in five dimensions is $[\kappa] = -3/2$ and $e = \det e_M^A$ is the determinant of the vielbein. Varying the action w.r.t. C one finds the equation of motion $\det e_M^A = 0$. This is obviously unphysical.

Another drawback is the presence of the auxiliary spinor λ for which it is not possible to write down an action.

We can solve these problems in three different ways; all these solutions lead to different off-shell formulations of supergravity. One has to introduce a compensator multiplet which allows the breaking of the gauged $SU(2)_\mathcal{R}$ (see the discussion in section 4.2.). This multiplet introduces the required $(8+8)$ components to the theory. The compensator multiplet may be the nonlinear multiplet from section 3.4, the hypermultiplet from section 3.5 or the tensor multiplet from section 3.3. The resulting off-shell supergravity theories are denoted by version *I*, *II* and *III*, respectively. All these theories lead upon elimination of the auxiliary fields to the same on-shell theory.

2.1 Basics of spinors and supersymmetry in five dimensions

This beginning section is concerned with the collection of some elementary properties of spinors and supersymmetry in five dimensions (see also [10, 77, 81] and the appendix of the present work).

Spinors in five dimensions are four component objects and a priori complex and unconstrained quantities, i.e. Dirac spinors. The Dirac matrices in five dimensions satisfy the relation

$$(C\gamma^A)^T = -C\gamma^A$$

where C is the antisymmetric charge conjugation matrix. If one imposes the standard Majorana condition $\bar{\psi} = \psi^T C$, one finds that the canonical lagrangian for spinors of this type

$$\mathscr{L} \sim i\bar{\psi}\gamma^A \partial_A \psi$$

vanishes identically (our notations and conventions are collected in appendix A). We conclude, that there exist no Majorana spinors in five dimensions.

In order to impose a reality constraint, we have to consider a family of spinors ψ^i, with $i = 1, \ldots \mathcal{N}$, and require after introducing a symplectic form Ω_{ij} (i.e. with $\Omega^T = -\Omega$) the *symplectic Majorana condition*

$$\bar{\psi}_i = \Omega_{ij}(\psi^j)^T C, \tag{2.2}$$

where the Dirac conjugated spinor $\bar{\psi}_i$ is defined as usual by eq. (A.2).

From iterating the symplectic Majorana condition (2.2), it follows that symplectic Majorana spinors always have to appear pairwise, that is $\mathcal{N} = 2, 4, \ldots$. Except for the hyperinos in section 3.5, which satisfy a different reality constraint, all spinors in this work are symplectic Majorana spinors.

The structures we have found in analyzing spinors reappear in the supersymmetry algebra. The supersymmetry algebra is

$$\{Q^i_{\underline{\alpha}}, Q^j_{\underline{\beta}}\} = \Omega^{ij}(\gamma^A C)_{\underline{\alpha\beta}} P_A + C_{\underline{\alpha\beta}} Z^{ij}, \tag{2.3}$$

where the $Q^i_{\underline{\alpha}}$ are of course the supercharges. P_A is the generator of translations and the Z^{ij} are the central charges, with

$$Z^{ij} = -Z^{ji}.$$

The underlined greek indices are spinor indices, i.e. $\underline{\alpha}, \underline{\beta} = 1, \ldots 4$. The indices i, j count the number $\mathcal{N}$ of supersymmetries, for this work $\mathcal{N} = 2$, i.e. $i, j = 1, 2$ and the symplectic form reduces in this case to the totally antisymmetric symbol in two dimensions,

$$\Omega^{ij} = \varepsilon^{ij} \qquad \text{for} \qquad \mathcal{N} = 2.$$

Thus, the central charge may be written as $Z^{ij} = Z\varepsilon^{ij}$.

For arbitrary $\mathcal{N}$ the algebra (2.3) has an automorphism group $USp(\mathcal{N})_{\mathcal{R}}$, which is usually called $\mathcal{R}$-symmetry in the physics literature. Since $USp(2)$ is isomorphic to $SU(2)$, one may speak in the $\mathcal{N} = 2$ case also about $SU(2)_{\mathcal{R}}$ objects, as will be done in this work.

Let us close this section with the remark that it is also common practice to work in five dimensions with Dirac spinors and not to invoke the symplectic Majorana constraint, see e.g. [8, 14, 2].

2.2 The multiplet of currents

An intrinsic feature of supersymmetric field theories is that the Noether currents are parts of a supermultiplet, the so-called multiplet of currents [38]. This multiplet is a genuine off-shell multiplet. Since globally supersymmetric field theories are translational invariant, the multiplet of currents must contain the associated Noether current, that is, the energy-stress tensor Θ_{AB}. If one gauges these symmetries, the Noether currents couple in a minimal way to the corresponding gauge field. In the case of the energy-stress tensor to the (linearized) metric. If one requires supersymmetry of this coupling, one finds the linearized transformation laws of the the minimal multiplet. This has been realized in [63] for the first time.

Excellent discussions of current multiplets and their usage for the construction of locally supersymmetric off-shell theories can be found in [79] and [80].

For the present work the relevant multiplet is the $\mathcal{N} = 2$ multiplet of currents in five dimensions. It has been discussed in large parts in superspace, see [54, 53].

Our starting point is the abelian $\mathcal{N} = 2$ super Yang-Mills theory in five dimensions. This theory contains the photon B_A (with field strength $G_{AB} = 2\partial_{[A}B_{B]}$), the gaugino Ω and a real scalar M. The lagrangian density [57]

$$\mathscr{L} = -\frac{1}{4}G_{AB}G^{AB} + \frac{1}{2}\partial_A M\partial^A M + \frac{i}{2}\bar{\Omega}\gamma^A\partial_A\Omega \tag{2.4}$$

is invariant under the rigid supersymmetry transformations

$$\begin{aligned}\delta B_A &= \frac{i}{\sqrt{2}}\bar{\varepsilon}\gamma_A\Omega, \qquad \delta M = \frac{i}{\sqrt{2}}\bar{\varepsilon}\Omega\\ \delta\Omega &= \frac{1}{2\sqrt{2}}\gamma^{AB}\varepsilon G_{AB} - \frac{1}{\sqrt{2}}\gamma^A\varepsilon\partial_A M.\end{aligned} \tag{2.5}$$

The field equations are (our conventions are collected in appendix A)

$$\partial^A\partial_A M = \not{\partial}\Omega = \partial_A G^{AB} = 0. \tag{2.6}$$

The Noether currents of this theory are easily deduced using standard methods. They are:

- the (symmetrized) energy-stress tensor

$$\Theta_{AB} = -G_{AC}G_B{}^C + \frac{1}{4}\eta_{AB}G_{CD}G^{CD} + \partial_A M\partial_B M - \frac{1}{2}\eta_{AB}(\partial M)^2 + \frac{i}{4}\bar{\Omega}(\gamma_A\partial_B + \gamma_B\partial_A)\Omega,$$

- the current associated with the $SU(2)_{\mathcal{R}}$ automorphism of the supersymmetry algebra under which only the gaugino transforms,

$$\vec{J}_A = -\frac{1}{4}\bar{\Omega}\vec{\tau}\gamma_A\Omega,$$

and

- the supercurrent, following from the invariance of the lagrangian (2.4) under the supersymmetry variations (2.5),

$$J^A = -\frac{1}{\sqrt{2}}\left(\gamma^B\gamma^A\Omega\partial_B M + \frac{1}{2}\gamma^{BC}\gamma^A\Omega G_{BC}\right).$$

These currents are conserved, i.e.

$$\partial_A J^A = \partial_A \Theta^{AB} = \partial_A \vec{J}^A = 0,$$

if the equations of motion (2.6) are valid.

It is now rather easy to construct the complete multiplet of currents. From [54] we know that the lowest dimensional component of this multiplet is a scalar C' of dimension $[C'] = 3$. We start by writing down the most general ansatz bilinear in the fields, which is gauge invariant and has the right dimension, i.e. $C' \sim M^2$. Then we act repeatedly with the supersymmetry transformations (2.5) on this expression, using the equations of motion (2.6). One then finds the complete multiplet of currents:

$$\begin{aligned}
\delta C' &= \frac{i}{2}\bar{\varepsilon}\zeta \\
\delta\zeta &= \frac{i}{2}\vec{\tau}\varepsilon\vec{X} + \frac{1}{2}\gamma^{AB}\varepsilon w_{AB} - \frac{i}{2}\vec{\tau}\gamma^A\varepsilon\vec{J}_A - \gamma^A\varepsilon\partial_A C' \\
\delta\vec{X} &= \frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^A J_A - \bar{\varepsilon}\vec{\tau}\partial\!\!\!/\zeta \\
\delta w_{AB} &= \frac{i}{4}\bar{\varepsilon}\gamma_{ABC}J^c - i\bar{\varepsilon}\gamma_{[A}\partial_{B]}\zeta \\
\delta\vec{J}_A &= -\frac{1}{2}\bar{\varepsilon}\vec{\tau}J_A \\
\delta J^A &= \gamma_B\varepsilon\Theta^{AB} + \varepsilon J^A_{(1)} + i\gamma^B\vec{\tau}\varepsilon\partial_B\vec{J}^A - \frac{1}{2}\varepsilon^{ABCDE}\gamma_B\varepsilon\partial_C w_{DE} \\
\delta J^A_{(1)} &= -\frac{i}{2}\bar{\varepsilon}\partial\!\!\!/ J^A \\
\delta\Theta^{AB} &= -\frac{i}{2}\bar{\varepsilon}\gamma^{C(A}\partial_C J^{B)}.
\end{aligned}$$

Besides the currents which we have already discussed, there are further fields in the multiplet of currents: a scalar C', a spinor ζ, an $SU(2)_{\mathcal{R}}$ triplet and Lorentz scalar $\vec{X}$ and an antisymmetric two form w_{AB}. These fields are defined by

$$C' = \frac{1}{2}M^2, \qquad \zeta = \sqrt{2}\Omega M, \qquad \vec{X} = \frac{1}{4}\bar{\Omega}\vec{\tau}\Omega, \qquad w_{AB} = MG_{AB} - \frac{i}{8}\bar{\Omega}\gamma_{AB}\Omega$$

and

$$J^A_{(1)} = -\frac{1}{8}\varepsilon^{ABCDE}G_{BC}G_{DE} - \partial_B(G^{AB}M).$$

Note, that this quantity is conserved, $\partial \cdot J_{(1)} = 0$.

2.3 The linearized minimal multiplet

The coupling term, which couples the multiplet of currents to the minimal multiplet and defines in this way the minimal multiplet, is

$$\sim \kappa \int d^5x \left(\frac{1}{8} h_{MN} \Theta^{MN} + \frac{i}{4} \bar{J}^M \psi_M - 4C'C \right.$$
$$\left. - 2i\bar{\zeta}\lambda - \frac{1}{2} w_{AB} v^{AB} + \vec{X}\vec{t} + \frac{1}{2\sqrt{3}} A_M J^M_{(1)} + \frac{1}{4} \vec{J}^M \vec{V}_M \right). \tag{2.7}$$

h_{MN} is the linearized metric

$$g_{MN} = \eta_{MN} + \kappa h_{MN}$$

and the other fields which belong to the minimal multiplet have been listed in table 2.1. If one requires supersymmetric invariance of the coupling term (2.7) one deduces the linearized, that is, the rigid supersymmetry transformation laws of the fields of the minimal multiplet. One finds

$$\begin{aligned}
\delta h_{MN} &= -i\bar{\varepsilon}\gamma_M \psi_N + (M \leftrightarrow N) \\
\delta \psi_M &= -\frac{1}{4}\gamma^{PN}\varepsilon \partial_P h_{MN} + \frac{1}{2}\gamma_{MAB}\varepsilon v^{AB} \\
&+ 2i\vec{\tau}\gamma_M \varepsilon \vec{t} - \frac{i}{2}\vec{\tau}\varepsilon \vec{V}_M + \frac{1}{\sqrt{3}}\gamma^N \varepsilon \partial_N A_M && \text{(2.8a)} \\
\delta A_M &= -\frac{\sqrt{3}i}{2}\bar{\varepsilon}\psi_M \\
\delta \vec{V}_M &= \bar{\varepsilon}\vec{\tau}\gamma^N \partial_N \psi_M - 4\bar{\varepsilon}\vec{\tau}\gamma_M \lambda && \text{(2.8b)} \\
\delta v^{AB} &= \frac{i}{4}\varepsilon^{ABCDE}\bar{\varepsilon}\gamma_C \partial_D \psi_E + 2i\bar{\varepsilon}\gamma^{AB}\lambda \\
\delta \lambda &= \frac{1}{4}\gamma_A \varepsilon \partial_B v^{AB} + \varepsilon C - \frac{i}{2}\vec{\tau}\gamma^A \varepsilon \partial_A \vec{t} \\
\delta C &= -\frac{i}{2}\bar{\varepsilon}\gamma^A \partial_A \lambda \\
\delta \vec{t} &= \bar{\varepsilon}\vec{\tau}\lambda.
\end{aligned}$$

The conservation of the Noether currents Θ^{AB}, $\vec{J}_A$, J_A and $J^A_{(1)}$ induces various gauge symmetries on the fields they are coupled to. These standard symmetries for the linearized supergravity fields are

$$\delta h_{MN} = \partial_M \xi_N + (M \leftrightarrow N), \qquad \delta \psi_M = \partial_M \eta, \qquad \delta A_M = \partial_M \Lambda. \tag{2.9}$$

Besides these symmetries, one finds a local $U(1)^3$ which follows from the conservation of $\vec{J}^A$. The associated gauge field is $\vec{V}_M$,

$$\delta \vec{V}_M = \partial_M \vec{\xi}. \tag{2.10}$$

This $U(1)^3$ will be knitted together with the global $SU(2)_{\mathcal{R}}$ automorphisms in next order in the coupling constant, in our case κ, to give a local $SU(2)_{\mathcal{R}}$ automorphism symmetry. Schematically,

$$U(1)^3_{local} \times SU(2)_{\mathcal{R},global} \stackrel{\mathcal{O}(\kappa)}{\longrightarrow} SU(2)_{\mathcal{R},local}.$$

This mechanism is well known from the Noether procedure, where a gauge theory with gauge group $U(1)^n$ and a global $SU(n)$ symmetry gives in first nontrivial order in the coupling constant a gauge theory with gauge group $SU(n)$.

It should be stressed, that $\vec{V}_M$ is an auxiliary field, so that the $\mathcal{R}$-symmetry of our theory will be gauged by an auxiliary field.

The next step is to use the symmetries (2.9) to bring the transformation laws in a manifest covariant form. For example, using the invariance under the second transformation in (2.9), we add a term of the form

$$\Delta\psi_M = -\partial_M \left(\frac{1}{\sqrt{3}} \gamma^N \varepsilon A_N \right)$$

to the transformation law of the gravitino in eq. (2.8a). The last term in this expression then combines to the linearized gravitino field strength. Similarly, the transformations of the form (2.10) are added to the supersymmetry transformation laws of $\vec{V}_M$ in (2.8b) to complete the first term to the linearized gravitino field strength. The supercovariantization of this modified multiplet is then easily seen and will be performed in the next section.

2.4 The full minimal multiplet

The next step is to promote the variation parameter of the supersymmetry transformations ε to a local quantity. The transformation laws are then covariantized w.r.t. local Lorentz transformations and local $SU(2)_{\mathcal{R}}$ transformations, and finally supercovariantized.

The notion of supercovariance is defined by the property, that the supersymmetry variation of a supercovariant quantity contains no derivatives of the variation parameter ε. This is achieved by the addition of terms proportional to the gravitino: in general, the supercovariant derivative of a generic field ϕ is given by

$$\widehat{\partial}_M \phi = \partial_M \phi - \kappa \delta_Q(\psi_M)\phi. \tag{2.11}$$

Here, the hat on the object denotes its supercovariance and $\delta_Q(\psi_M)$ is a supersymmetry variation with parameter ψ_M. Note, that a supercovariant derivative of the gravitino is not possible, since this field transforms as gauge field under supersymmetry. Instead, we can define the supercovariant field strength

$$\widehat{\mathcal{R}}_{MN} = \kappa \delta_Q(\psi_{[N})\psi_{M]}. \tag{2.12}$$

The next step is to calculate the gauge algebra, i.e. the commutator of two supersymmetry transformations on all fields. It is convenient, to start with the lowest dimensional field.

Closure of the algebra requires new terms of the order κ and higher in the transformation laws. After a long and tedious calculation one finds the following result:

$$\begin{aligned}
\delta e_M^A &= -i\kappa\bar{\varepsilon}\gamma^A\psi_M \\
\delta\psi_M &= \frac{1}{\kappa}\mathscr{D}_M\varepsilon + \frac{1}{2}\gamma_{MAB}\varepsilon v^{AB} + 2i\vec{\tau}\gamma_M\varepsilon\vec{t} - \frac{1}{\sqrt{3}}\gamma^N\varepsilon\widehat{F}_{MN} \\
\delta A_M &= -\frac{\sqrt{3}i}{2}\bar{\varepsilon}\psi_M \\
\delta\vec{V}_M &= 2\bar{\varepsilon}\vec{\tau}\gamma^N\widehat{\mathcal{R}}_{NM} - 4\bar{\varepsilon}\vec{\tau}\gamma_M\lambda \\
&- 2\kappa\bar{\varepsilon}\vec{\tau}\gamma^{AB}\psi_M v_{AB} + \frac{\kappa}{\sqrt{3}}\bar{\varepsilon}\vec{\tau}\gamma^{AB}\psi_M\widehat{F}_{AB} - 12i\kappa\bar{\varepsilon}\psi_M\vec{t} \\
\delta v^{AB} &= \frac{i}{4}\bar{\varepsilon}\gamma^{ABCD}\widehat{\mathcal{R}}_{CD} + 2i\bar{\varepsilon}\gamma^{AB}\lambda \\
\delta\lambda &= \frac{1}{4}\gamma_A\varepsilon\widehat{\mathscr{D}}_B v^{AB} + \varepsilon C - \frac{i}{2}\vec{\tau}\gamma^A\varepsilon\widehat{\mathscr{D}}_A\vec{t} - \frac{i\kappa}{2\sqrt{3}}\vec{\tau}\gamma^{AB}\varepsilon\vec{t}\widehat{F}_{AB} \\
&+ i\kappa\gamma^{AB}\vec{\tau}\varepsilon v_{AB}\vec{t} + \frac{\kappa}{48}\gamma^{ABCD}\varepsilon\widehat{F}_{AB}\widehat{F}_{CD} \\
\delta C &= -\frac{i}{2}\bar{\varepsilon}\gamma^A\widehat{\mathscr{D}}_A\lambda - \frac{3i\kappa}{4}\bar{\varepsilon}\gamma^{AB}\lambda v_{AB} - 11\kappa\bar{\varepsilon}\vec{\tau}\lambda\vec{t} - \frac{\kappa}{2}\bar{\varepsilon}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\vec{t} \\
\delta\vec{t} &= \bar{\varepsilon}\vec{\tau}\lambda.
\end{aligned}$$

$\mathscr{D}_M$ is a Lorentz- and $SU(2)$ covariant derivative, e.g.

$$\mathscr{D}(\widehat{\omega})_M\psi_N = (\partial_M + \frac{1}{4}\widehat{\omega}_M{}^{AB}\gamma_{AB} - \frac{i\kappa}{2}\vec{\tau}\vec{V}_M)\psi_N,$$

and the spin connection $\widehat{\omega}_M{}^{AB}$ appearing in covariant derivatives is always supercovariant. Explicit expressions for the spin connection and its supercovariantization are collected in appendix A. As mentioned above, hatted quantities denote supercovariance, the supercovariant gravitino field strength is, for example,

$$\widehat{\mathcal{R}}_{MN} = \mathscr{D}_{[M}\psi_{N]} + \frac{\kappa}{2}\gamma_{AB[M}\psi_{N]}v^{AB} + 2i\kappa\gamma_{[M}\vec{\tau}\psi_{N]}\vec{t} - \frac{\kappa}{\sqrt{3}}\gamma^P\psi_{[N}\widehat{F}_{M]P}, \tag{2.13}$$

cf. eq. (2.12). The other supercovariant derivatives which appear in the transformation laws are

$$\begin{aligned}
\widehat{\mathscr{D}}_M\lambda &= \mathscr{D}_M\lambda - \frac{\kappa}{4}\gamma_A\psi_M\widehat{\mathscr{D}}_B v^{AB} - C\psi_M + \frac{i\kappa}{2}\vec{\tau}\gamma^A\psi_M\widehat{\mathscr{D}}_A\vec{t} \\
&+ \frac{i\kappa^2}{2\sqrt{3}}\vec{\tau}\gamma^{AB}\psi_M\vec{t}\widehat{F}_{AB} - i\kappa^2\gamma^{AB}\vec{\tau}\psi_M v_{AB}\vec{t} - \frac{\kappa^2}{48}\varepsilon^{ABCDE}\gamma_A\psi_M\widehat{F}_{BC}\widehat{F}_{DE} \\
\widehat{\mathscr{D}}_M\vec{t} &= \partial_M\vec{t} - \kappa\bar{\lambda}\vec{\tau}\psi_M + \kappa\vec{V}_M \times \vec{t} \\
\widehat{\mathscr{D}}_M v^{AB} &= \mathscr{D}_M v^{AB} - \frac{i\kappa}{4}\varepsilon^{ABCDE}\bar{\psi}_M\gamma_C\widehat{\mathcal{R}}_{DE} - 2i\kappa\bar{\psi}_M\gamma^{AB}\lambda.
\end{aligned}$$

Complicated but useful formulae are the supersymmetry variation of the supercovariant spin connection

$$\delta\widehat{\omega}_{MAB} = -i\kappa(2\bar{\varepsilon}\gamma_{[A}\widehat{\mathcal{R}}_{B]M} - \bar{\varepsilon}\gamma_M\widehat{\mathcal{R}}_{AB}) + 2i\kappa^2(2i\bar{\varepsilon}\vec{\tau}\gamma_{AB}\psi_M\vec{t} + \frac{1}{\sqrt{3}}\bar{\varepsilon}\psi_M\widehat{F}_{AB} + \frac{1}{2}\bar{\varepsilon}\gamma_{ABCD}\psi_M v^{CD})$$

and the variation of the supercovariant gravitino field strength

$$\begin{aligned}\delta\widehat{\mathcal{R}}_{AB} &= \frac{1}{8\kappa}\gamma_{CD}\varepsilon\widehat{R}(\widehat{\omega})^{CD}{}_{AB} - \frac{\kappa}{3}\gamma^{CD}\varepsilon\widehat{F}_{AD}\widehat{F}_{BC} - 4\kappa\gamma_{AB}\varepsilon\vec{t}^2 + 6\kappa\gamma_{[AB}\varepsilon v_C{}^D v_{D]}{}^C \\ &+ 2i\kappa\gamma_{ABCD}\vec{\tau}\varepsilon\vec{t}v^{CD} + \frac{\kappa}{\sqrt{3}}\gamma^{CDE}{}_{[A}\varepsilon\widehat{F}_{B]C}v_{DE} - \frac{4i\kappa}{\sqrt{3}}\gamma^C{}_{[B}\vec{\tau}\varepsilon\widehat{F}_{A]C}\vec{t} + \frac{1}{2}\gamma_{CD[B}\varepsilon\widehat{\mathscr{D}}_{A]}v^{CD} \\ &+ 2i\vec{\tau}\gamma_{[B}\varepsilon\widehat{\mathscr{D}}_{A]}\vec{t} - \frac{i}{4}\vec{\tau}\varepsilon\widehat{\vec{G}}(V)_{AB} - \frac{1}{\sqrt{3}}\gamma^C\varepsilon\widehat{\mathscr{D}}_{[A}\widehat{F}_{B]C},\end{aligned}$$

with the supercovariant $SU(2)$ field strength

$$\begin{aligned}\widehat{\vec{G}}(V)_{MN} &= \partial_M\vec{V}_N - \partial_N\vec{V}_M + \kappa\vec{V}_M\times\vec{V}_N + 4\kappa\bar{\psi}_{[M}\vec{\tau}\gamma^P\widehat{\mathcal{R}}_{N]P} + 8\kappa\bar{\psi}_{[M}\vec{\tau}\gamma_{N]}\lambda \\ &+ 2\kappa^2\bar{\psi}_M\vec{\tau}\gamma^{CD}\psi_N v_{CD} - \frac{\kappa^2}{\sqrt{3}}\bar{\psi}_M\vec{\tau}\gamma^{AB}\psi_N\widehat{F}_{AB} + 12i\kappa^2\bar{\psi}_M\psi_N\vec{t}\end{aligned}$$

and the supercovariant Riemann curvature tensor

$$\begin{aligned}\widehat{R}(\widehat{\omega})^{AB}{}_{MN} =& R(\widehat{\omega})^{AB}{}_{MN} + 4i\kappa^2\bar{\psi}_{[M}\gamma^{[A}\widehat{\mathcal{R}}^{B]}{}_{N]} - 2i\kappa^2\bar{\psi}_{[M}\gamma_{N]}\widehat{\mathcal{R}}^{AB} \\ &+ 4\kappa^3\bar{\psi}_M\vec{\tau}\gamma^{AB}\psi_N\vec{t} - \frac{2i\kappa^3}{\sqrt{3}}\bar{\psi}_M\psi_N\widehat{F}^{AB} - i\kappa^3\bar{\psi}_M\gamma^{ABCD}\psi_N v_{CD}.\end{aligned} \tag{2.14}$$

The definition of the curvature tensor is presented in the appendix, eq. (A.12). Furthermore useful is the variation of the supercovariant graviphoton field strength, explicitly defined in eq. (A.9),

$$\delta\widehat{F}_{AB} = -i\sqrt{3}\bar{\varepsilon}\widehat{\mathcal{R}}_{AB}.$$

The commutator of two supersymmetry transformations is

$$\begin{aligned}[\delta_Q(\eta),\delta_Q(\varepsilon)] =& \delta_Q(i\kappa\bar{\varepsilon}\gamma^M\eta\psi_M) + \delta_{g.c.}(i\bar{\eta}\gamma^M\varepsilon) + \delta_z(-\frac{\sqrt{3}i}{2\kappa}\bar{\varepsilon}\eta + i\bar{\varepsilon}\gamma^M\eta A_M) \\ &+ \delta_{Lt}(i\bar{\eta}\gamma^M\varepsilon\widehat{\omega}_{MAB} - 4\kappa\bar{\eta}\vec{\tau}\gamma_{AB}\varepsilon\vec{t} + \frac{2i}{\sqrt{3}}\kappa\bar{\eta}\varepsilon\widehat{F}_{AB} + i\kappa\bar{\eta}\gamma_{ABCD}\varepsilon v^{CD}) \\ &+ \delta_{\mathcal{R}}(-i\kappa\bar{\eta}\gamma^M\varepsilon\vec{V}_M + 12i\kappa\bar{\eta}\varepsilon\vec{t} + 2\kappa\bar{\eta}\vec{\tau}\gamma_{AB}\varepsilon v^{AB} - \frac{\kappa}{\sqrt{3}}\bar{\eta}\vec{\tau}\gamma^{AB}\varepsilon\widehat{F}_{AB}),\end{aligned} \tag{2.15}$$

where $\delta_Q, \delta_{g.c.}, \delta_{Lt}, \delta_{\mathcal{R}}, \delta_z$ denote supersymmetry, general coordinate, local Lorentz, local $SU(2)$ and local [2] $U(1)$ transformations, respectively. The expressions in the brackets

[2]The graviphoton transforms as gauge field under this $U(1)$ and is the only field which transforms under this symmetry. However, we will see later that matter multiplets exist which are charged under this central charge.

denote the parameter of the transformation. The gauge algebra (2.15) is of central importance for any off-shell supergravity theory. Every multiplet, which is to be coupled to this theory, has to be a representation of this algebra, that is, it must satisfy this algebra.

We have calculated the algebra on e_M^A, ψ_M, A_M, $\vec{V}_M$, v_{AB}, $\vec{t}$ and λ. We have not verified closure of the algebra on C, but since closure on the other fields fixes the transformation laws completely, there are no possibilities left for further modifications.

The multiplet we have constructed in this section is similar to the minimal multiplet in four dimensions [6].

2.5 Superconformal substructure

Although the minimal multiplet is irreducible in the framework of Poincaré supersymmetry, it turns out to be reducible if one considers conformal invariance. In the framework of conformal supersymmetry, the minimal multiplet is a gauge fixed version of the Weyl multiplet, coupled to a Maxwell multiplet. The latter multiplet is introduced as a compensator multiplet for the superconformal symmetries. The Weyl multiplet is in some sense the superconformal analogon of the minimal multiplet, i.e. it contains the vielbein, the gravitino, etc. By definition, it is the smallest supermultiplet which contains a spin 2 particle.

The superconformal substructure of the minimal multiplet is the topic of this section.

Besides the translations (with generator P_A), the Lorentz transformations (M_{AB}), which form the Poincaré group, the conformal group contains dilatations (D) and special conformal transformations (K_A). If one enlarges this set by the inclusion of supercharges (Q) and special supercharges (S) together with the $SU(2)_{\mathcal{R}}$ generators (T) of the $\mathcal{R}$-symmetry, one obtains the superconformal algebra. The superconformal $\mathcal{N}=2$ algebra in five dimensions is presented in appendix B. The Weyl multiplet contains the gauge fields for the superconformal group, i.e. to each generator there is a gauge field associated. Unfortunately, this is not the complete field content: It turns out, that the number of bosonic and fermionic degrees of freedom do not match. Further, the symmetries are internal gauge symmetries. For example, the P_A transformations are in no connection with the reparametrizations of the manifold. To achieve this, one imposes a set of so called *conventional constraints*. By solving these constraints, various gauge fields are expressed through other fields. For example, by requiring the conventional constraint

$$R(P)_{MN}{}^A = 0,$$

one finds the second order formalism spin connection, or, more precise, its supercovariantization, eqs. (A.8, A.10), as a function of the vielbeine. $R(P)_{MN}{}^A$ is the field strength of the gauge field of the translations. After having imposed the maximal number of conventional constraints, one finds that the number of field components is of course strongly reduced, but nevertheless there is no Fermi-Bose matching yet. To achieve this, further fields, conformal matter fields have to be introduced.

The gauge fields are

$$e_M^A,\ \omega_M{}^{AB},\ b_M,\ f_M^A,\ \psi_M,\ \phi_M,\ \vec{V}_M$$

for

$$P_A,\ M_{AB},\ D,\ K_A,\ Q,\ S,\ T$$

transformations. The constraints then resolve $\omega_M{}^{AB}$, f_M^A, ϕ_M through other fields. The remaining gauge fields of the Weyl multiplet are then

$$(e_M^A,\ b_M,\ \psi_M,\ \vec{V}_M).$$

To get the minimal multiplet, a Maxwell multiplet with field content

$$(M,\ B_M,\ \Omega,\ \vec{X})$$

has to be added (cf. table 3.1). This multiplet is used as a conformal compensator multiplet. Thus one chooses the Poincaré gauge, schematically

$$M = const, \qquad \Omega = 0, \qquad b_M = 0.$$

This gauge breaks the superconformal symmetry to the desired Poincaré supersymmetry of the minimal multiplet. What are the additional fields present? It is easy to see, that the field content is now

$$(e_M^A,\ \psi_M,\ \vec{V}_M,\ B_M,\ \vec{X}).$$

Let us note that the $U(1)$ gauge symmetry of the Maxwell multiplet has become the central charge transformation of the Poincaré algebra. The gauge field B_M is then the graviphoton A_M. The field $\vec{X}$ is to be identified with the auxiliary field $\vec{t}$. By comparison with table 2.1 one notes the conformal matter fields: Besides the antisymmetric tensor field v_{AB} and the real scalar C one has to add a spinor λ to the Weyl multiplet. The independent fields of the Weyl multiplet are then

$$(e_M^A,\ b_M,\ \psi_M,\ \vec{V}_M,\ v_{AB},\ C,\ \lambda).$$

For the development of a superconformal multiplet calculus in $\mathcal{N} = 2$ supersymmetry powerful techniques have been developed [24, 26, 15, 25, 27, 3], see also the review articles [18, 65, 16]. These methods represent alternatives to the method which is used in the present work. In general, due to the higher degree of symmetry, the effort for the construction of superconformal multiplets is less then in our case. However, the construction of super Poincaré invariant actions is more tedious.

Field	Name	Properties	
e_M^A	Vielbein	P_A Gauge field, independent	Weyl multiplet
$\omega_M{}^{AB}$	Connection	M_{AB} Gauge field, dependent	
b_M		D Gauge field, independent	
f_M^A		K_A Gauge field, dependent	
ψ_M	Gravitino	Q Gauge field, independent	
ϕ_M		S Gauge field, dependent	
v_{AB}		Matter field	
λ		Matter field	
C		Matter field	
B_M		$U(1)$ Gauge field, independent	Maxwell multiplet
M			
Ω	Gaugino		

Table 2.2: The decomposition of the minimal multiplet under the superconformal group.

Special complications arise due to the fact that the superconformal algebra forbids central charges. When constructing multiplets like the hypermultiplet which is centrally charged, exotic concepts have to be introduced, see e.g. [20].

The results of the decomposition of the minimal multiplet performed in this section are collected in table 2.2.

Chapter 3

Matter Multiplets

In this chapter we construct the local supersymmetry transformation laws of all matter multiplets which exist in five dimensional $\mathcal{N} = 2$ supersymmetry.[1] The requirement for all these multiplets is that they satisfy the gauge algebra (2.15), i.e. they have to form representations of $\mathcal{N} = 2$ supersymmetry.

3.1 The super Yang-Mills multiplet

The field content of the super Yang-Mills multiplet is given in table 3.1 (The index $\mathcal{A}$ is a gauge group index). We have found the transformation laws of local supersymmetry by starting from the rigid transformation laws given in [57]. By enforcing the commutator algebra (2.15) we then extended these formulas to the locally supersymmetric case. Of course, the gauge algebra has to be modified: in addition to the terms given in (2.15), one finds field dependent terms which represent gauge transformations:

$$[\delta_Q(\eta), \delta_Q(\varepsilon)] = \ldots + \delta_{gauge}(ig\bar{\varepsilon}\gamma^M \eta B_M^{\mathcal{A}} - ig\bar{\varepsilon}\eta M^{\mathcal{A}}). \tag{3.1}$$

The dots represent the transformations as given in (2.15), $\delta_{gauge}(\alpha^{\mathcal{A}})$ is a gauge transformation with parameter $\alpha^{\mathcal{A}}$ and g is the gauge coupling constant which has mass dimension $[g] = -1/2$ in five dimensions. The indices $\mathcal{A}, \mathcal{B}$ run over the adjoint of the gauge group $\mathfrak{G}$.

The supersymmetry transformation rules we find are:

$$\begin{aligned}
\delta B_M^{\mathcal{A}} &= i\bar{\varepsilon}\gamma_M\Omega^{\mathcal{A}} - i\bar{\varepsilon}\psi_M M^{\mathcal{A}} \\
\delta M^{\mathcal{A}} &= i\bar{\varepsilon}\Omega^{\mathcal{A}} \\
\delta\Omega^{\mathcal{A}} &= \frac{1}{4}\gamma^{AB}\varepsilon\widehat{G}_{AB}^{\mathcal{A}} - \frac{1}{2}\gamma^A\varepsilon\widehat{\mathscr{D}}_A M^{\mathcal{A}} + i\vec{\tau}\varepsilon\vec{X}^{\mathcal{A}} - \frac{1}{2\sqrt{3}}\gamma^{AB}\varepsilon\widehat{F}_{AB}M^{\mathcal{A}} \\
\delta\vec{X}^{\mathcal{A}} &= -\frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^A\widehat{\mathscr{D}}_A\Omega^{\mathcal{A}} - \frac{g}{2}f_{\mathcal{BC}}{}^{\mathcal{A}}\bar{\varepsilon}\vec{\tau}\Omega^{\mathcal{B}}M^{\mathcal{C}} + \frac{1}{4}\bar{\varepsilon}\vec{\tau}\gamma^{AB}\Omega^{\mathcal{A}}v_{AB} + 4i\bar{\varepsilon}\Omega^{\mathcal{A}}\vec{t} - i\bar{\varepsilon}\vec{\tau}\tilde{\tau}\Omega^{\mathcal{A}}\tilde{t},
\end{aligned} \tag{3.2}$$

[1]Note that we restrict our attention to matter multiplets with $(8+8)$ components. We are not interested in multiplets with more components, like the somewhat exotic "self dual" tensor multiplet which is discussed on-shell in [47, 46, 48].

Type	Field	$SU(2)_{\mathcal{R}}$	Dimension	Components		
				bosonic		fermionic
Gauge field	$B_M^{\mathcal{A}}$	**1**	3/2	5 − 1		
Scalar	$M^{\mathcal{A}}$	**1**	3/2	1		
Gaugino	$\Omega^{\mathcal{A}}$	**2**	2			8
Isotriplet	$\vec{X}^{\mathcal{A}}$	**3**	5/2	3		
				8	+	8

Table 3.1: Field content of the super Yang-Mills multiplet in $D = 5$.

where $f_{\mathcal{AB}}{}^{\mathcal{C}}$ are the structure constants of the Lie algebra

$$[T_{\mathcal{A}}, T_{\mathcal{B}}] = i f_{\mathcal{AB}}{}^{\mathcal{C}} T_{\mathcal{C}};$$

that is, we work with hermitian generators $T_{\mathcal{A}}$. In these formulas, the gravitational coupling constant κ is suppressed. The supercovariant objects as they appear in (3.2) are explicitly given by:

$$\begin{aligned}
\widehat{\mathscr{D}}_M M^{\mathcal{A}} &= \partial_M M^{\mathcal{A}} + g f_{\mathcal{BC}}{}^{\mathcal{A}} B_M^{\mathcal{B}} M^{\mathcal{C}} + i\bar{\Omega}^{\mathcal{A}}\psi_M \\
\widehat{G}_{MN}^{\mathcal{A}} &= \partial_M B_N^{\mathcal{A}} - \partial_N B_M^{\mathcal{A}} + g f_{\mathcal{BC}}{}^{\mathcal{A}} B_M^{\mathcal{B}} B_N^{\mathcal{C}} - 2i\bar{\psi}_{[M}\gamma_{N]}\Omega^{\mathcal{A}} + i\bar{\psi}_M\psi_N M^{\mathcal{A}} \\
\widehat{\mathscr{D}}_M \Omega^{\mathcal{A}} &= \partial_M \Omega^{\mathcal{A}} + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\Omega^{\mathcal{A}} - \frac{i}{2}\vec{V}_M\vec{\tau}\Omega^{\mathcal{A}} + g f_{\mathcal{BC}}{}^{\mathcal{A}} B_M^{\mathcal{B}}\Omega^{\mathcal{C}} \\
&- \frac{1}{4}\gamma^{AB}\psi_M \widehat{G}_{AB}^{\mathcal{A}} + \frac{1}{2}\gamma^A\psi_M\widehat{\mathscr{D}}_A M^{\mathcal{A}} - i\vec{\tau}\psi_M\vec{X}^{\mathcal{A}} + \frac{1}{2\sqrt{3}}\gamma^{AB}\psi_M\widehat{F}_{AB}M^{\mathcal{A}}.
\end{aligned}$$

3.2 The linear multiplet

The linear multiplet is a useful tool for the construction of supersymmetrically invariant actions [5]. The highest component of the linear multiplet is a scalar N which transforms in the rigidly supersymmetric case into a total derivative. Therefore, a supersymmetric functional is immediately found:

$$\sim \int d^5x\ N. \tag{3.3}$$

One easily extends this relation to the locally supersymmetric case (see section 4.1).

As will be shown in the later sections, all supermultiplets can be embedded in the linear multiplet. This property explains the importance of the linear multiplet: if we know this embedding explicitly and if an invariant for the linear multiplet is known, it is straightforward to write down the invariant for the embedded multiplet. By application of this technique, the construction of supersymmetric actions is enormously simplified. We will use it intensively in chapter 4.

The complete field content of the linear multiplet is collected in table 3.2 (The degree of freedom subtracted from the vectorfield follows from the constraint (3.6)). It contains

Type	Field	$SU(2)_{\mathcal{R}}$	Components	
			bosonic	fermionic
Isotriplet	$\vec{Y}$	**3**	3	
Fermion	ρ	**2**		8
Scalar	N	**1**	1	
Vector	W_A	**1**	$5-1$	
			8 +	8

Table 3.2: The field content of the linear multiplet in $D = 5$.

a vectorfield W_A, which is a conserved quantity in the case of global supersymmetry, i.e. $\partial \cdot W = 0$. For the locally supersymmetric theory, this relation will be modified, its final form is given in equation (3.6).

In the globally supersymmetric limit, the linear multiplet is dual to the Maxwell multiplet. Thus, by requiring invariance of the coupling term[2]

$$\sim \int d^5x \left(B_A W^A + MN + i\bar{\Omega}\rho - \vec{X}\vec{Y} \right), \tag{3.4}$$

one derives the supersymmetry transformation laws of the linear multiplet. Subsequently, this transformation laws are completed to the full, nonlinear transformation laws by requiring that the commutator of two supersymmetry variations satisfies the algebra (2.15).

In addition, the fields of the linear multiplet may transform under a nonabelian gauge symmetry. It follows from the reality properties of the fields that the representation of the gauge group under which the linear multiplet transforms, has to be real. If such a symmetry is included, the fields transform like

$$\delta N = i\alpha^{\mathcal{A}} T_{\mathcal{A}} N, \qquad \text{etc.}$$

under an infinitesimal gauge transformation, where the $T_{\mathcal{A}}$ are the relevant representation matrices and $\alpha^{\mathcal{A}}$ is the parameter of the gauge transformation. If the linear multiplet is charged under the gauge group, the gauge algebra is of course extended by the expression (3.1).

Furthermore, it is possible to allow for a nontrivial central charge under which the linear multiplet may transform. The only requirement is, that this central charge is central in the strict sense, i.e. the central charge commutes with all other symmetries.

A remark on our notation: If we write for the central charge transformation of a field $\delta_z N$, we mean a transformation with unit parameter, that is $\delta_z N \equiv \delta_z(1)N$. In general, a generic field ϕ transforms as[3]

$$\delta_z(z)\phi = z\delta_z\phi$$

[2]From this point of view the conservation of the vectorfield W_A is a direct consequence of the fact, that B_A is the gauge field of a $U(1)$ symmetry.

[3]The only exception is the graviphoton which transforms as $\delta_z(z)A_M = \partial_M z$ under central charge transformations, i.e. as gauge field.

under central charge transformations.

The supersymmetry transformation rules we find are

$$
\begin{aligned}
\delta\vec{Y} &= \bar{\varepsilon}\vec{\tau}\rho && \text{(3.5a)}\\
\delta\rho &= \gamma^A\varepsilon W_A + \varepsilon N - \frac{i}{2}\gamma^A\vec{\tau}\varepsilon\widehat{\mathscr{D}}_A\vec{Y} + i\gamma^{AB}\vec{\tau}\varepsilon v_{AB}\vec{Y} \\
&+ 6i\vec{\tau}\varepsilon(\vec{t}\times\vec{Y}) - \frac{i}{2\sqrt{3}}\gamma^{AB}\vec{\tau}\varepsilon\widehat{F}_{AB}\vec{Y} + \frac{g}{2}\vec{\tau}\varepsilon M\vec{Y} - \frac{\sqrt{3}i}{4}\vec{\tau}\varepsilon\delta_z\vec{Y} && \text{(3.5b)}\\
\delta N &= -\frac{i}{2}\bar{\varepsilon}\gamma^A\widehat{\mathscr{D}}_A\rho - \frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\vec{Y} \\
&- 8\bar{\varepsilon}\vec{\tau}\lambda\vec{Y} - \frac{3i}{4}\bar{\varepsilon}\gamma^{AB}\rho v_{AB} - 3\bar{\varepsilon}\vec{\tau}\rho\vec{t} - \frac{ig}{2}\bar{\varepsilon}\vec{\tau}\Omega\vec{Y} && \text{(3.5c)}\\
\delta W^A &= \frac{i}{2}\bar{\varepsilon}\gamma^{AB}\widehat{\mathscr{D}}_B\rho + \frac{1}{2}\bar{\varepsilon}\vec{\tau}\gamma^{ABC}\widehat{\mathcal{R}}_{BC}\vec{Y} - \frac{i}{2}\bar{\varepsilon}\gamma^{ABC}\rho v_{BC} + \frac{3i}{2}\bar{\varepsilon}\gamma_B\rho v^{AB} + 4\bar{\varepsilon}\vec{\tau}\gamma^A\rho\vec{t} \\
&+ \frac{i}{2\sqrt{3}}\bar{\varepsilon}\gamma^{ABC}\rho\widehat{F}_{BC} - \frac{i}{2\sqrt{3}}\bar{\varepsilon}\gamma_B\rho\widehat{F}^{AB} + \frac{ig}{2}\bar{\varepsilon}\vec{\tau}\gamma^A\Omega\vec{Y} + \frac{g}{2}\bar{\varepsilon}\gamma^A M\rho - \frac{\sqrt{3}i}{4}\bar{\varepsilon}\gamma^A\delta_z\rho.
\end{aligned}
$$

Here we have used Lie algebra valued expressions for the fields which belong to the super Yang-Mills multiplet, e.g. $\Omega = \Omega^A T_A$. The explicit expressions for the supercovariant objects which appear in the transformation laws are:

$$
\begin{aligned}
\widehat{\mathscr{D}}_M\vec{Y} &= \partial_M\vec{Y} + \vec{V}_M\times\vec{Y} - igB_M\vec{Y} - A_M\delta_z\vec{Y} - \bar{\rho}\vec{\tau}\psi_M \\
\widehat{\mathscr{D}}_M\rho &= \partial_M\rho + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\rho - \frac{i}{2}\vec{V}_M\vec{\tau}\rho - igB_M\rho - A_M\delta_z\rho - \gamma^A\psi_M W_A \\
&- \psi_M N + \frac{i}{2}\gamma^A\vec{\tau}\psi_M\widehat{\mathscr{D}}_A\vec{Y} - i\gamma^{AB}\vec{\tau}\psi_M v_{AB}\vec{Y} - 6i\vec{\tau}\psi_M(\vec{t}\times\vec{Y}) \\
&+ \frac{i}{2\sqrt{3}}\gamma^{AB}\vec{\tau}\psi_M\widehat{F}_{AB}\vec{Y} - \frac{g}{2}\vec{\tau}\psi_M M\vec{Y} + \frac{\sqrt{3}i}{4}\vec{\tau}\psi_M\delta_z\vec{Y} \\
\widehat{\mathscr{D}}_M N &= \partial_M N - igB_M N - A_M\delta_z N + \frac{i}{2}\bar{\psi}_M\gamma^A\widehat{\mathscr{D}}_A\rho + \frac{1}{2}\bar{\psi}_M\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\vec{Y} \\
&+ \bar{\psi}_M\vec{\tau}\lambda\vec{Y} + \frac{3i}{4}\bar{\psi}_M\gamma^{AB}\rho v_{AB} + 3\bar{\psi}_M\vec{\tau}\rho\vec{t} + \frac{ig}{2}\bar{\psi}_M\vec{\tau}\Omega\vec{Y}
\end{aligned}
$$

and the constraint which has been mentioned before, following from the requirement of closure of the gauge algebra on N and W_A is

$$
\widehat{\mathscr{D}}_A W^A + \frac{i}{2}\bar{\rho}\gamma^{AB}\widehat{\mathcal{R}}_{AB} - 4\sqrt{3}\vec{t}\delta_z\vec{Y} - \frac{\sqrt{3}}{2}\delta_z N - 8igM\vec{Y}\vec{t} + ig\vec{X}\vec{Y} + g\bar{\Omega}\rho - igMN = 0. \quad (3.6)
$$

Type	Field	$SU(2)_{\mathcal{R}}$	Components	
			bosonic	fermionic
Isotriplet	$\vec{Y}$	**3**	3	
Fermion	ρ	**2**		8
Tensor field	B_{MNP}	**1**	4	
Scalar	N	**1**	1	
			8 +	8

Table 3.3: Field content of the tensor multiplet in $D = 5$.

The supercovariant derivative on W^A which appears here, is defined by

$$\begin{aligned}\widehat{\mathscr{D}}_M W^A &= \partial_M W^A + \widehat{\omega}_M{}^{AB} W_B - igB_M W^A - A_M \delta_z W^A - \frac{1}{2}\bar{\psi}_M \gamma^{ABC} \vec{\tau} \widehat{\mathcal{R}}_{BC} \vec{Y} \\ &+ \frac{i}{2}\bar{\psi}_M \gamma^{ABC} \rho v_{BC} - \frac{3i}{2}\bar{\psi}_M \gamma_B \rho v^{AB} - \frac{i}{2\sqrt{3}}\bar{\psi}_M \gamma^{ABC} \rho \widehat{F}_{BC} - \frac{i}{2}\bar{\psi}_M \gamma^{AB} \widehat{\mathscr{D}}_B \rho \\ &+ \frac{i}{2\sqrt{3}}\bar{\psi}_M \gamma_B \rho \widehat{F}^{AB} - \frac{ig}{2}\bar{\psi}_M \vec{\tau} \gamma^A \Omega \vec{Y} - \frac{g}{2}\bar{\psi}_M \gamma^A M \rho + \frac{\sqrt{3}i}{4}\bar{\psi}_M \gamma^A \delta_z \rho - 4\bar{\psi}_M \vec{\tau} \gamma^A \rho \vec{t}.\end{aligned}$$

3.3 The tensor multiplet

In the last section the linear multiplet has been treated. Supersymmetry required the constraint (3.6). For vanishing gauge group and vanishing central charge transformations, this constraint may be solved explicitly by Poincaré duality.[4] The resulting multiplet is then the tensor multiplet whose field content is collected in table 3.3.

The constraint (3.6) is solved by introducing an antisymmetric 3-form potential B_{MNP} through the equation

$$W^A = \frac{1}{12} e^A_M \varepsilon^{MNPQR} \widehat{\mathscr{D}}_N B_{PQR}. \tag{3.7}$$

The field W^A which is constrained in the case of the linear multiplet, may now be interpreted as supercovariant field strength of the tensor potential B_{MNP}.

The introduction of the tensor field implies a new gauge symmetry, namely invariance under tensor gauge transformations

$$\delta_\Lambda(\lambda) B_{MNP} = 3\partial_{[M} \lambda_{NP]}. \tag{3.8}$$

If one solves the constraint (3.6) by the supersymmetric Poincaré dualization, eq. (3.7), one finds the transformation law

$$\delta B_{MNP} = -i\bar{\varepsilon}\gamma_{MNP}\rho - 3\bar{\varepsilon}\vec{\tau}\gamma_{[NP}\psi_{M]}\vec{Y}$$

[4] This has been worked out for the case of conformal supergravity in four dimensions in [21, 27], for the six dimensional case see [3].

Type	Field	$SU(2)_{\mathcal{R}} \times SU(2)'$	Dimension	Components		
				bosonic		fermionic
Scalar	$\phi^i{}_\alpha$	$(\mathbf{2},\mathbf{2})$	0	3		
Spinor	χ	$(\mathbf{2},\mathbf{1})$	2			8
Scalar	φ	$(\mathbf{1},\mathbf{1})$	5/2	1		
Vector	V_A	$(\mathbf{1},\mathbf{1})$	5/2	$5-1$		
				8	+	8

Table 3.4: Field content of the nonlinear multiplet in $D = 5$.

which together with the previously given transformation laws (3.5a - 3.5c) forms the tensor multiplet (of course with $g = \delta_z = 0.$). The supercovariant derivative in (3.7) is given explicitly by

$$\widehat{\mathscr{D}}_{[M}B_{NPQ]} = \partial_{[M}B_{NPQ]} + i\bar{\rho}\gamma_{[NPQ}\psi_{M]} - \frac{3}{2}\bar{\psi}_{[N}\vec{\tau}\gamma_{PQ}\psi_{M]}\vec{Y}$$

and the gauge algebra has to be extended by the tensor gauge transformations (3.8); one finds

$$[\delta_Q(\eta), \delta_Q(\varepsilon)] = \ldots + \delta_\Lambda(i\bar{\varepsilon}\gamma^P\eta B_{MNP} - \bar{\varepsilon}\vec{\tau}\gamma_{MN}\eta\vec{Y}).$$

The dots represent the terms, which are already present in the original gauge algebra (2.15).

3.4 The nonlinear multiplet

As has been discussed in section 2, the minimal multiplet as it stands is to small to form a supergravity multiplet. It has to be extended by a matter multiplet with certain properties to be discussed in chapter 4. There are three possibilities, one of them being the nonlinear multiplet which will be discussed in this section. The name of this multiplet comes from the fact that the supersymmetry transformation laws are nonlinear in fields which belong to the nonlinear multiplet[5]. The nonlinear multiplet presented in this section is the five dimensional Poincaré variant of the superconformal nonlinear multiplet in four [27] and six [3] dimensions.

The nonlinear multiplet contains $(8 + 8)$ components. They are collected in table 3.4 (The subtracted degree of freedom follows from the constraint (3.9)). Together with the $(40 + 40)$ components of the minimal multiplet it will form version I of $\mathcal{N} = 2$ off-shell supergravity in five dimensions.

[5]Of course, most local supersymmetry variations are nonlinear in fields, but this is usually due to fields which belong to the supergravity multiplet. For the fields which belong to matter multiplets, the transformation laws are usually linear.

The scalar $\phi^i{}_\alpha$ transforms from the left under the local $SU(2)_\mathcal{R}$ (that is, this symmetry acts on the index i, consistent with our notations) and in addition we have introduced a global $SU(2)'$ under which $\phi^i{}_\alpha$ transforms from the right (acting on the index α). $\phi^i{}_\alpha$ is an element of the group[6] $SU(2)$, i.e. denoting by $\phi^\alpha{}_i$ the hermitian conjugate of $\phi^i{}_\alpha$, the following relations hold [27]:

$$\phi^i{}_\alpha \phi^\alpha{}_j = \delta^i_j, \qquad \phi^\alpha{}_i \phi^i{}_\beta = \delta^\alpha_\beta, \qquad \phi^\alpha{}_i = \varepsilon_{ij} \varepsilon^{\alpha\beta} \phi^j{}_\beta.$$

Using the $\phi^i{}_\alpha$ one can convert local $SU(2)_\mathcal{R}$ indices to global ones.

The nonlinear multiplet is also a possible starting point for the construction of off-shell supergravity with gauged $SU(2)_\mathcal{R}$. Here we mean a theory where the complete $\mathcal{R}$-symmetry is gauged by a propagating vectorfield which belongs to an additional super Yang-Mills multiplet. For the discussion of this theory in the four dimensional case, see [17]. Effectively, the auxiliary field $\vec{V}_M$ is replaced by a dynamical field. To construct such a theory, the global $SU(2)'$ acting on the index α has to be gauged by the introduction of a super Yang-Mills multiplet. Further, if one identifies an $SO(2)$ subgroup of the gauged $SU(2)'$ with the $U(1)$ for which the graviphoton is the gauge field, one obtains *gauged supergravity*, i.e. supergravity with a cosmological constant whose ground state is an anti de Sitter space. Theories of this kind are consequently also called *adS* supergravities. Originally, this procedure has been worked out in the four dimensional case by de Wit et al. in [27].

We have constructed this multiplet by the following ansatz which is motivated from the literature, [27, 3]:

$$\delta\phi^i{}_\alpha \sim (\vec{\tau})^i{}_j \phi^j{}_\alpha \bar{\varepsilon} \vec{\tau} \chi$$

and a general ansatz for $\delta\chi$, bilinear in χ and $\phi^i{}_\alpha$. By implementing the algebra (2.15), including the modifications (3.1) all the remaining terms are easily found.

As already mentioned, the $SU(2)'$ may be gauged by a super Yang-Mills multiplet. This possibility is included in our construction. We denote the required super Yang-Mills multiplet by

$$(\vec{B}_M,\ \vec{M},\ \vec{\Omega},\ \tilde{\vec{X}}),$$

where the arrows on the fields denote the **3** of $SU(2)'$ and the tilde on $\tilde{\vec{X}}$ denotes the **3** of the automorphism group $SU(2)_\mathcal{R}$. The gauge coupling constant of the $SU(2)'$ is g.

[6]This $SU(2)$ ist not to be confused with the $SU(2)_\mathcal{R}$ $\mathcal{R}$-symmetry. The new $SU(2)$ is completely independent.

The supersymmetry transformation laws we find are:

$$
\begin{aligned}
\delta\phi^i{}_\alpha &= -i\kappa(\vec{\tau})^i{}_j\phi^j{}_\alpha\bar{\varepsilon}\vec{\tau}\chi \\
\delta\chi^i &= \frac{1}{2}\gamma^A\varepsilon^i V_A - \frac{1}{2}\varepsilon^i\varphi - \frac{1}{2\kappa}\phi^i{}_\alpha\widehat{\mathscr{D}}_A\phi^\alpha{}_j\gamma^A\varepsilon^j + 2i\kappa\chi^i\bar{\chi}\varepsilon + 3i(\vec{\tau})^i{}_j\varepsilon^j\vec{t} \\
&- \frac{i}{4}\kappa\gamma^{AB}\varepsilon^i\bar{\chi}\gamma_{AB}\chi + \gamma^{AB}\varepsilon^i(-\frac{1}{4\sqrt{3}}\widehat{F}_{AB} + \frac{1}{2}v_{AB}) - \frac{ig}{4\kappa}\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\varepsilon^j\vec{M} \\
\delta\varphi &= i\bar{\varepsilon}\gamma^A\widehat{\mathscr{D}}_A\chi + i\bar{\varepsilon}_i\gamma^A\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_A\phi^\alpha{}_j - \frac{i}{2}\bar{\varepsilon}\gamma^{AB}\widehat{\mathcal{R}}_{AB} - 2i\bar{\varepsilon}\lambda + \frac{i\kappa}{2}\bar{\varepsilon}\gamma^{AB}\chi v_{AB} - i\kappa\bar{\varepsilon}\chi\varphi \\
&+ \frac{i\kappa}{2\sqrt{3}}\bar{\varepsilon}\gamma^{AB}\chi\widehat{F}_{AB} - i\kappa\bar{\varepsilon}\gamma^A\chi V_A + \frac{g}{2}\bar{\varepsilon}_i\chi^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\vec{M} + \frac{g}{2\kappa}\bar{\varepsilon}_i\vec{\Omega}^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j \\
\delta V_A &= i\bar{\varepsilon}\gamma_A{}^B\widehat{\mathscr{D}}_B\chi + i\bar{\varepsilon}_i\gamma_A{}^B\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_B\phi^\alpha{}_j + i\bar{\varepsilon}_i\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_A\phi^\alpha{}_j - \frac{i}{2}\bar{\varepsilon}\gamma_A{}^{BC}\widehat{\mathcal{R}}_{BC} - i\bar{\varepsilon}\gamma^B\widehat{\mathcal{R}}_{AB} \\
&- 2i\bar{\varepsilon}\gamma_A\lambda + i\kappa\bar{\varepsilon}\gamma^B\chi v_{AB} + 2\kappa\bar{\varepsilon}\vec{\tau}\gamma_A\chi\vec{t} + \frac{i\kappa}{2\sqrt{3}}\bar{\varepsilon}\gamma_{ABC}\chi\widehat{F}^{BC} - i\kappa\bar{\varepsilon}\chi V_A \\
&- i\kappa\bar{\varepsilon}\gamma_{AB}\chi V^B - i\kappa\bar{\varepsilon}\gamma_A\chi\varphi + \frac{g}{2}\bar{\varepsilon}_i\gamma_A\chi^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\vec{M} + \frac{g}{2\kappa}\bar{\varepsilon}_i\gamma_A\vec{\Omega}^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j.
\end{aligned}
$$

Note that the transformation law of $\phi^i{}_\alpha$ is a field dependent $SU(2)_{\mathcal{R}}$ gauge transformation.

The supercovariant derivatives are given by

$$
\begin{aligned}
\widehat{\mathscr{D}}_M\phi^i{}_\alpha &= \partial_M\phi^i{}_\alpha - \frac{i\kappa}{2}(\vec{\tau})^i{}_j\phi^j{}_\alpha\vec{V}_M + i\kappa^2(\vec{\tau})^i{}_j\phi^j{}_\alpha\bar{\psi}_M\vec{\tau}\chi + \frac{ig}{2}\phi^i{}_\beta(\vec{\tau})^\beta{}_\alpha\vec{B}_M \\
\widehat{\mathscr{D}}_M\chi^i &= \partial_M\chi^i + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\chi^i - \frac{i\kappa}{2}(\vec{\tau})^i{}_j\chi^j\vec{V}_M - \frac{\kappa}{2}\gamma^A\psi^i_M V_A + \frac{\kappa}{2}\psi^i_M\varphi \\
&+ \frac{i\kappa^2}{4}\gamma^{AB}\psi^i_M\bar{\chi}\gamma_{AB}\chi + \frac{1}{2}\phi^i{}_\alpha\widehat{\mathscr{D}}_A\phi^\alpha{}_j\gamma^A\psi^j_M - 2i\kappa^2\chi^i\bar{\chi}\psi_M - 3i\kappa(\vec{\tau})^i{}_j\psi^j_M\vec{t} \\
&- \kappa\gamma^{AB}\psi^i_M(-\frac{1}{4\sqrt{3}}\widehat{F}_{AB} + \frac{1}{2}v_{AB}) + \frac{ig}{4}\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\psi^j_M\vec{M}.
\end{aligned}
$$

Closure of the algebra on φ and V_A requires the following constraint:

$$
\begin{aligned}
&C - \frac{1}{16\kappa}\widehat{R}(\widehat{\omega})_{AB}{}^{AB} - \frac{1}{4}\widehat{\mathscr{D}}_A V^A - \frac{1}{8\kappa}\widehat{\mathscr{D}}^M\phi^i{}_\alpha\widehat{\mathscr{D}}_M\phi^\alpha{}_i - \frac{i\kappa}{2}\bar{\chi}\gamma^M\widehat{\mathscr{D}}_M\chi + \frac{i\kappa}{4}\bar{\chi}\gamma^{AB}\widehat{\mathcal{R}}_{AB} \\
&+ 2i\kappa\bar{\chi}\lambda + 5\kappa\vec{t}^2 - \frac{\kappa}{24}\widehat{F}_{AB}\widehat{F}^{AB} + \frac{\kappa}{4}v_{AB}v^{AB} - \frac{i\kappa}{2}\bar{\chi}_i\gamma^M\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_M\phi^\alpha{}_j \\
&- \kappa^2\frac{i}{4}\bar{\chi}\gamma^{AB}\chi v_{AB} - \frac{i\kappa^2}{4\sqrt{3}}\bar{\chi}\gamma^{AB}\chi\widehat{F}_{AB} + \frac{\kappa}{4}V_A V^A - \frac{\kappa}{4}\varphi^2 - \frac{g}{8\kappa}\vec{\tilde{X}}(\tilde{\tau})^j{}_i\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j \\
&- \frac{g}{4}\bar{\chi}_i(\kappa\chi^j\vec{M} + 2\vec{\Omega}^j)\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j + \frac{g}{4}(\tilde{\tau})^j{}_i\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\vec{M}\tilde{t} + \frac{g^2}{16\kappa}\vec{M}^2 = 0.
\end{aligned}
\tag{3.9}
$$

A useful formula in this context is the supersymmetry variation of the supercovariant curvature tensor, defined in eq. (2.14):

$$\begin{aligned}
\delta\widehat{R}(\widehat{\omega})^{AB}{}_{CD} &= -4i\kappa\bar{\varepsilon}\gamma^{[A}\widehat{\mathscr{D}}_{[C}\widehat{\mathcal{R}}^{B]}{}_{D]} - 2i\kappa\bar{\varepsilon}\gamma_{[C}\widehat{\mathscr{D}}_{D]}\widehat{\mathcal{R}}^{AB} - 8\kappa^2\bar{\varepsilon}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{CD}\vec{t} \\
&- 4i\kappa^2(\frac{1}{2}v_{EF}\bar{\varepsilon}\gamma_{[C}{}^{EF} + 2i\bar{\varepsilon}\vec{\tau}\gamma_{[C}\vec{t} - \frac{1}{\sqrt{3}}\bar{\varepsilon}\gamma^E\widehat{F}_{E[C)})\gamma^{[A}\widehat{\mathcal{R}}^{B]}{}_{D]} \\
&+ 2i\kappa^2(\frac{1}{2}v_{EF}\bar{\varepsilon}\gamma_{[C}{}^{EF} + 2i\bar{\varepsilon}\vec{\tau}\gamma_{[C}\vec{t} - \frac{1}{\sqrt{3}}\bar{\varepsilon}\gamma^E\widehat{F}_{E[C)})\gamma_{D]}\widehat{\mathcal{R}}^{AB} \\
&+ 2i\kappa^2\bar{\varepsilon}\gamma^{ABEF}\widehat{\mathcal{R}}_{CD}v_{EF} + \frac{4i\kappa^2}{\sqrt{3}}\bar{\varepsilon}\widehat{\mathcal{R}}_{CD}\widehat{F}^{AB}.
\end{aligned}$$

Further, the following standard symmetries which are valid for Riemann tensors in spaces without torsion are valid for the supercovariant Riemann tensor:

$$\widehat{R}(\widehat{\omega})_{[MNP]Q} = 0, \qquad \widehat{R}(\widehat{\omega})_{ABCD} = \widehat{R}(\widehat{\omega})_{CDAB}$$

The last formula in this section is the explicit expression for the supercovariant derivative on V_A:

$$\begin{aligned}
\widehat{\mathscr{D}}_M V^A &= \partial_M V^A + \widehat{\omega}_M{}^{AB}V_B - i\kappa\bar{\psi}_M\gamma^{AB}\widehat{\mathscr{D}}_B\chi - i\kappa\bar{\psi}_{Mi}\gamma^{AB}\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_B\phi^\alpha{}_j + i\kappa^2\bar{\psi}_M\chi V^A \\
&+ \frac{i\kappa}{2}\bar{\psi}_M\gamma^{ABC}\widehat{\mathcal{R}}_{BC} + i\kappa\bar{\psi}_M\gamma_B\widehat{\mathcal{R}}^{AB} + 2i\kappa\bar{\psi}_M\gamma^A\lambda - i\kappa^2\bar{\psi}_M\gamma_B\chi v^{AB} \\
&- 2\kappa^2\bar{\psi}_M\vec{\tau}\gamma^A\chi\vec{t} - \frac{i\kappa^2}{2\sqrt{3}}\bar{\psi}_M\gamma^{ABC}\chi\widehat{F}_{BC} + i\kappa^2\bar{\psi}_M\gamma^{AB}\chi V_B + i\kappa^2\bar{\psi}_M\gamma^A\chi\varphi \\
&- \frac{g\kappa}{2}\bar{\psi}_{Mi}\gamma^A\chi^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j\vec{M} - \frac{g}{2}\bar{\psi}_{iM}\gamma^A\vec{\Omega}^j\phi^i{}_\alpha(\vec{\tau})^\alpha{}_\beta\phi^\beta{}_j - i\kappa\bar{\psi}_{Mi}\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}^A\phi^\alpha{}_j.
\end{aligned}$$

3.5 The hypermultiplet

Our presentation of the hypermultiplet follows the papers [20] on conformal hypermultiplets and [7]. The structure of the hypermultiplet is in four, five and six dimensions almost the same. This is due to the fact that the hypermultiplet contains no field which carries vector indices.

Let us stress that the approach within Poincaré supersymmetry [7] is more powerful than the superconformal one [20]. The latter suffers from the fact that the superconformal algebra allows no central charge (cf. appendix B) and due to this, one has to cope with somewhat artificially looking constructions such as the appearance of infinitely many fields at intermediate steps. Nevertheless, in the literature this multiplet has been studied mostly within the superconformal framework [20, 3, 56].

The on-shell version of this multiplet and its coupling to an arbitrary number of abelian vector multiplets was described in [69].

The hypermultiplet contains a scalar A_i^α which transforms as a doublet under the $\mathcal{R}$-symmetry. The other index, $\alpha = 1, \dots 2r$, is an index which may transform under a gauge

Type	Field	$SU(2)_{\mathcal{R}}$	Dimension	Components	
				bosonic	fermionic
Scalar	A_i^α	$\mathbf{2}$	3/2	$4r$	
Dirac Fermion	ζ^α	$\mathbf{1}$	2		$8r$
Scalar	F_i^α	$\mathbf{2}$	5/2	$4r$	
				$8r$ +	$8r$

Table 3.5: Field content of the hypermultiplet in $D = 5$.

group. The reason for the range of this index will become clear below. If A_i^α is charged under the gauge group, its transformation is defined to be

$$\delta A_i^\alpha = i\alpha^{\mathcal{A}}(t_{\mathcal{A}})^\alpha{}_\beta A_i^\beta.$$

Here the $t_{\mathcal{A}}$'s generate the relevant representation and $\alpha^{\mathcal{A}}$ is the transformation parameter.

Besides this field, the hypermultiplet contains a fermion ζ^α, the so called hyperino, which is a priori a Dirac spinor and which fulfills a reality constraint to be specified below. The hypermultiplet is completed by the auxiliary field F_i^α. The field content of the hypermultiplet is collected in table 3.5.

In principle there are two equivalent ways of treating the hypermultiplet: The first possibility is to view the fields of this multiplet as complex and unconstrained fields; this would be the so-called complex hypermultiplet [7]. Or, one may impose certain reality constraints, which then leads to the real hypermultiplet [20]. We will follow the latter possibility. The reality constraint on the scalar is

$$A_\alpha^i \equiv A_i^{\alpha *} = \varepsilon^{ij}\rho_{\alpha\beta}A_j^\beta, \tag{3.10}$$

where we have defined implicitly the notation for the complex conjugated scalar. Further, we have introduced a metric $\rho_{\alpha\beta}$. Consistency requires

$$\rho_{\alpha\beta}\rho^{\beta\gamma} = -\delta_\alpha^\gamma, \qquad \text{where} \qquad \rho^{\alpha\beta} \equiv \rho_{\alpha\beta}^*.$$

Since $|\det\rho|^2 = \det(-\mathbb{1})$, it follows that the gauge group index has to run over an even number of values. It can be shown [20], that by field redefinitions $\rho_{\alpha\beta}$ may be brought to the form

$$\rho = diag\ (\varepsilon \oplus \varepsilon \oplus \ldots). \tag{3.11}$$

Here we use ε, the totally antisymmetric symbol in two dimensions. From the transformation law (see below)

$$\delta A_i^\alpha = i\bar{\varepsilon}_i\zeta^\alpha$$

and the reality constraint (3.10) it follows for the fermion, that

$$\bar{\zeta}_\alpha = \rho_{\alpha\beta}\zeta^{\beta T}C, \qquad \text{i.e.} \qquad \zeta^\alpha = -\rho^{\alpha\beta}\gamma^0 C^{-1}\zeta_\beta^*, \tag{3.12}$$

where $\bar{\zeta}_\alpha \equiv (\zeta^\alpha)^\dagger \gamma^0$ is the Dirac conjugated spinor. The symmetries for bilinears which contain fermions of this kind are collected in the appendix, eqs. (A.6) and (A.7).

Consistency requires that the generators of the gauge group satisfy

$$\rho_{\alpha\beta}(t_{\mathcal{A}})^\beta{}_\gamma = -(t_{\mathcal{A}})_\alpha{}^\beta \rho_{\beta\gamma}, \qquad \text{with} \qquad (t_{\mathcal{A}})_\alpha{}^\beta = (t_{\mathcal{A}})^{\alpha *}{}_\beta. \tag{3.13}$$

Using (3.11) one easily sees that the $t_{\mathcal{A}}$'s must be defined over the field $\mathbb{H}$ of quaternions; that is, the $t_{\mathcal{A}}$'s generate a subgroup of $Gl(r, \mathbb{H})$.

A crucial property of the hypermultiplet is that it is charged under central charge transformations. If one defines two objects

$$\begin{aligned}
\Delta^\alpha \;&\equiv\; \frac{i}{2}\gamma^A \widehat{\mathscr{D}}_A \zeta^\alpha + \frac{i\kappa}{4}\gamma^{AB}\zeta^\alpha v_{AB} - 2i\kappa\lambda^i A_i^\alpha \\
&+\; \frac{i\kappa}{4\sqrt{3}}\gamma^{AB}\zeta^\alpha \widehat{F}_{AB} - \frac{i\kappa}{2}\gamma^{AB}\widehat{\mathcal{R}}^i_{AB} A_i^\alpha + g\Omega^{i\alpha}{}_\beta A_i^\beta + \frac{g}{2}M^\alpha{}_\beta \zeta^\beta \\
\mathscr{X}_i^\alpha \;&\equiv\; 2gM^\alpha{}_\beta F_i^\beta - 2i\kappa A_i^\alpha C + \frac{i}{8}\widehat{R}(\widehat{\omega})_{AB}{}^{AB} A_i^\alpha + ig(\vec{\tau})^j{}_i \vec{X}^\alpha{}_\beta A_j^\beta \\
&-\; \frac{ig^2}{2}(M^2)^\alpha{}_\beta A_i^\beta - \frac{i}{2}\widehat{\mathscr{D}}^A \widehat{\mathscr{D}}_A A_i^\alpha + ig\bar{\Omega}_i^\alpha{}_\beta \zeta^\beta + 4\kappa\vec{t} F_j^\alpha (\vec{\tau})^j{}_i - 2\kappa\bar{\lambda}_i \zeta^\alpha \\
&-\; 10i\kappa^2 \vec{t}^2 A_i^\alpha - 2ig\kappa(\vec{\tau})^j{}_i \vec{t} M^\alpha{}_\beta A_j^\beta - \frac{i\kappa^2}{2} v_{AB} v^{AB} A_i^\alpha + \frac{i\kappa^2}{12}\widehat{F}_{AB}\widehat{F}^{AB} A_i^\alpha,
\end{aligned}$$

the central charge transformation laws of the hypermultiplet are easily written down:

$$\delta_z A_i^\alpha = -\frac{4\kappa}{\sqrt{3}} F_i^\alpha, \qquad \delta_z \zeta^\alpha = -\frac{4i\kappa}{\sqrt{3}}\Delta^\alpha, \qquad \delta_z F_i^\alpha = -\frac{2i\kappa}{\sqrt{3}}\mathscr{X}_i^\alpha. \tag{3.14}$$

Under supersymmetry, the fields transform as follows:

$$\begin{aligned}
\delta A_i^\alpha &= i\bar{\varepsilon}_i \zeta^\alpha \\
\delta \zeta^\alpha &= -\gamma^A \varepsilon^i \widehat{\mathscr{D}}_A A_i^\alpha + 2\varepsilon^i F_i^\alpha + 6i\kappa(\vec{\tau}\varepsilon)^i A_i^\alpha \vec{t} \\
&+ \kappa\gamma^{AB}\varepsilon^i A_i^\alpha v_{AB} - \frac{\kappa}{2\sqrt{3}}\gamma^{AB}\varepsilon^i A_i^\alpha \widehat{F}_{AB} - ig\varepsilon^i (M)^\alpha{}_\beta A_i^\beta \\
\delta F_i^\alpha &= -\bar{\varepsilon}_i \Delta^\alpha,
\end{aligned}$$

with the supercovariant derivatives

$$\begin{aligned}
\widehat{\mathscr{D}}_M \zeta^\alpha &= \partial_M \zeta^\alpha + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\zeta^\alpha - ig(B_M)^\alpha{}_\beta \zeta^\beta + \frac{4i\kappa}{\sqrt{3}}\Delta^\alpha A_M \\
&+ \kappa\gamma^A \psi_M^i \widehat{\mathscr{D}}_A A_i^\alpha - 2\kappa\psi_M^i F_i^\alpha - 6i\kappa(\vec{\tau}\psi_M)^i A_i^\alpha \vec{t} \\
&- \kappa^2 \gamma^{AB}\psi_M^i A_i^\alpha v_{AB} + \frac{\kappa^2}{2\sqrt{3}}\gamma^{AB}\psi_M^i A_i^\alpha \widehat{F}_{AB} + ig\kappa\psi_M^i (M)^\alpha{}_\beta A_i^\beta \\
\widehat{\mathscr{D}}_M A_i^\alpha &= \partial_M A_i^\alpha + \frac{i\kappa}{2}A_j^\alpha (\vec{\tau})^j{}_i \vec{V}_M - ig(B_M)^\alpha{}_\beta A_i^\beta + \frac{4\kappa}{\sqrt{3}}A_M F_i^\alpha - i\kappa\bar{\psi}_{Mi}\zeta^\alpha \\
\widehat{\mathscr{D}}_M F_i^\alpha &= \partial_M F_i^\alpha + \frac{i\kappa}{2}F_j^\alpha (\vec{\tau})^j{}_i \vec{V}_M - ig(B_M)^\alpha{}_\beta F_i^\beta + \frac{2i\kappa}{\sqrt{3}}\mathscr{X}_i^\alpha A_M + \kappa\bar{\psi}_{Mi}\Delta^\alpha.
\end{aligned}$$

Again we use Lie-algebra valued expressions for the fields which belong to the vector multiplet, e.g. $(B_M)^\alpha{}_\beta \equiv B_M^{\mathcal{A}}(t_{\mathcal{A}})^\alpha{}_\beta$.

Chapter 4

Supersymmetric Actions

Using the multiplets we presented in chapter 2 and 3, we develop in this chapter a tensor calculus and present supersymmetric actions. In particular, we construct three inequivalent formulations of off-shell supergravity. These are *all* minimal versions, i.e. versions with $(48+48)$ components, as can be easily seen by the following argument: in order to extend the minimal multiplet to a consistent formulation of off-shell supergravity, a matter multiplet with at least $(8+8)$ components has to be included. This multiplet must contain a Lorentz scalar which transforms nontrivially under the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry, since it is required to fix this symmetry. It is then an easy task to search the possible multiplets for such a scalar. One finds, that as in four dimensions [21], there are three candidates for such a compensator multiplet: the nonlinear multiplet (leads to version *I*), the hypermultiplet (version *II*) and the tensor multiplet (version *III*).

4.1 Two fundamental invariants

In this section we present two fundamental invariants. They are the elementary building blocks of the actions we present in the following sections.

We start with the coupling term of a linear multiplet with a Maxwell multiplet. Here, the linear multiplet is uncharged under the gauge group $U(1)$ and central charge transformations. The action

$$S = \int d^5x \; e\mathscr{L}$$

with lagrangian

$$\begin{aligned}\mathscr{L} = {} & 2\vec{X}\vec{Y} - 2i\bar{\Omega}\rho - 2MN - 2B^A W_A - \bar{\Omega}\vec{\tau}\gamma^M\psi_M\vec{Y} - i\bar{\rho}\gamma^M\psi_M M \\ & + i\bar{\rho}\gamma^{MN}\psi_M B_N + \frac{1}{2}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{Y}M - \frac{1}{2}\bar{\psi}_B\vec{\tau}\gamma^{ABC}\psi_C B_A\vec{Y} - 16\vec{Y}\vec{t}M\end{aligned} \tag{4.1}$$

is invariant under supersymmetry. This is the extension of (3.4) to local supersymmetry.

As already mentioned several times, it is possible to embed many multiplets in the linear multiplet. This means, that if an action for the linear multiplet is known, an action for the embedded multiplet is easily found. Thus we have to derive an action for the linear multiplet. This can be achieved by observing [56], that the supergravity multiplet contains a Maxwell multiplet:

$$(B_M,\ M,\ \Omega,\ \vec{X}) = \left(A_M,\ \frac{\sqrt{3}}{2},\ 0,\ 0 \right) \tag{4.2}$$

Now we can find the generalization of the rigidly supersymmetric invariant (3.3) to local supersymmetry. We use the embedding (4.2) in the action formula (4.1) to derive

$$\begin{aligned} \mathscr{L}_{lin} = -4N + 2i\bar{\psi}_A\gamma^A\rho - 32\vec{t}\vec{Y} + \bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{Y} \\ - \frac{2}{\sqrt{3}}\left(4W^A + 2i\bar{\rho}\gamma^{AB}\psi_B + \bar{\psi}_B\vec{\tau}\gamma^{ABC}\psi_C\vec{Y}\right)A_A. \end{aligned} \tag{4.3}$$

This holds also in the case when the linear multiplet is centrally charged. Please note that the invariance under these transformations is non-trivial since the gauge field of this $U(1)$, the graviphoton, appears explicitly in (4.3). To proof invariance under central charge transformations one has to use the constraint of the linear multiplet, eq. (3.6).

In (4.3) we have assumed that the fields of the linear multiplet are gauge singlets. It turns out that this requirement is satisfied in all cases we will investigate.

Finally, note the similarity of our result with the equivalent result in four dimensions, [6].

4.2 The supergravity action

The embedding of the minimal multiplet in the linear multiplet is given by

$$(\vec{Y},\ \rho,\ N,\ W^A) = \left(\vec{t},\ \lambda,\ C,\ \frac{1}{4}\widehat{\mathscr{D}}_B v^{AB} + \frac{1}{48}\varepsilon^{ABCDE}\widehat{F}_{BC}\widehat{F}_{DE} \right). \tag{4.4}$$

It is now very easy to write down an action for the minimal multiplet. If one uses the embedding (4.4) in the action formula for the linear multiplet, eq. (4.3), one finds

$$\begin{aligned} \mathscr{L}_{min} = -\frac{4}{\kappa}C - 2i\bar{\lambda}\gamma^M\psi_M + \kappa\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} - \frac{1}{\sqrt{3}}F_{AB}v^{AB} \\ - \frac{\kappa}{6\sqrt{3}}\varepsilon^{ABCDE}A_AF_{BC}F_{DE} + \frac{i\kappa}{8\sqrt{3}}\varepsilon^{ABCDE}\bar{\psi}_A\gamma_B\psi_CF_{DE} - 32\vec{t}\vec{t}. \end{aligned} \tag{4.5}$$

Of course, this can only be a first step towards a physically significant action, since, as stated in the beginning of section 2, the variation of the action w.r.t. C leads to the unphysical equation of motion $\det e_M^A = 0$. However, something like this was expected to happen, since the minimal multiplet contains only $(40+40)$ of the required $(48+48)$ components.

An alternative reason for the inconsistency of (4.5) may be found in the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry which is gauged by the auxiliary field $\vec{V}_M$. This symmetry has to be fixed. To achieve that, an additional multiplet, a so-called compensator multiplet, has to be introduced. A suitable multiplet has to fulfill the following requirements: it must contain a propagating scalar which transforms in a nontrivial representation of the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry. Using this field one may fix the gauge and one finds the desired result. If the scalar transforms in the fundamental **2** of the $SU(2)_{\mathcal{R}}$, this symmetry can be fixed completely (this is the case for versions *I* and *II*). However, if the scalar transforms in the vector representation **3**, this is not possible; in this case a residual $U(1)$ survives. This $U(1)$ is still gauged by an auxiliary field (version *III*). In all three cases gauged versions are possible, i.e. a $U(1)$ subgroup of the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry is gauged via the (propagating) graviphoton A_M. This gauging is performed especially in the case of version *III* straightforwardly.

Inspection of the matter multiplets in chapter 3 shows, that the linear multiplet (or equivalently the tensor multiplet), the nonlinear multiplet and the hypermultiplet fulfill the upper requirements. Inclusion of each of these multiplets leads to different formulations of off-shell supergravity. We will discuss all these three variants in turn.

For version *I* the required $(8+8)$ components are added using the nonlinear multiplet. We treat the case of ungauged $SU(2)'$ first. In this case we substitute in the action (4.5) for the field C the constraint (3.9). After partial integration one finds

$$
\begin{aligned}
\mathscr{L} = &-\frac{1}{4\kappa^2}R(\widehat{\omega})_{AB}{}^{AB} + \frac{i\kappa}{8\sqrt{3}}\bar{\psi}_A\gamma^{ABCD}\psi_B(2\widehat{F}_{CD}-F_{CD}) - \frac{i\kappa}{4}\bar{\psi}_A\gamma^{ABCD}\psi_B v_{CD} \\
&+\frac{3i\kappa}{2}\bar{\psi}_A\psi_B v^{AB} - \frac{1}{6}\widehat{F}_{AB}\widehat{F}^{AB} - \frac{1}{\sqrt{3}}\widehat{F}_{AB}v^{AB} + v_{AB}v^{AB} - \frac{i}{2}\bar{\psi}_A\gamma^{ABC}\widehat{\mathcal{R}}_{BC} \\
&-\frac{\kappa}{6\sqrt{3}}\varepsilon^{ABCDE}A_A F_{BC}F_{DE} + 8i\bar{\chi}\lambda + 3\kappa\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} - \frac{i\kappa}{2\sqrt{3}}\bar{\chi}\gamma^M\gamma^{AB}\psi_M\widehat{F}_{AB} \\
&-6\kappa\bar{\chi}\vec{\tau}\gamma^M\psi_M\vec{t} + i\kappa\bar{\chi}\gamma^M\gamma^{AB}\psi_M v_{AB} + \frac{i}{2}\bar{\psi}_{Mi}\gamma^{MNP}\psi_N^j\phi^i{}_\alpha\widehat{\mathscr{D}}_P\phi^\alpha{}_j \\
&-i\kappa\bar{\psi}_A\gamma^A\gamma^B\chi V_B - i\kappa\bar{\psi}_M\gamma^M\chi\varphi + 2i\bar{\chi}\gamma^{AB}\widehat{\mathcal{R}}_{AB} - 2i\bar{\chi}\gamma^M\widehat{\mathscr{D}}_M\chi \\
&-12\vec{t}^2 + V_A V^A - \varphi^2 - i\kappa\bar{\chi}\gamma^{AB}\chi(v_{AB} + \frac{1}{\sqrt{3}}\widehat{F}_{AB}) \\
&+i\bar{\psi}_{Ai}\gamma^{AB}\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_B\phi^\alpha{}_j + i\bar{\psi}_{Ai}\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}^A\phi^\alpha{}_j - \frac{1}{2\kappa^2}\widehat{\mathscr{D}}_M\phi^i{}_\alpha\widehat{\mathscr{D}}^M\phi^\alpha{}_i \\
&-2i\kappa\bar{\chi}_i\gamma^M\chi^j\phi^i{}_\alpha\widehat{\mathscr{D}}_M\phi^\alpha{}_j - \kappa^2\bar{\psi}_P\gamma^P\psi_M\bar{\chi}\gamma^{MN}\psi_N + \frac{\kappa^2}{2}\bar{\psi}_N\gamma^P\psi_M\bar{\chi}\gamma^{MN}\psi_P \\
&-\frac{\kappa^2}{4}\bar{\psi}_A\gamma^{ABCD}\psi_B\bar{\chi}\gamma_{CD}\chi - \frac{\kappa^2}{2}\bar{\psi}_N\psi_M\bar{\chi}\gamma^{MN}\chi + 2\kappa^2\bar{\psi}_M\gamma^{MN}\chi\bar{\chi}\psi_N.
\end{aligned}
\tag{4.6}
$$

Note that the kinetic terms for gravitino and auxiliary spinor χ are nondiagonal. The redefinition

$$\lambda' = \lambda + \frac{1}{4}\gamma^{AB}\widehat{\mathcal{R}}_{AB} \tag{4.7}$$

diagonalizes these terms. Further, by replacing v_{AB} by

$$v'_{AB} = v_{AB} - \frac{1}{2\sqrt{3}}\widehat{F}_{AB}, \tag{4.8}$$

one finds a canonical kinetic term for the graviphoton. Finally, the local $SU(2)_{\mathcal{R}}$ $\mathcal{R}$ symmetry has to be fixed. This is achieved by setting

$$\phi^i{}_\alpha = \delta^i_\alpha, \tag{4.9}$$

so that a gauge equivalent formulation of the former theory is obtained. The three bosonic degrees of freedom of the scalar $\phi^i{}_\alpha$ are absorbed by the former $SU(2)_{\mathcal{R}}$ gauge field $\vec{V}_M$. To stay in the gauge (4.9), a field dependent $SU(2)_{\mathcal{R}}$ transformation with parameter

$$\vec{\xi} = 2\kappa\bar{\varepsilon}\vec{\tau}\chi$$

has to be added to the supersymmetry transformation laws. This introduces a term of the form $\sim \partial\varepsilon$ into the supersymmetry transformation law of the $\vec{V}_M$, which may be eliminated by defining a new vector field

$$\vec{V}'_M = \vec{V}_M - 2\kappa\bar{\psi}_M\vec{\tau}\chi. \tag{4.10}$$

To gain a better understanding of the lagrangian density (4.6), we decovariantize this expression and use the definitions (4.7 – 4.10). Finally we decompose the lagrangian

$$\mathscr{L}_{VI} = \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3. \tag{4.11}$$

The on-shell part is [10, 8]

$$\begin{aligned}\mathscr{L}_1 = &-\frac{1}{4\kappa^2}R(\widetilde{\omega})_{AB}{}^{AB} - \frac{i}{2}\bar{\psi}_P\gamma^{PMN}\mathscr{D}(\frac{\widehat{\omega}+\widetilde{\omega}}{2})_M\psi_N - \frac{1}{4}F_{AB}F^{AB}\\ &-\frac{\kappa}{6\sqrt{3}}\varepsilon^{ABCDE}A_AF_{BC}F_{DE} - \frac{\sqrt{3}i\kappa}{16}(F^{AB}+\widehat{F}^{AB})\bar{\psi}_C\gamma^{[C}\gamma_{AB}\gamma^{D]}\psi_D\end{aligned} \tag{4.12}$$

and we use the same conventions as [29]. On-shell $\widetilde{\omega}_{MAB}$ is the solution to its own field equation, [10]. An explicit expression is given in the appendix, eq. (A.11). the covariant derivative is only covariant under local Lorentz transformations.

The off-shell lagrangians are:

$$\begin{aligned}
\mathscr{L}_2 \;&=\; v'_{AB}v^{AB'} + 8i\bar{\chi}\lambda' - 12\vec{t}\vec{t} + V_A V^A - \varphi^2 - \frac{1}{4}\vec{V}'_A\vec{V}'^A \\
&+\; 2i\kappa\bar{\chi}\gamma^M\gamma^{AB}\psi_M v'_{AB} + 2i\kappa\bar{\chi}\psi_M V^M - 2i\bar{\chi}\gamma^M\mathscr{D}_M\chi \\
&+\; \kappa\bar{\chi}\vec{\tau}\gamma^{MN}\psi_M\vec{V}'_N - i\kappa\bar{\chi}\gamma^{AB}\chi(v'_{AB} + \frac{\sqrt{3}}{2}\widehat{F}_{AB}) - 12\kappa\bar{\chi}\vec{\tau}\gamma^M\psi_M\vec{t}
\end{aligned}$$

and

$$\mathscr{L}_3 = -\frac{\kappa^2}{4}\bar{\psi}_M\gamma^{MN}\gamma^{AB}\psi_N\bar{\chi}\gamma_{AB}\chi + 2\kappa^2\bar{\psi}_M\gamma^{MN}\chi\bar{\chi}\psi_N.$$

The field equations for the auxiliary fields are

$$v'_{AB} = \chi = \lambda' = \vec{t} = V_A = \varphi = \vec{V}'_A = 0$$

and on-shell only $\mathscr{L}_1$ remains.

In the formulation we have discussed here, the C field has been replaced by lower dimensional fields. The Einstein-Hilbert and Rarita-Schwinger terms have been introduced by this replacement to the action[1]. The λ' field plays the role of a lagrange multiplicator for χ .

For completeness, let us give the supersymmetry transformation laws under which the lagrangian (4.11) is invariant. Explicit expressions for the appearing supercovariant objects are collected in the appendix, eqs. (A.9, A.13).

The gravitino field strength which appears here is given by

$$\begin{aligned}
\widehat{\mathcal{R}}_{MN} =& \mathscr{D}_{[M}\psi_{N]} - \frac{i\kappa}{2}\vec{\tau}\psi_{[N}\vec{V}'_{M]} + \frac{\kappa}{2}\gamma_{[M}{}^{AB}\psi_{N]}(v'_{AB} + \frac{1}{2\sqrt{3}}\widehat{F}_{AB}) \\
&+2i\kappa\gamma_{[M}\vec{\tau}\psi_{N]}\vec{t} - \frac{\kappa}{\sqrt{3}}\gamma^P\psi_{[N}\widehat{F}_{M]P} - i\kappa^2\vec{\tau}\psi_{[N}\bar{\psi}_{M]}\vec{\tau}\chi.
\end{aligned}$$

[1]Both terms appear in the constraint (3.9): The Einstein-Hilbert term explicitly and the Rarita-Schwinger term implicitly in the supercovariant derivative on V_A.

Let us now give the transformation laws:

$$
\begin{aligned}
\delta e_M^A &= -i\kappa\bar{\varepsilon}\gamma^A\psi_M && (4.13a)\\
\delta A_M &= -\frac{i\sqrt{3}}{2}\bar{\varepsilon}\psi_M && (4.13b)\\
\delta\psi_M &= \frac{1}{\kappa}\mathscr{D}(\widehat{\omega})_M\varepsilon + \frac{1}{4\sqrt{3}}\gamma_{MAB}\varepsilon\widehat{F}^{AB} - \frac{1}{\sqrt{3}}\gamma^N\varepsilon\widehat{F}_{MN} \\
&- \frac{i}{2}\vec{\tau}\varepsilon\vec{V}'_M + 2i\gamma_M\vec{\tau}\varepsilon\vec{t} + \frac{1}{2}\gamma_{MAB}\varepsilon v'^{AB} + i\kappa\vec{\tau}\psi_M\bar{\varepsilon}\vec{\tau}\chi - i\kappa\vec{\tau}\varepsilon\bar{\chi}\vec{\tau}\psi_M && (4.13c)\\
\delta\chi &= -\frac{1}{2}\varepsilon\varphi + 3i\vec{\tau}\varepsilon\vec{t} + \frac{1}{2}\gamma^A\varepsilon V_A - \frac{i}{4}\gamma^A\vec{\tau}\varepsilon\vec{V}'_A + \frac{1}{2}\gamma^{AB}\varepsilon v'_{AB} + i\kappa\chi\bar{\chi}\varepsilon && (4.13d)\\
\delta\vec{t} &= \bar{\varepsilon}\vec{\tau}\lambda' - \frac{1}{4}\bar{\varepsilon}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB} + 2\kappa\vec{t}\times(\bar{\varepsilon}\vec{\tau}\chi) && (4.13e)\\
\delta\varphi &= i\bar{\varepsilon}\gamma^M\widehat{\mathscr{D}}_M\chi - 2i\bar{\varepsilon}\lambda' + \frac{i\kappa}{2}\bar{\varepsilon}\gamma^{AB}\chi v'_{AB} + \frac{i\kappa\sqrt{3}}{4}\bar{\varepsilon}\gamma^{AB}\chi\widehat{F}_{AB} - i\kappa\bar{\varepsilon}\chi\varphi - i\kappa\bar{\varepsilon}\gamma^A\chi V_A && (4.13f)\\
\delta V_A &= i\bar{\varepsilon}\gamma_A{}^B\widehat{\mathscr{D}}_B\chi - \frac{\kappa}{2}\bar{\varepsilon}\vec{\tau}\chi\vec{V}'_A - 2i\bar{\varepsilon}\gamma_A\lambda' + i\kappa\bar{\varepsilon}\gamma^B\chi v'_{AB} + \frac{i\kappa}{2\sqrt{3}}\bar{\varepsilon}\gamma^B\chi\widehat{F}_{AB} \\
&+ 2\kappa\bar{\varepsilon}\vec{\tau}\gamma_A\chi\vec{t} + \frac{i\kappa}{2\sqrt{3}}\bar{\varepsilon}\gamma_{ABC}\chi\widehat{F}^{BC} - i\kappa\bar{\varepsilon}\gamma_A\gamma^B\chi V_B - i\kappa\bar{\varepsilon}\gamma_A\chi\varphi && (4.13g)\\
\delta v'^{AB} &= -\frac{i}{4}\bar{\varepsilon}\gamma^{[A}\gamma^{B]CD}\widehat{\mathcal{R}}_{CD} - \frac{3i}{2}\bar{\varepsilon}\gamma^{[A}\gamma_C\widehat{\mathcal{R}}^{B]C} + 2i\bar{\varepsilon}\gamma^{AB}\lambda' && (4.13h)\\
\delta\vec{V}'_A &= -4\bar{\varepsilon}\vec{\tau}\gamma_A\lambda' + \bar{\varepsilon}\vec{\tau}\gamma_A{}^{BC}\widehat{\mathcal{R}}_{BC} + 2\bar{\varepsilon}\vec{\tau}\widehat{\mathscr{D}}_A\chi + i\kappa\bar{\chi}\varepsilon\vec{V}'_A - \kappa\bar{\chi}\vec{\tau}\gamma_{ABC}\varepsilon v'^{BC} \\
&- \frac{\kappa}{2\sqrt{3}}\bar{\chi}\vec{\tau}\gamma_{ABC}\varepsilon\widehat{F}^{BC} - 4i\kappa\bar{\chi}\vec{\tau}\tilde{\tau}\gamma_A\varepsilon\tilde{t} + \frac{2\kappa}{\sqrt{3}}\bar{\chi}\vec{\tau}\gamma^B\varepsilon\widehat{F}_{AB} - \kappa(\bar{\varepsilon}\vec{\tau}\chi)\times\vec{V}'_A && (4.13i)\\
\delta\lambda' &= \varepsilon\left(\frac{1}{4}\widehat{\mathscr{D}}_A V^A + \frac{\kappa}{16}\vec{V}'_A\vec{V}'^A + \frac{i\kappa}{2}\bar{\chi}\gamma^M\widehat{\mathscr{D}}_M\chi + \frac{i\kappa}{4}\bar{\chi}\gamma^{AB}\widehat{\mathcal{R}}_{AB} - \frac{\kappa}{4}V_AV^A\right.\\
&+ \left. 15\kappa\vec{t}\vec{t} + \frac{i\kappa^2}{4}\bar{\chi}\gamma^{AB}\chi(v'_{AB} + \frac{\sqrt{3}}{2}\widehat{F}_{AB}) + \frac{\kappa}{4}\varphi^2 + \frac{\kappa}{2}v'_{AB}v'^{AB} + \frac{\kappa}{2\sqrt{3}}v'^{AB}\widehat{F}_{AB} - 2i\kappa\bar{\chi}\lambda'\right)\\
&- \frac{1}{2}\gamma_A\varepsilon(\widehat{\mathscr{D}}_B v^{AB'} - \frac{\kappa}{2}\varepsilon^{ABCDE}v'_{BC}v'_{DE} - \frac{\kappa}{\sqrt{3}}\varepsilon^{ABCDE}\widehat{F}_{BC}v'_{DE}) + \frac{3i\kappa}{2}\gamma^A\vec{\tau}\varepsilon(\vec{V}'_A\times t)\\
&+ \gamma^{AB}\vec{\tau}\varepsilon\left(-\frac{i\kappa}{16}(\vec{V}'_A\times\vec{V}'_B) - 2i\kappa v'_{AB}\vec{t} + \frac{i\sqrt{3}\kappa}{2}\widehat{F}_{AB}\vec{t} - \frac{i}{8}\widehat{\mathscr{D}}_A\vec{V}'_B - \frac{i\kappa}{4}\bar{\chi}\vec{\tau}\widehat{\mathcal{R}}_{AB}\right)\\
&+ \frac{3i}{2}\gamma^A\vec{\tau}\varepsilon\widehat{\mathscr{D}}_A\vec{t} + \frac{1}{4}\gamma^{ABC}\varepsilon\widehat{\mathscr{D}}_A v'_{BC} + i\kappa\vec{\tau}\lambda'\bar{\varepsilon}\vec{\tau}\chi - \frac{\kappa}{\sqrt{3}}\gamma^{AB}\varepsilon\widehat{F}_{CA}v'^C{}_B. && (4.13j)
\end{aligned}
$$

Since there are some typos in the literature, let us also give the on-shell transformation laws under which $\mathscr{L}_1$ is invariant:

$$\begin{aligned}
\delta e_M^A =& - i\bar{\varepsilon}\gamma^A\psi_M \\
\delta\psi_M =& (\partial_M + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB})\varepsilon + \frac{1}{4\sqrt{3}}\gamma_{MAB}\varepsilon\widehat{F}^{AB} - \frac{1}{\sqrt{3}}\gamma^N\varepsilon\widehat{F}_{MN} \\
\delta A_M =& - \frac{\sqrt{3}i}{2}\bar{\varepsilon}\psi_M.
\end{aligned}$$

Of course, the gauge algebra of this transformations closes only modulo field equations (the relevant commutator is the one on ψ_M which closes up to $\sim \gamma_M{}^{NP}\widehat{\mathcal{R}}_{NP}$ terms).

4.3 The action for the super Yang-Mills multiplet

To construct an action for the super Yang-Mills multiplet we proceed along the lines of the construction of the supergravity action as presented in section 4.2. As in the case discussed there it is possible to embed the Super Yang-Mills multiplet in a linear multiplet. The correspondence is rather complicated:

$$\begin{aligned}
\vec{Y}^{\mathcal{AB}} =& \frac{1}{2}(M^{\mathcal{A}}\vec{X}^{\mathcal{B}} + M^{\mathcal{B}}\vec{X}^{\mathcal{A}} + \frac{1}{2}\bar{\Omega}^{\mathcal{A}}\vec{\tau}\Omega^{\mathcal{B}} - 4M^{\mathcal{A}}M^{\mathcal{B}}\vec{t}), \\
\rho^{\mathcal{AB}} =& \frac{i}{4}\tau\Omega^{\mathcal{B}}\vec{X}^{\mathcal{A}} - \frac{1}{4}\gamma^A\widehat{\mathscr{D}}_A\Omega^{\mathcal{B}}M^{\mathcal{A}} - \frac{g}{4}f_{\mathcal{CD}}{}^{\mathcal{B}}\Omega^{\mathcal{C}}M^{\mathcal{D}}M^{\mathcal{A}} \\
&+ \frac{1}{8}\gamma^{AB}\Omega^{\mathcal{B}}v_{AB}M^{\mathcal{A}} - \frac{i}{2}\vec{\tau}\Omega^{\mathcal{B}}\vec{t}M^{\mathcal{A}} - \frac{1}{16}\gamma^{AB}\Omega^{\mathcal{B}}\widehat{G}^{\mathcal{A}}_{AB} \\
&- \frac{1}{8}\gamma^A\Omega^{\mathcal{B}}\widehat{\mathscr{D}}_AM^{\mathcal{A}} + \frac{1}{8\sqrt{3}}\gamma^{AB}\Omega^{\mathcal{B}}\widehat{F}_{AB}M^{\mathcal{A}} - \lambda M^{\mathcal{A}}M^{\mathcal{B}} + (\mathcal{A}\leftrightarrow\mathcal{B}) \\
N^{\mathcal{AB}} =& - \frac{1}{4}\vec{X}^{\mathcal{A}}\vec{X}^{\mathcal{B}} + \frac{1}{8}\widehat{\mathscr{D}}_A(\widehat{\mathscr{D}}^AM^{\mathcal{A}})M^{\mathcal{B}} - \frac{1}{4\sqrt{3}}\widehat{F}^{AB}\widehat{G}^{\mathcal{A}}_{AB}M^{\mathcal{B}} - \frac{1}{4}\widehat{G}^{\mathcal{A}}_{AB}v^{AB}M^{\mathcal{B}} \\
&+ \frac{1}{2\sqrt{3}}\widehat{F}_{AB}v^{AB}M^{\mathcal{A}}M^{\mathcal{B}} + \frac{1}{32}\widehat{G}^{\mathcal{A}}_{AB}\widehat{G}^{AB\mathcal{B}} + \frac{1}{16}\widehat{\mathscr{D}}_AM^{\mathcal{A}}\widehat{\mathscr{D}}^AM^{\mathcal{B}} \\
&+ \frac{1}{8}\widehat{F}_{AB}\widehat{F}^{AB}M^{\mathcal{A}}M^{\mathcal{B}} - M^{\mathcal{A}}M^{\mathcal{B}}C - \frac{i}{8}\bar{\Omega}^{\mathcal{A}}\gamma^A\widehat{\mathscr{D}}_A\Omega^{\mathcal{B}} - \frac{3}{4}\bar{\Omega}^{\mathcal{A}}\vec{\tau}\Omega^{\mathcal{B}}\vec{t} \\
&- \frac{ig}{8}f_{\mathcal{CD}}{}^{\mathcal{A}}\bar{\Omega}^{\mathcal{C}}\Omega^{\mathcal{D}}M^{\mathcal{B}} + \frac{i}{16}\bar{\Omega}^{\mathcal{A}}\gamma^{AB}\Omega^{\mathcal{B}}v_{AB} - \frac{i}{8}\bar{\Omega}^{\mathcal{A}}\gamma^{AB}\widehat{\mathcal{R}}_{AB}M^{\mathcal{B}} \\
&+ 2i\bar{\lambda}\Omega^{\mathcal{A}}M^{\mathcal{B}} + \frac{i}{16\sqrt{3}}\bar{\Omega}^{\mathcal{A}}\gamma^{AB}\Omega^{\mathcal{B}}\widehat{F}_{AB} - 2\vec{X}^{\mathcal{B}}\vec{t}M^{\mathcal{A}} + (\mathcal{A}\leftrightarrow\mathcal{B}) \\
W_A^{\mathcal{AB}} =& \frac{1}{8}\widehat{\mathscr{D}}^B\widehat{G}^{\mathcal{A}}_{AB}M^{\mathcal{B}} - \frac{1}{4\sqrt{3}}\widehat{\mathscr{D}}^B\widehat{F}_{AB}M^{\mathcal{A}}M^{\mathcal{B}} - \frac{1}{2\sqrt{3}}\widehat{F}_{AB}\widehat{\mathscr{D}}^BM^{\mathcal{A}}M^{\mathcal{B}} \\
&+ \frac{g}{8}f_{\mathcal{CD}}{}^{\mathcal{B}}\widehat{\mathscr{D}}_AM^{\mathcal{C}}M^{\mathcal{D}}M^{\mathcal{A}} - \frac{1}{2}v_{AB}\widehat{\mathscr{D}}^BM^{\mathcal{A}}M^{\mathcal{B}} - \frac{1}{64}\varepsilon_{ABCDE}\widehat{G}^{BC\mathcal{A}}\widehat{G}^{DE\mathcal{B}} \\
&- \frac{1}{4}\widehat{\mathscr{D}}^Bv_{AB}M^{\mathcal{A}}M^{\mathcal{B}} - \frac{i}{8}\bar{\Omega}^{\mathcal{A}}\gamma_{AB}\widehat{\mathscr{D}}^B\Omega^{\mathcal{B}} + \frac{ig}{8}f_{\mathcal{CD}}{}^{\mathcal{A}}\bar{\Omega}^{\mathcal{C}}\gamma_A\Omega^{\mathcal{B}}M^{\mathcal{D}} \\
&+ \frac{1}{8}\widehat{G}^{\mathcal{A}}_{AB}\widehat{\mathscr{D}}^BM^{\mathcal{B}} + \frac{ig}{8}f_{\mathcal{CD}}{}^{\mathcal{A}}\bar{\Omega}^{\mathcal{C}}\gamma_A\Omega^{\mathcal{D}}M^{\mathcal{B}} + \frac{i}{8}\bar{\Omega}^{\mathcal{A}}\gamma_A{}^{BC}\widehat{\mathcal{R}}_{BC}M^{\mathcal{B}} + (\mathcal{A}\leftrightarrow\mathcal{B}).
\end{aligned} \tag{4.14}$$

Note the similarity of the lowest component to the four dimensional result of Breitenlohner and Sohnius [6].

The fields of the linear multiplet now transform in the $(ad \times ad)_S$ representation of the gauge group. A gauge singlet can be projected out by contracting indices $\mathcal{A}$ and $\mathcal{B}$ in (4.14). We write this as a trace over generators. The lagrangian density for the super Yang-Mills multiplet is then found by plugging the embedding (4.14) in the lagrangian (4.3) for the linear multiplet:

$$\begin{aligned}\mathscr{L}_{YM} = tr\Big(&-\frac{1}{4}\widehat{G}_{AB}\widehat{G}^{AB} + \frac{1}{2}\widehat{\mathscr{D}}_A M\widehat{\mathscr{D}}^A M + 2\vec{X}\vec{X} + i\bar{\Omega}\gamma^M\widehat{\mathscr{D}}_M\Omega + 64MM\vec{t}\vec{t} \\ &- 16M\vec{X}\vec{t} + 2(\widehat{G}_{AB} - \frac{1}{\sqrt{3}}M\widehat{F}_{AB})(v^{AB} + \frac{1}{2\sqrt{3}}\widehat{F}^{AB})M + 8CM^2 \\ &+ \frac{1}{4\sqrt{3}}\varepsilon^{ABCDE}A_A G_{BC} G_{DE} - 16i\bar{\lambda}\Omega M - \frac{i}{2}\bar{\Omega}\gamma^{AB}\Omega v_{AB} - 2\bar{\Omega}\vec{\tau}\Omega\vec{t} \\ &+ g[\bar{\Omega},\Omega]M - \bar{\psi}_A\vec{\tau}\gamma^A\Omega\vec{X} + 8\bar{\Omega}\vec{\tau}\gamma^A\psi_A\vec{t}M - \frac{i}{2}\bar{\Omega}\gamma^A\gamma^B\psi_B\widehat{\mathscr{D}}_A M \\ &- \frac{i}{4}\bar{\psi}_A\gamma^A\gamma^{BC}\Omega\widehat{G}_{BC} + 4i\bar{\Omega}\gamma_A\psi_B v^{AB}M + \frac{i}{2\sqrt{3}}\bar{\psi}_A\gamma^A\gamma^{BC}\Omega\widehat{F}_{BC}M \\ &+ \frac{i}{4}\bar{\psi}_M\gamma^{MNAB}\psi_N\widehat{G}_{AB}M - 4i\bar{\psi}_A\gamma^A\lambda M^2 - 2\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t}M^2 \\ &- \frac{i}{4\sqrt{3}}\bar{\psi}_M\gamma^{MNAB}\psi_N\widehat{F}_{AB}M^2 - iv^{AB}\bar{\psi}_A\psi_B M^2 + \frac{1}{4}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\bar{\Omega}\vec{\tau}\Omega \\ &+ \frac{1}{4}\bar{\Omega}\gamma^{AB}\Omega\bar{\psi}_A\psi_B - \frac{1}{2}\bar{\psi}_N\gamma^A\psi_M\bar{\Omega}\gamma^{MN}\psi_A M + \bar{\psi}_P\gamma^P\psi_M\bar{\Omega}\gamma^{MN}\psi_N M \\ &- \bar{\Omega}\gamma^{MN}\psi_N\bar{\Omega}\psi_M + \frac{1}{2}\bar{\Omega}\gamma^{MNP}\psi_N\bar{\psi}_M\psi_P M + \frac{1}{8}\bar{\psi}_M\gamma^{MNPQ}\psi_N\bar{\psi}_P\psi_Q M^2\Big)\end{aligned} \tag{4.15}$$

Again, as in the case of the minimal multiplet, a term linear in C (the last term in the second line) appears. We could remove it as in the case of pure supergravity by introducing the nonlinear multiplet.

Let us comment on a generalization of the embedding (4.14). Over the years a lot of effort has been undertaken to construct the most general locally supersymmetric coupling of matter to supergravity. For the $\mathcal{N}=2$ supergravity in five dimensions this was performed by Gunaydin and collaborators in a series of papers [44, 43, 45, 47, 48, 46]. Their construction is a brute force construction, i.e. they write down the most general (on-shell) transformation laws and lagrangian and by requiring invariance of the action and closure of the gauge algebra they deduce a set of equations which constrain the geometry of the theory. The origin of these constraints is rather nebulous, so one may hope, that off-shell techniques could improve the understanding, as it did in four dimensional $\mathcal{N}=2$ supergravity [20]. The presumably necessary generalization of our embedding (4.14) was given recently by Kugo and Ohashi in [56]. These authors define an arbitrary function of degree two,

$$f(M) = f_0 + f_{0\mathcal{A}} M^{\mathcal{A}} + \frac{1}{2} f_{0\mathcal{A}\mathcal{B}} M^{\mathcal{A}} M^{\mathcal{B}}$$

and identify (after contracting gauge group indices) the lowest component of the linear multiplet

$$\vec{Y} = \alpha_1 f(M) \vec{t} + \alpha_2 f(M)_{\mathcal{A}} \vec{X}^{\mathcal{A}} + \alpha_3 f(M)_{\mathcal{A}\mathcal{B}} \bar{\Omega}^{\mathcal{A}} \vec{\tau} \Omega^{\mathcal{B}}$$

with constants α_i and the definitions

$$f_{\mathcal{A}} \equiv \frac{\partial f}{\partial M^{\mathcal{A}}}, \qquad f_{\mathcal{A}\mathcal{B}} \equiv \frac{\partial^2 f}{\partial M^{\mathcal{A}} \partial M^{\mathcal{B}}}.$$

The authors claim [55] that in this way they can reproduce the results of Gunaydin et al.

4.4 Gauged supergravity

In principle there exist two possible versions of gauged supergravity. First, one could gauge the complete $\mathcal{R}$-symmetry. To achieve this, an additional Super Yang-Mills multiplet with gauge group $SU(2)$ has to be introduced. The second possibility is the gauging of an $SO(2)$ subgroup of the $SU(2)_{\mathcal{R}}$, where the graviphoton, a field which belongs to the supergravity multiplet, plays the role of the gauge field. We will not discuss the first possibility in all details. The pioneering work in this direction has been done in the context of four dimensional conformal supergravity by de Wit [17].

For the second possibility, i.e. AdS supergravity, we will present detailed formulae. In this case we follow the work of de Wit et al. on four dimensional conformal supergravity, [27].

As already stated, there exist three independent off-shell versions of $\mathcal{N} = 2$ supergravity in five dimensions. All of them may be gauged. In this section the gauged variant of version I is discussed [82].

To gauge a part or the whole automorphism group, the nonlinear multiplet in its gauged form is required. One starts by replacing the C field in the lagrangian for the super Yang-Mills multiplet (4.15) by the constraint of the linear multiplet, eq. (3.9). We remind the reader that the gauge group is $SU(2)'$ and that we denote the vector multiplet which gauges this symmetry by

$$(\vec{B}_M,\ \vec{M},\ \vec{\Omega},\ \vec{\tilde{X}}).$$

The arrows on these fields denote the vector representation of the $SU(2)'$ and the twiddle on $\vec{\tilde{X}}$ denotes the **3** of the $SU(2)_{\mathcal{R}}$.

The next step is to break the gauge group $SU(2)_{\mathcal{R}} \times SU(2)'$ to a residual $SU(2)$. This is achieved by choosing the gauge (4.9)

$$\phi^i{}_\alpha = \delta^i_\alpha. \tag{4.16}$$

To stay in this gauge, a field dependent $SU(2)_{\mathcal{R}}$ transformation with parameter

$$\vec{\xi} = 2\bar{\varepsilon}\vec{\tau}\chi + \vec{\eta}$$

has to be added to the supersymmetry transformation laws of the fields. $\vec{\eta}$ is the transformation parameter for the $SU(2)'$. As a consequence, there are now two fields which transform as gauge fields of the residual $SU(2)$, namely $\vec{B}_M$ and $\vec{V}_M$. Further, there is a derivative term $\sim \partial\varepsilon$ in the supersymmetry transformation law of $\vec{V}_M$. All this can be corrected by defining a new vector field

$$\vec{B}'_M = \vec{V}_M - g\vec{B}_M + 2\bar{\psi}_M\vec{\tau}\chi. \tag{4.17}$$

$\vec{B}'_M$ is now inert under gauge transformations and has the dimension of an auxiliary field.

To gauge an $SO(2)$ subgroup of the $SU(2)_{\mathcal{R}}$ automorphism group one could now proceed by setting

$$\vec{\Omega} = \vec{\vec{X}} = 0, \qquad \vec{M} = \frac{1}{\sqrt{2}}(0,1,0)^T, \qquad \vec{B}_M = \sqrt{\frac{2}{3}}(0, A_M, 0)^T. \tag{4.18}$$

Inspection of the supersymmetry transformation laws for the super Yang-Mills multiplet shows that this choice is supersymmetric. Furthermore, one should perform the following redefinitions, which lead to canonical kinetic terms, cf. eqs. (4.7) and (4.8):

$$\lambda' = \lambda + \frac{1}{4}\gamma^{AB}\widehat{\mathcal{R}}_{AB}, \qquad v'_{AB} = v_{AB} - \frac{1}{2\sqrt{3}}\widehat{F}_{AB}. \tag{4.19}$$

However, there is a shortcut to this procedure: One starts with (4.18) and the redefinitions (4.19). After replacing the field C by the constraint (3.9), the original $SU(2)_{\mathcal{R}} \times SU(2)'$ is broken to $SO(2)$ by fixing the gauge through (4.16). To stay in this gauge, an $SU(2)_{\mathcal{R}}$ transformation with parameter

$$\vec{\xi} = 2\bar{\varepsilon}\vec{\tau}\chi + (0, \eta^2, 0)^T$$

has to be added to the supersymmetry transformation laws of the fields. As in the case of the gauged $SU(2)$ which we have discussed before, there are now two gauge fields for the $SO(2)$ present, namely V^2_M and A_M. This problem can be solved by defining

$$\vec{V}'_M = \vec{V}_M - g\sqrt{\frac{2}{3}}(0, A_M, 0)^T - 2\bar{\psi}_M\vec{\tau}\chi.$$

This field is invariant under the $SO(2)$. If one replaces

$$\vec{t} \to \vec{t} + \frac{g}{12\sqrt{2}}(0,1,0)^T$$

the lagrangian becomes

$$\mathscr{L} = \mathscr{L}_1 + \mathscr{L}_2 + \mathscr{L}_3 + \frac{g'^2}{\kappa^4} - \sqrt{3}\frac{g'}{\kappa}\bar{\chi}\tau^2\chi - \frac{\sqrt{3}g'}{4\kappa}\bar{\psi}_A\tau^2\gamma^{AB}\psi_B$$

where we have restored the gravitational coupling constant κ. $\mathscr{L}_1+\mathscr{L}_2+\mathscr{L}_3$ is the lagrangian density of the ungauged theory, cf. eq. (4.11), where the derivatives on the spinor doublets ψ_M and χ are replaced by gauge covariant derivatives,

$$\partial_M \to \partial_M - ig' A_M \tau^2. \tag{4.20}$$

Further, we have defined a new gauge coupling constant, $g' = g/\sqrt{6}$. On-shell only

$$\mathscr{L}_{gauged} = \mathscr{L}_1 + \frac{g'^2}{\kappa^4} - \frac{\sqrt{3}g'}{4\kappa}\bar{\psi}_A \tau^2 \gamma^{AB}\psi_B. \tag{4.21}$$

remains. The on-shell lagrangian of the gauged supergravity has been given for the first time in [14].

4.5 An action for hypermultiplets

If we want to embed the hypermultiplet in the linear multiplet we have to take into account that the hypermultiplet is charged under central charge transformations.

One starts with the lowest dimensional fields of two hypermultiplets $(A_i^\alpha,\ \zeta^\alpha,\ F_i^\alpha)$ and $(A_i^{\alpha'},\ \zeta^{\alpha'},\ F_i^{\alpha'})$, and combines them such that they form an isovector $\vec{Y}$:

$$\vec{Y}^{\alpha\beta} = -\frac{\sqrt{3}i}{8} A_i^\alpha A_j^{\beta'} (\vec{\tau}\varepsilon)^{ij}, \tag{4.22}$$

where ε denotes the totally antisymmetric symbol in two dimensions. $\vec{Y}$ is the lowest component of a linear multiplet. If one uses the supersymmetry transformation law of the A_i^α to compute the variation of $\vec{Y}^{\alpha\beta}$, one finds successively all component fields of the linear multiplet:

$$\begin{aligned}
(\vec{Y}^{\alpha\beta},\ \rho^{i\alpha\beta},\ N^{\alpha\beta},\ W_A^{\alpha\beta}) &= \\
= -\tfrac{\sqrt{3}i}{8}\Big(& A_i^\alpha A_j^{\beta'} (\vec{\tau}\varepsilon)^{ij}, \\
& i\varepsilon^{ij}(\zeta^\alpha A_j^{\beta'} + \zeta^{\beta'} A_j^\alpha), \\
& i(F_j^\alpha A^{\beta' j} + F_j^{\beta'} A^{j\alpha}) + \frac{1}{2}\rho^{\alpha\gamma}\bar{\zeta}_\gamma \zeta^{\beta'} + \\
& + \frac{g}{2}(M^\alpha{}_\gamma A_j^\gamma A^{j'\beta} + M^\beta{}_\gamma A_j^{\gamma'} A^{j\alpha}) - 6\vec{t} A_i^\alpha A_j^{\beta'} (\vec{\tau}\varepsilon)^{ij}, \\
& -\frac{i}{2}\varepsilon^{ij}(\widehat{\mathscr{D}}_A A_i^\alpha A_j'^\beta - A_i^\alpha \widehat{\mathscr{D}}_A A_j'^\beta) + \frac{1}{2}\rho^{\alpha\gamma}\bar{\zeta}_\gamma \gamma_A \zeta^{\beta'}\Big).
\end{aligned} \tag{4.23}$$

With the embedding (4.23) and using the action formula for the linear multiplet, eq. (4.3), it is straightforward to construct an action for the hypermultiplet. First, however, a gauge singlet has to be projected out. This is achieved by introducing a constant and a priori arbitrary matrix $\eta_{\alpha\beta}$. One then defines the gauge invariant fields

$$\vec{Y} = \vec{Y}^{\alpha\beta}\eta_{\alpha\beta}, \qquad \text{etc.} \tag{4.24}$$

To ensure the gauge invariance of this expression, the metric has to satisfy the following condition:

$$\eta_{\alpha\gamma}(t_{\mathcal{A}})^{\gamma}{}_{\beta} + (t_{\mathcal{A}})^{\gamma}{}_{\alpha}\eta_{\gamma\beta} = 0.$$

The reality properties of the fields of the linear multiplet lead to

$$\eta_{\alpha\beta} = \rho_{\gamma\alpha}\eta^{\gamma\delta}\rho_{\delta\beta}, \qquad \text{with} \qquad \eta^{\alpha\beta} \equiv \eta^{*}_{\alpha\beta}.$$

Using (3.11) one easily sees that η may be interpreted as quaternionic $r \times r$ matrix.

We are now in the position to study invariants. If one sets the primed multiplet equal the unprimed one, i.e.

$$(A_i^{\alpha'},\ \zeta^{\alpha'},\ F_i^{\alpha'}) = (A_i^{\alpha},\ \zeta^{\alpha},\ F_i^{\alpha}), \tag{4.25}$$

and uses the well known action formula for the linear multiplet, one finds a mass term for the hypermultiplet. In this case $\eta_{\alpha\beta}$ is of course symmetric, as can be seen from eq. (4.22).

In order to generate a kinetic term for the hypermultiplet, one uses the fact that the variation of a hypermultiplet under central charge transformations forms again a hypermultiplet. Thus one chooses

$$(A_i^{\alpha'}, \zeta^{\alpha'}, F_i^{\alpha'}) = \delta_z(A_i^{\alpha},\ \zeta^{\alpha},\ F_i^{\alpha}). \tag{4.26}$$

Again one forms by contracting indices with the metric $\eta_{\alpha\beta}$ gauge singlets, cf. eq. (4.24). Next, one decomposes the metric $\eta_{\alpha\beta}$ into its symmetric and antisymmetric parts. As in four dimensions, [20] one finds that the symmetric part is a total derivative and so does not contribute.

If we confine ourselves to actions which are at most quadratic in derivatives, the most general action is given by a superposition of the two possibilities (4.25) and (4.26).

First we shall work out the simple case, eq. (4.25). As mentioned before, the metric is in this case symmetric, $\eta_{\alpha\beta} = \eta_{\beta\alpha}$. For the action one finds

$$\begin{aligned}\mathscr{L} = \frac{\sqrt{3}i}{8}\big(&+2\bar{\zeta}_{\gamma}\zeta^{\beta}\rho^{\alpha\gamma} + 8\vec{t}A_i^{\alpha}A_j^{\beta}(\vec{\tau}\varepsilon)^{ij} + 4g\varepsilon^{ij}M^{\alpha}{}_{\gamma}A_i^{\gamma}A_j^{\beta} + 4\varepsilon^{ij}\bar{\psi}_{iA}\gamma^{A}\zeta^{\alpha}A_j^{\beta}\\ &-\bar{\psi}_{A}\vec{\tau}\gamma^{AB}\psi_{B}A_i^{\alpha}A_j^{\beta}(\vec{\tau}\varepsilon)^{ij} - \frac{8i}{\sqrt{3}}\varepsilon^{ij}\widehat{\mathscr{D}}_{A}A_i^{\alpha}A_j^{\beta}A^{A} + \frac{4}{\sqrt{3}}\bar{\zeta}_{\gamma}\gamma_{A}\zeta^{\beta}\rho^{\alpha\gamma}A^{A}\\ &-\frac{8i}{\sqrt{3}}\varepsilon^{ij}\bar{\psi}_{iB}\gamma^{AB}\zeta^{\alpha}A_j^{\beta}A_{A} + \frac{2}{\sqrt{3}}\bar{\psi}_{B}\vec{\tau}\gamma^{ABC}\psi_{C}A_i^{\alpha}A_j^{\beta}(\vec{\tau}\varepsilon)^{ij}A_{A} + 8i\varepsilon^{ij}F_i^{\alpha}A_j^{\beta}\big)\eta_{\alpha\beta}.\end{aligned} \tag{4.27}$$

In the second case, eq. (4.26), one finds

$$\begin{aligned}
\mathscr{L}_{Hyper} = \Big\{ & 2(1-\frac{4}{3}A_A A^A)F^i_\alpha F^\beta_i + \frac{3}{4}\bar{\psi}_A\gamma^{[A}\psi_B\bar{\zeta}_\alpha\gamma^{BC]}\psi^i_C A^\beta_i - \frac{g^2}{2}A^i_\alpha A^\gamma_i (M^2)^\beta{}_\gamma \\
& - 2g(\vec{\tau})^i{}_j A^\delta_i A^j_\alpha M^\beta{}_\delta \vec{t} + \frac{i}{2}\bar{\zeta}_\alpha\gamma^M\mathscr{D}_M\zeta^\beta + \frac{i}{4}\bar{\zeta}_\alpha\gamma^{AB}\zeta^\beta v_{AB} + \frac{i}{4\sqrt{3}}\bar{\zeta}_\alpha\gamma^{AB}\zeta^\beta\widehat{F}_{AB} \\
& + g(\vec{\tau})^i{}_j A^j_\alpha \vec{X}^\beta{}_\gamma A^\gamma_i - \frac{1}{2}\bar{\zeta}_\alpha\gamma^{MN}\psi^i_N\bar{\psi}_{Mi}\zeta^\beta + \frac{g}{2}\bar{\zeta}_\alpha\zeta^\delta M^\beta{}_\delta + \frac{1}{2}\mathscr{D}_A A^i_\alpha\mathscr{D}^A A^\beta_i \\
& + \frac{1}{2}\bar{\zeta}_\alpha\psi^i_A\bar{\psi}^A_i\zeta^\beta + 2g\bar{\zeta}_\alpha\Omega^{i\beta}{}_\gamma A^\gamma_i - 2(\vec{\tau})^i{}_j\bar{\zeta}_\alpha\gamma^A\psi^j_A\vec{t}A^\beta_i - g\bar{\zeta}_\alpha\gamma^M\psi^i_M M^\beta{}_\gamma A^\gamma_i \\
& + i\bar{\zeta}_\alpha\gamma_B\psi^i_A A^\beta_i v^{AB} - \frac{i}{2\sqrt{3}}\bar{\zeta}_\alpha\gamma^{MAB}\psi^i_M A^\beta_i\widehat{F}_{AB} + \frac{g}{4}(\vec{\tau})^i{}_j\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B M^\beta{}_\delta A^j_\alpha A^\delta_i \\
& + 4i\bar{\lambda}_i\zeta^\beta A^i_\alpha - \frac{i}{4}(\vec{\tau})^i{}_j\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_B\mathscr{D}_C A^\beta_i A^j_\alpha - \frac{1}{4}(\vec{\tau})^i{}_j\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_B\bar{\psi}_{iC}\zeta^\beta A^j_\alpha \\
& + A^i_\alpha A^\beta_i\big[-2C + \frac{1}{8}R(\widehat{\omega})_{AB}{}^{AB} - 10\vec{t}^2 - \frac{1}{2}v_{AB}v^{AB} + \frac{1}{12}\widehat{F}_{AB}\widehat{F}^{AB} + i\bar{\psi}_A\gamma^A\lambda \\
& + \frac{i}{4}\bar{\psi}_P\gamma^{PMN}\mathscr{D}_M\psi_N + \frac{1}{2}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} + \frac{i}{8\sqrt{3}}\bar{\psi}_A\gamma^{ABCD}\psi_B\widehat{F}_{CD} + \frac{i}{4}\bar{\psi}_A\psi_B v^{AB}\big] \\
& + i\bar{\zeta}_\alpha\gamma^M\gamma^N\psi^i_M\mathscr{D}_N A^\beta_i - i\bar{\zeta}_\alpha\gamma^{MN}\mathscr{D}_M\psi^i_N A^\beta_i - \frac{g}{2}(\vec{\tau})^i{}_j\bar{\psi}_M\vec{\tau}\gamma^M\Omega^\beta{}_\delta A^\delta_i A^j_\alpha\Big\} d_\beta{}^\alpha.
\end{aligned} \tag{4.28}$$

The metric $\eta_{\alpha\beta}$ is now, as discussed before, antisymmetric. The covariant derivatives which appear here are not covariant w.r.t. central charge transformations. In the action formula (4.28) we have defined a new symbol, [20]:

$$d^\alpha{}_\beta = -\eta^{\alpha\gamma}\rho_{\gamma\beta}.$$

One easily checks the following properties [20]:

$$\begin{aligned}
d_\alpha{}^\beta \equiv d^{\alpha *}_{\ \beta} = d^\beta{}_\alpha, &\qquad \text{hermiticity} \\
d_\alpha{}^\beta = \rho_{\gamma\alpha}\rho^{\delta\beta}d^\gamma{}_\delta, &\qquad \text{quaternionic} \\
t^\gamma{}_\alpha d_\gamma{}^\beta + d_\alpha{}^\gamma t_\gamma{}^\beta = 0. &\qquad \text{gauge invariant}
\end{aligned} \tag{4.29}$$

It can be shown [20], that one may choose

$$d_\alpha{}^\beta = \begin{bmatrix} 1_p & 0 \\ 0 & -1_q \end{bmatrix}$$

without violating (3.11). p and q are even valued numbers and of course have to satisfy $p+q=2r$. It follow from (3.13) and (4.29) that the generators must form a subgroup of $USp(p,q)$.

It is argued in [20], that the mass term (4.27) may be absorbed by a field redefinition.

4.6 An alternative action for supergravity

In the last section we have constructed an action for the hypermultiplet, which contains the kinetic terms for the vielbein and the gravitino, i.e. the Einstein-Hilbert and Rarita-Schwinger action. This suggests the possibility that one may find an alternative formulation of off-shell supergravity. The procedure is similar to the treatment in four dimensional conformal supergravity [27, 21] and leads to version II of the three possible off-shell supergravities.

To find the action in question, we restrict the index $\alpha = 1, 2$. Further, we choose $p = 0,\ q = 2$, that is

$$d_\alpha{}^\beta = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{4.30}$$

what fixes the gauge group to be $USp(2) = SU(2)$. If the gauge coupling constant vanishes, $g = 0$, a global $SU(2)$ remains which acts on the index α.

The lagrangian of the theory is

$$\mathscr{L}_{VII} = \mathscr{L}_{min} + \mathscr{L}_{Hyper}$$

where $\mathscr{L}_{min}$ is the lagrangian of the minimal multiplet from section 4.2, eq. (4.5), and $\mathscr{L}_{Hyper}$ is the lagrangian for the hypermultiplet, eq. (4.28). To break the local $SU(2)_\mathcal{R}$ $\mathcal{R}$-symmetry to a global $SU(2)_\mathcal{R}$, we choose the gauge

$$A_i^\alpha = e^u \delta_i^\alpha,$$

where u is a new real scalar. The lagrangian becomes

$$\begin{aligned}
\mathscr{L}_{VII} \;=\;& (1-e^{2u})(-4C - 2i\bar\lambda\gamma^M\psi_M + \bar\psi_A\vec\tau\gamma^{AB}\psi_B\vec t) - (32-20e^{2u})\vec t^2 - \frac{1}{\sqrt3}F_{AB}v^{AB} \\
&- \frac14 e^{2u}R(\widehat\omega)_{AB}{}^{AB} - \frac{i}{2}e^{2u}\bar\psi_P\gamma^{PMN}\mathscr{D}_M\psi_N - \frac{1}{6\sqrt3}\varepsilon^{ABCDE}A_AF_{BC}F_{DE} - \frac16 e^{2u}\widehat F_{AB}\widehat F^{AB} \\
&- \frac{i}{8\sqrt3}\bar\psi_A\gamma^{ABCD}\psi_B(F_{CD} + 2e^{2u}\widehat F_{CD}) + e^{2u}v_{AB}v^{AB} - \frac{i}{2}e^{2u}\bar\psi_A\psi_Bv^{AB} - \frac{i}{2}\bar\zeta\gamma^M\partial_M\zeta \\
&+ 2e^u\bar\zeta\vec\tau\gamma^M\psi_M\vec t - \frac{i}{4\sqrt3}\bar\zeta\gamma^{AB}\zeta\widehat F_{AB} - 2F_i^\alpha F_\alpha^i(1 - \frac43 A_AA^A) + \frac12\bar\zeta_\alpha\gamma^{AB}\psi_B^i\bar\psi_{Ai}\zeta^\alpha \\
&- \frac{i}{4}\bar\zeta\gamma^{AB}\zeta v_{AB} - 4ie^u\bar\lambda\zeta - ie^u\bar\zeta\gamma_B\psi_Av^{AB} - \frac34 e^u\bar\psi_A\gamma^{[A}\psi_B\bar\zeta\gamma^{BC]}\psi_C - \frac12\bar\zeta_\alpha\psi_A^i\bar\psi_i^A\zeta^\alpha \\
&+ \frac14 e^u\bar\psi_A\vec\tau\gamma^{ABC}\psi_B\bar\psi_C\vec\tau\zeta - e^{2u}\partial_Mu\partial^Mu + \frac{i}{2\sqrt3}e^u\bar\zeta\gamma^{MAB}\psi_M\widehat F_{AB} + \frac12 e^u\bar\zeta\vec\tau\psi_M\vec V^M \\
&- ie^u\bar\zeta\gamma^M\gamma^N\psi_M\partial_Nu + ie^u\bar\zeta\gamma^{MN}\mathscr{D}_M\psi_N - \frac14 e^{2u}\vec V_M\vec V^M.
\end{aligned}$$

The appearing covariant derivatives are only covariant w.r.t. local Lorentz transformations. λ plays the rôle of the lagrange multiplier for ζ. In contrary to the action (4.11), our first result, the C field is not replaced by lower dimensional fields. After elimination of the auxiliary fields by their equations of motion one ends up, as expected, with the on-shell theory given in (4.12).

In analogy to the construction of the alternative off-shell formulation of supergravity from this chapter, we can find an alternative formulation of gauged off-shell supergravity using the hypermultiplet. We start again with the lagrangian (4.28) and choose as gauge group $SU(2)'$, i.e. $\alpha, \beta = 1, 2$ and $d_\alpha{}^\beta$ as in (4.30). Our lagrangian is now

$$\mathscr{L} = \mathscr{L}_{YM} + \mathscr{L}_{Hyper}$$

with $\mathscr{L}_{YM}$ being the lagrangian density for the super Yang-Mills multiplet, eq. (4.15), with gauge group $SU(2)'$. If we choose again

$$A_i^\alpha = e^u \delta_i^\alpha,$$

the gauge group $SU(2)_\mathcal{R} \times SU(2)'$ is broken to a residual $SU(2)$. We proceed analogously to the case discussed in section 4.4. One then finds a supergravity theory where a $U(1)$ subgroup of the $SU(2)_\mathcal{R}$ symmetry is gauged by the graviphoton. We leave it to the reader to perform this little exercise.

4.7 The action for tensor multiplets

An action for the tensor multiplet is found as follows. We start with the supersymmetric coupling term between the linear multiplet and a Maxwell multiplet, eq. (4.1). If one solves the constraint (3.6) on the linear multiplet by introducing the tensor field B_{MNP} (using relation (3.7)), one easily derives from (4.1)

$$\begin{aligned}\mathscr{L} = {} & 2\vec{X}\vec{Y} - 2i\bar{\Omega}\rho - 2MN - \frac{1}{12}\varepsilon^{ABCDE}G_{AB}B_{CDE} - 16\vec{Y}\vec{t}M \\ & - i\bar{\rho}\gamma^M\psi_M M + \frac{1}{2}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{Y}M - \bar{\Omega}\vec{\tau}\gamma^M\psi_M\vec{Y}\end{aligned} \tag{4.31}$$

as the coupling term between tensor multiplet and Maxwell multiplet. The next step is to embed the tensor multiplet in a Maxwell multiplet. For the globally supersymmetric case this is rather trivial; one finds

$$(M,\ G_{AB},\ \Omega,\ \vec{X}) = \left(N,\ \ -2\partial_{[A}W_{B]},\ \ -\frac{1}{2}\gamma^A\partial_A\rho,\ \frac{1}{4}\partial_A\partial^A\vec{Y}\right) \tag{4.32}$$

and the Bianchi identity for the field strength tensor $\partial_{[A}G_{BC]} = 0$ is identically fulfilled. Using this embedding in (4.31), of course dropping the terms which contain fields which belong to the minimal multiplet, one finds an action for the tensor multiplet:

$$\mathscr{L} = -\frac{1}{2}\partial_A\vec{Y}\partial^A\vec{Y} - i\bar{\rho}\gamma^A\partial_A\rho - 2N^2 + \frac{1}{3}\partial^A B^{BCD}\partial_{[A}B_{BCD]}.$$

The embedding for the locally supersymmetric case should then be a straightforward generalization of (4.32). However, one finds that this is not possible. To find a supersymmetric embedding one has to move on to the so called *improved tensor multiplet* [22, 21]. The embedding as the resulting action is then non-polynomial in the fields. One defines

$$Y = (\vec{Y}\vec{Y})^{1/2}$$

and finds the following, rather complicated expression:

$$\begin{aligned}
M =& Y^{-1}N + \frac{1}{4}Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y} + 6Y^{-1}\vec{Y}\vec{t} \\
\widehat{G}_{AB} =& -2\widehat{\mathscr{D}}_{[A}(W_{B]}Y^{-1} + \frac{1}{4}\bar{\rho}\vec{\tau}\gamma_{B]}\rho\vec{Y}Y^{-3}) - i\bar{\rho}\widehat{\mathcal{R}}_{AB}Y^{-1} \\
& + \frac{1}{2}\widehat{\vec{G}}(V)_{AB}\vec{Y}Y^{-1} - \frac{1}{2}\vec{Y}(\widehat{\mathscr{D}}_A\vec{Y} \times \widehat{\mathscr{D}}_B\vec{Y})Y^{-3} \\
\Omega =& Y^{-1}\big(-\frac{1}{2}\gamma^A\widehat{\mathscr{D}}_A\rho + \frac{i}{2}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\vec{Y} + 2i\vec{\tau}\lambda\vec{Y} - \frac{1}{4}\gamma^{AB}\rho v_{AB} \\
& - \frac{1}{4\sqrt{3}}\gamma^{AB}\rho\widehat{F}_{AB}\big) + Y^{-3}\big(-\frac{i}{2}\gamma^A\vec{\tau}\rho W_A\vec{Y} + \frac{i}{2}\vec{\tau}\rho N\vec{Y} \\
& + \frac{1}{4}\tilde{\tau}\vec{\tau}\gamma^A\rho\widehat{\mathscr{D}}_A\tilde{Y}\vec{Y} - \frac{i}{4}\vec{\tau}\rho\bar{\rho}\vec{\tau}\rho + \frac{3i}{4}\vec{\tau}\rho\bar{\rho}\tilde{\tau}\rho\vec{Y}\tilde{Y}Y^{-2} + 3i\vec{\tau}\rho\vec{Y}(\tilde{Y}\tilde{t})\big) \\
\vec{X} =& Y^{-1}\Big(\frac{1}{4}\widehat{\mathscr{D}}^A\widehat{\mathscr{D}}_A\vec{Y} - \frac{1}{8}\widehat{R}(\widehat{\omega})_{AB}{}^{AB}\vec{Y} + 2\vec{Y}C - \frac{1}{12}\widehat{F}_{AB}\widehat{F}^{AB}\vec{Y} + \frac{1}{2}v_{AB}v^{AB}\vec{Y} \\
& + 12\vec{t}(\tilde{t}\tilde{Y}) + 10\vec{Y}\tilde{t}^2 + 2\vec{t}N - 2\bar{\lambda}\vec{\tau}\rho - \frac{1}{4}\bar{\rho}\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\Big) + Y^{-3}\Big(-\frac{1}{2}W_A^2\vec{Y} \\
& + \frac{1}{2}N^2\vec{Y} + \frac{i}{4}\bar{\rho}\tilde{\tau}\vec{\tau}\gamma^A\widehat{\mathscr{D}}_A\rho\tilde{Y} + \frac{i}{8\sqrt{3}}\bar{\rho}\gamma^{AB}\rho\widehat{F}_{AB}\vec{Y} + 18\vec{Y}(\tilde{Y}\tilde{t})^2 + 6\vec{Y}(\tilde{Y}\tilde{t})N \\
& - \frac{1}{2}\vec{Y} \times \widehat{\mathscr{D}}_A\vec{Y}W^A - \frac{1}{4}\widehat{\mathscr{D}}_A\vec{Y}\widehat{\mathscr{D}}^A\tilde{Y}\tilde{Y} + \frac{1}{8}\vec{Y}\widehat{\mathscr{D}}_A\tilde{Y}\widehat{\mathscr{D}}^A\tilde{Y} + \frac{1}{2}\vec{t}\bar{\rho}\tilde{\tau}\rho\tilde{Y} - \frac{3}{2}\bar{\rho}\vec{\tau}\rho\tilde{Y}\tilde{t} \\
& - \frac{1}{4}\bar{\rho}\vec{\tau}\rho N + \frac{1}{2}\bar{\rho}\tilde{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB}\vec{Y}\tilde{Y} + \frac{1}{4}\bar{\rho}\vec{\tau}\gamma^A\rho W_A + 2\bar{\lambda}\tilde{\tau}\rho\vec{Y}\tilde{Y} + \frac{i}{8}\bar{\rho}\gamma^{AB}\rho v_{AB}\vec{Y} \\
& + \frac{1}{8}\bar{\rho}\vec{\tau}\gamma^A\rho \times \widehat{\mathscr{D}}_A\vec{Y}\Big) + Y^{-5}\Big(\frac{3}{4}\bar{\rho}\tilde{\tau}\rho N\vec{Y}\tilde{Y} - \frac{3}{4}\bar{\rho}\tilde{\tau}\gamma^A\rho\vec{Y}\tilde{Y}W_A + \frac{9}{2}\bar{\rho}\vec{\vec{\tau}}\rho\tilde{t}\tilde{Y}\vec{Y}\vec{\vec{Y}} \\
& - \frac{3}{8}\vec{Y} \times \widehat{\mathscr{D}}_A\vec{Y}\bar{\rho}\tilde{\tau}\gamma^A\rho\tilde{Y} - \frac{3}{16}\bar{\rho}\tilde{\tau}\rho\bar{\rho}\tilde{\tau}\rho\vec{Y} - \frac{3}{8}\bar{\rho}\vec{\tau}\rho\bar{\rho}\tilde{\tau}\rho\tilde{Y}\Big) + \frac{15}{16}\bar{\rho}\tilde{\tau}\rho\bar{\rho}\vec{\vec{\tau}}\rho\vec{Y}\tilde{Y}\vec{\vec{Y}}Y^{-7}.
\end{aligned} \tag{4.33}$$

One may check, that G_{AB} is closed, i.e. $dG = 0$, but not exact, $G \neq dB$. This cohomology problem and its consequences are discussed in detail in [21] for the four dimensional superconformal case, whereto we refer the interested reader.

It is now quite simple to write down an action for the tensor multiplet. One simply plugs in the embedding (4.33) in the action formula (4.31). The action becomes after partial integration

$$
\begin{aligned}
\mathscr{L}_{tensor} = & -\frac{1}{4}YR(\widehat{\omega})^{AB}{}_{AB} + 4YC - \frac{1}{6}Y\widehat{F}_{AB}\widehat{F}^{AB} + Yv_{AB}v^{AB} + 20Y\vec{t}^2 \\
& + Y^{-1}W_AW^A - Y^{-1}N^2 - 36Y^{-1}(\vec{t}\vec{Y})^2 - \frac{1}{4}Y^{-1}\widehat{\mathscr{D}}_A\vec{Y}\widehat{\mathscr{D}}^A\vec{Y} \\
& - 4Y^{-1}\bar{\lambda}\vec{\tau}\rho\vec{Y} - \frac{1}{12}Y^{-1}\varepsilon^{MNPQR}\vec{Y}\vec{V}_M\partial_N B_{PQR} - 2iY\bar{\psi}_A\gamma^A\lambda \\
& - \frac{i}{2}Y^{-1}\bar{\rho}\gamma^A\mathscr{D}_A\rho - \frac{i}{4\sqrt{3}}Y^{-1}\bar{\rho}\gamma^{AB}\rho\widehat{F}_{AB} - 3Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y}\tilde{t}\tilde{Y} \\
& - \frac{1}{2}Y^{-3}\bar{\rho}\vec{\tau}\rho\vec{Y}N - \frac{i}{4}Y^{-1}\bar{\rho}\gamma^{AB}\rho v_{AB} + \frac{1}{4}Y^{-3}\bar{\rho}\vec{\tau}\gamma^A\rho(\vec{Y}\times\widehat{\mathscr{D}}_A\vec{Y}) \\
& + \frac{1}{2}Y^{-1}\vec{Y}\bar{\psi}_A\vec{\tau}\gamma^{AB}\mathscr{D}_B\rho + iY^{-1}\bar{\rho}\psi_AW^A - 12Y^{-1}N\vec{t}\vec{Y} \\
& + \frac{1}{2}Y^{-3}\bar{\rho}\vec{\tau}\gamma^A\rho\vec{Y}W_A - \frac{1}{2}Y^{-1}\bar{\rho}\vec{\tau}\gamma^{MN}\mathscr{D}_M\psi_N\vec{Y} \\
& - \frac{i}{2}Y\bar{\psi}_P\gamma^{PMN}\mathscr{D}_M\psi_N - Y\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} - \frac{i}{2}Y\bar{\psi}_A\psi_Bv^{AB} \\
& + \frac{1}{24}Y^{-3}\varepsilon^{MNPQR}\vec{Y}(\partial_M\vec{Y}\times\partial_N\vec{Y})B_{PQR} - \frac{i}{4\sqrt{3}}Y\bar{\psi}_A\gamma^{ABCD}\psi_B\widehat{F}_{CD} \\
& + Y^{-1}\bar{\rho}\vec{\tau}\gamma_B\psi_Av^{AB}\vec{Y} + 2iY^{-1}\bar{\rho}\gamma^A\psi_A\vec{t}\vec{Y} + 2Y^{-1}\bar{\psi}_A\vec{\tau}\gamma^A\rho(\vec{t}\times\vec{Y}) \\
& - \frac{1}{2\sqrt{3}}Y^{-1}\bar{\rho}\vec{\tau}\gamma^{ABC}\psi_A\widehat{F}_{BC}\vec{Y} - \frac{1}{4}Y^{-1}\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_B(\vec{Y}\times\widehat{\mathscr{D}}_C\vec{Y}) \\
& - \frac{1}{2}Y^{-3}\bar{\psi}_A\vec{\tau}\gamma^{AB}\rho\vec{Y}\tilde{Y}\widehat{\mathscr{D}}_B\tilde{Y} + \frac{1}{8}Y^{-3}(\bar{\rho}\vec{\tau}\rho)^2 - \frac{3}{8}Y^{-5}(\bar{\rho}\vec{\tau}\rho\vec{Y})^2 \\
& + \frac{1}{8}Y^{-3}\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\bar{\rho}\tilde{\tau}\rho\vec{Y}\tilde{Y} + \frac{i}{4}Y^{-3}\bar{\psi}_A\vec{\tau}\tilde{\tau}\gamma^A\rho\bar{\rho}\tilde{\tau}\rho\vec{Y} - \frac{i}{2}Y^{-3}\bar{\psi}_A\gamma^A\rho\bar{\rho}\vec{\tau}\rho\vec{Y} \\
& - \frac{i}{4}Y^{-3}\bar{\rho}\vec{\tau}\gamma_B\rho\bar{\rho}\gamma^{AB}\psi_A\vec{Y} - \frac{1}{8}Y^{-3}\bar{\rho}\vec{\tau}\gamma_B\rho\bar{\psi}_A\tilde{\tau}\gamma^{ABC}\psi_C\vec{Y}\tilde{Y} \\
& + \frac{1}{2}Y^{-1}\bar{\rho}\psi_B\bar{\rho}\gamma^{AB}\psi_A - \frac{i}{4}Y^{-1}\bar{\rho}\psi_B\bar{\psi}_A\vec{\tau}\gamma^{ABC}\psi_C\vec{Y}.
\end{aligned}
\tag{4.34}
$$

Please note the standard term linear in C.

4.8 A third supergravity action

We now turn our attention to the construction of the third and final version of off-shell supergravity. This theory is based on the action for the tensor multiplet from the last section. The compensator for the $SU(2)_\mathcal{R}$ $\mathcal{R}$-symmetry is the lowest component $\vec{Y}$ of the tensor multiplet. As has already been mentioned, it is not possible to break the whole $SU(2)_\mathcal{R}$ when the relevant scalar is a triplet under the $SU(2)_\mathcal{R}$; a residual $U(1)$, gauged by an auxiliary field, remains. This is the case for the tensor multiplet. Concerning this point the construction is then similar to the *new minimal formulation* of $\mathcal{N}=1$ supergravity in four dimensions [72, 71, 70] and the non-minimal variant of this theory [73]. In these cases there is a $U(1)$ $\mathcal{R}$-symmetry gauged by an auxiliary field.

We now present the explicit construction. The action (4.34) satisfies already several requirements for a consistent supergravity theory; it contains Einstein-Hilbert and Rarita-Schwinger terms. What remains to be done is the gauge fixing. We set

$$\vec{Y} = e^u(0,\ 1,\ 0)^T, \tag{4.35}$$

where we have introduced a new scalar u. This gauge leaves, as mentioned before, an $SO(2)$ subgroup of the original $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry intact. Nevertheless the above lagrangian is unphysical, since variation w.r.t. C gives the field equation

$$ee^u = 0$$

which leads to inconsistencies. This problem may be solved elegantly by taking

$$\mathscr{L} = \mathscr{L}_{tensor} + \mathscr{L}_{min},$$

with $\mathscr{L}_{min}$ from eq. (4.5), as the starting point. The terms proportional to C in the action are then

$$\mathscr{L} \sim (1 - e^u)C$$

and variation w.r.t. C gives the consistent equation of motion $u = 0$. If one eliminates all the auxiliary fields, the on-shell action (4.12) remains.

Before moving on to the gauged theory, we shall compare the field contents of the three different versions of off-shell supergravity: version *I* from section 4.2, version *II* from section 4.6 and version *III* from this section. All these variants are of course minimal in the sense that they contain $(48+48)$ components. The elementary building block is always the minimal multiplet augmented by three different matter multiplet which are used as compensators for the gauged $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry : For version *I* this compensator is taken to be the nonlinear multiplet, for version *II* we used the hypermultiplet and in the third version it is the tensor multiplet. After the gauge fixing, the field contents of the various theories are

$$\begin{aligned}
\text{Version } I &: \ (e_M^A,\ \psi_M,\ A_M \mid \vec{t},\ v_{AB},\ \vec{V}_M,\ \lambda,\ \varphi,\ \chi,\ V_A) \\
\text{Version } II &: \ (e_M^A,\ \psi_M,\ A_M \mid \vec{t},\ v_{AB},\ \vec{V}_M,\ \lambda,\ C,\ \zeta,\ u,\ F_i^\alpha) \\
\text{Version } III &: \ (e_M^A,\ \psi_M,\ A_M \mid \vec{t},\ v_{AB},\ \vec{V}_M,\ \lambda,\ C,\ \rho,\ u,\ N,\ B_{MNP})
\end{aligned}$$

We now discuss the gauged version *III*. It can be formulated rather elegant. The lagrangian is simply

$$\mathscr{L}_{VIII} = \mathscr{L}_{tensor} + \mathscr{L}_{min} - \frac{\sqrt{3}}{4} g' \mathscr{L}_{lin} \tag{4.36}$$

with $\mathscr{L}_{min}$ from equation (4.5) and $\mathscr{L}_{lin}$ given in (4.3). g' is the cosmological constant. After fixing the $SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry by choosing the gauge (4.35), a gauged $SO(2)$ remains off-shell which is gauged by the auxiliary field $\vec{V}^2_M$. Further, there is a $U(1)$ for which the gauge field is the graviphoton A_M. It is the only field which transforms off-shell under this symmetry. Stated a bit more unclear: Compared with the construction in section 4.4 the unbroken $SO(2) \subset SU(2)_{\mathcal{R}}$ is off-shell completely independent from the $U(1)$ for which the graviphoton is the gauge field. In the case of the construction in section 4.4 the graviphoton is already off-shell the gauge field for the unbroken $SO(2) \subset SU(2)_{\mathcal{R}}$.

Unfortunately, the lagrangian (4.36) turns out to be quite complicated, but since it is the starting point for our constructions in the next chapter, we present it in full detail:

$$\begin{aligned}
\mathscr{L}_{VIII} =& e^u(-\frac{1}{4}R(\widehat{\omega})_{AB}{}^{AB} + 4C - \frac{1}{6}\widehat{F}_{AB}\widehat{F}^{AB} + v_{AB}v^{AB} + 20\vec{t}\vec{t} - 36(t^2)^2 \\
& - \frac{1}{4}\partial^A u \partial_A u - \frac{1}{4}V^1_A V^{A1} - \frac{1}{4}V^3_A V^{A3} + 8\sqrt{3}g' t^2 - \frac{i}{2}\bar{\psi}_P\gamma^{PMN}\mathscr{D}_M\psi_N \\
& - 2i\bar{\psi}_A\gamma^A\lambda - \frac{\sqrt{3}g'}{4}\bar{\psi}_A\tau^2\gamma^{AB}\psi_B - \frac{i}{2}\bar{\psi}_A\psi_B v^{AB}) - 12Nt^2 - 4C \\
& - \frac{1}{\sqrt{3}}F_{AB}v^{AB} - \frac{1}{6\sqrt{3}}\varepsilon^{ABCDE}A_A F_{BC}F_{DE} - 4\bar{\lambda}\tau^2\rho - 2i\bar{\lambda}\gamma^A\psi_A \\
& - \frac{1}{12}\varepsilon^{MNPQR}(V^2_M - 2g'A_M)\partial_N B_{PQR} - 32\vec{t}\vec{t} + \sqrt{3}g'N \\
& + \frac{1}{2}\bar{\rho}\tau^2\psi_A\partial^A u - \frac{1}{2}\bar{\rho}\tau^1\psi^M V^3_M + \frac{1}{2}\bar{\rho}\tau^3\psi^M V^1_M + \frac{1}{2}\bar{\psi}_A\tau^2\gamma^{AB}\mathscr{D}_B\rho \\
& - \frac{1}{2}\bar{\rho}\tau^2\gamma^{MN}\mathscr{D}_M\psi_N + \bar{\rho}\tau^2\gamma_B\psi_A v^{AB} - \frac{1}{2\sqrt{3}}\bar{\rho}\tau^2\gamma^{ABC}\psi_A\widehat{F}_{BC} \\
& + 2i\bar{\rho}\gamma^A\psi_A t^2 - 2\bar{\psi}_A\tau^1\gamma^A\rho t^3 + 2\bar{\psi}_A\tau^3\gamma^A\rho t^1 - \frac{1}{2}\bar{\psi}_A\tau^2\gamma^{AB}\rho\partial_B u \\
& - \frac{\sqrt{3}ig'}{2}\bar{\psi}_A\gamma^A\rho - \frac{i}{4\sqrt{3}}\bar{\psi}_A\gamma^{ABCD}\psi_B(e^u\widehat{F}_{CD} + \frac{1}{2}F_{CD}) \\
& + (1-e^u)\bar{\psi}_A\vec{\tau}\gamma^{AB}\psi_B\vec{t} + e^{-u}\big(-\frac{i}{4}\bar{\rho}\gamma^{AB}\rho(v_{AB} + \frac{1}{\sqrt{3}}\widehat{F}_{AB}) \\
& - 3\bar{\rho}\tau^2\rho t^2 + W_A W^A - N^2 - \frac{i}{2}\bar{\rho}\gamma^A\mathscr{D}_A\rho + i\bar{\rho}\psi_A W^A\big) \\
& - \frac{1}{2}e^{-2u}(\bar{\rho}\tau^2\rho N - \bar{\rho}\tau^2\gamma^A\rho W_A) + \mathscr{L}_{4F}.
\end{aligned} \tag{4.37}$$

The covariant derivatives $\mathscr{D}_M$ which appear here are now only covariant under local Lorentz transformations and $SO(2)$ transformations, e.g.

$$\mathscr{D}_M\rho = \partial_M\rho + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\rho - \frac{i}{2}\tau^2\rho V^2_M.$$

$\mathscr{L}_{4F}$ contains the four fermion terms,

$$\begin{aligned}\mathscr{L}_{4F} =& e^{-u}\big(-\frac{1}{4}\bar{\rho}\vec{\tau}\psi_A\bar{\rho}\vec{\tau}\psi^A+\frac{1}{16}\bar{\rho}\gamma_{CD}\rho\bar{\psi}_A\gamma^{AB}\gamma^{CD}\psi_B\big)+\frac{1}{4}e^{-2u}\big(-\bar{\rho}\tau^1\gamma^A\rho\bar{\rho}\tau^3\psi_A\\ &+\bar{\rho}\tau^3\gamma^A\rho\bar{\rho}\tau^1\psi_A+i\bar{\psi}_A\tau^2\vec{\tau}\gamma^A\rho\bar{\rho}\vec{\tau}\rho-2i\bar{\psi}_A\gamma^A\rho\bar{\rho}\tau^2\rho-i\bar{\rho}\tau^2\gamma_B\rho\bar{\rho}\gamma^{AB}\psi_A\big)\\ &-\frac{i}{4}\bar{\rho}\psi_B\bar{\psi}_A\tau^2\gamma^{ABC}\psi_C+\frac{1}{4}\bar{\psi}_A\tau^1\gamma^{ABC}\psi_B\bar{\rho}\tau^3\psi_C-\frac{1}{4}\bar{\psi}_A\tau^3\gamma^{ABC}\psi_B\bar{\rho}\tau^1\psi_C\\ &+\frac{1}{8}e^{-3u}\big((\bar{\rho}\vec{\tau}\rho)^2-3(\bar{\rho}\tau^2\rho)^2\big).\end{aligned} \tag{4.38}$$

After elimination of the auxiliary fields one finds the on-shell lagrangian (4.21), of course with the replacement (4.20). A cohomology problem appears in the course of elimination of the auxiliary fields. Namely, the field equation for the 3-form tensor field is

$$\partial_{[M}(V^2_{N]}-2g'A_{N]})=0.$$

For vanishing first cohomology group $H^1(\mathcal{M})$ the solution to this equation is $V_M=2g'A_M$. A possible additional gauge transformation can be absorbed by redefining rotations of the gravitino. However, for nontrivial cohomology there can appear additional terms,

$$V^2_M=2g'A_M+C_M,$$

where C_M is by definition closed but not exact. This would result in modifications of the quantum theory. For our purposes it is enough to set in this case $V_M=2g'A_M$, too.

The lagrangian (4.36) is the starting point for the orbifold construction of the next chapter.

Chapter 5

Supergravity on the Orbifold $S^1/\mathbb{Z}_2$

In this chapter we turn our interest to the construction of gauged five dimensional supergravity theories on the orbifold $S^1/\mathbb{Z}_2$. Of particular interest is the supersymmetrization of the Randall-Sundrum scenario. It turns out, that besides the cosmological constant which is already in the purely bosonic case present, an additional mass term for the gravitino has to be placed at the fixpoints.

We use the developed tensor calculus to supersymmetrize the brane world scenario off-shell. Besides the fact that this turns out to be rather simple, our approach has the advantage that the appearing structures are better seen. The presented techniques are very easily extended to include additional matter in the bulk and on the branes. We substantiate this statement, by discussing generalizations of the Randall-Sundrum model: we include supersymmetric gauge fields and chiral multiplets on the boundaries.

5.1 The geometry $S^1/\mathbb{Z}_2$

In visualizing the orbifold $S^1/\mathbb{Z}_2$ one should think of an interval. We give now a simple description of its construction. The starting point is the real axis $\mathbb{R}$. We identify the point x^5 with $x^5 + 2\ell$,

$$x^5 \sim x^5 + 2\ell,$$

and derive in this way the circle S^1. This circle is obviously orientable. As one easily understands, there is a $\mathbb{Z}_2$ mapping which reverses the orientation. It acts as reflection,

$$\mathbb{Z}_2 : \qquad x^5 \to -x^5$$

and has two fixpoints at $x^5 = 0$ and $x^5 = \ell$. According to common practice we will call fixpoints also boundaries or branes.

By modding out the $\mathbb{Z}_2$ symmetry, i.e. by identifying $x^5 \sim -x^5$, one derives something one may interpret as an unoriented circle, an object which is similar to an interval.

We now wish to define fields on the orbifold $S^1/\mathbb{Z}_2$. They must satisfy in any case the periodicity condition

$$\varphi(x^5 + 2\ell) = \varphi(x^5).$$

Further, the action of the $\mathbb{Z}_2$ mapping on bosonic fields φ is given by

$$\mathbb{Z}_2: \qquad \varphi(x^5) \to \mathcal{P}\varphi(-x^5) \qquad \text{with} \qquad \mathcal{P}^2 = 1.$$

In addition, we have to define the transformation law of a generic symplectic Majorana spinor ψ under the $\mathbb{Z}_2$:

$$\mathbb{Z}_2: \qquad \psi(x^5) \to \mathcal{P} i\tau^3\gamma^{\dot{5}}\psi(-x^5) \tag{5.1}$$

It is then clear that modding out the $\mathbb{Z}_2$ generates chiral fermions on the four dimensional fixpoints. Here and in what follows we use the convention that a dot on the index "5" denotes a Lorentz index.

5.2 General analysis and strategy

In chapter 4 we have constructed among others three independent off-shell formulations of $\mathcal{N} = 2$ supergravity in five dimensions. These formulations are the starting point for the construction of supergravity on the orbifold $S^1/\mathbb{Z}_2$, which is the topic of this chapter.[1]

The method we use is a straightforward generalization of the method presented in the excellent paper of E.A. Mirabelli and M.E. Peskin [57]. Before going into details let us give an argument, which indicates why supersymmetric theories on the orbifold $S^1/\mathbb{Z}_2$ should exist at all. The argumentation is based on the supersymmetry algebra (2.3). It is invariant under the discrete $\mathbb{Z}_2$ transformation

$$Q \to i\tau^3\gamma^{\dot{5}}Q, \qquad Z \to -Z \tag{5.2}$$

together with

$$P_a \to P_a, \qquad P_{\dot{5}} \to -P_{\dot{5}}.$$

If one mods out this $\mathbb{Z}_2$ symmetry the supersymmetry algebra becomes

$$\{Q^i_{\underline{\alpha}}, Q^j_{\underline{\beta}}\} = \varepsilon^{ij}(\gamma^a C)_{\underline{\alpha\beta}} P_a,$$

where the supercharges Q now satisfy the chirality constraint which follows from (5.2):

$$Q = i\tau^3\gamma^{\dot{5}}Q.$$

As can easily be seen from an analysis of the complete super Poincaré algebra and as may be expected from the projection condition $P_{\dot{5}} = 0$, it follows that by modding out the $\mathbb{Z}_2$ the five dimensional super Poincaré algebra becomes the four dimensional $\mathcal{N} = 1$ super Poincaré algebra.

This analysis suggests that modding out the $\mathbb{Z}_2$ symmetry applied to supersymmetric field theories is a consistent procedure.

[1]Of course, when we speak of supergravity on the orbifold $S^1/\mathbb{Z}_2$ we mean $S^1/\mathbb{Z}_2 \times M_4$.

As discussed in the last section, a generic bosonic field φ of our theory transforms like

$$\varphi(x^m, x^5) \to \mathcal{P}\varphi(x^m, -x^5), \qquad \text{with} \qquad \mathcal{P}^2 = 1,$$

i.e. $\mathcal{P} = \pm 1$, under the $\mathbb{Z}_2$ symmetry. Thus, at the fixpoints the fields with $\mathcal{P} = -1$ vanish. Supported by the analysis of the super Poincaré algebra presented above, one may assume that the remaining fields for which $\mathcal{P} = +1$, form a supermultiplet at the boundary. This has been noted for the first time by Mirabelli and Peskin in the before mentioned paper (in this context see also [49])[2]. However, this work is restricted to globally supersymmetric theories.

As will be shown explicitly in the next section, the minimal multiplet from section 2.4, which is the backbone of local supersymmetry in five dimensions, reduces at the fixpoints under the $\mathbb{Z}_2$ orbifold projection to a non-minimal[3] off-shell supergravity multiplet of $\mathcal{N} = 1$ supersymmetry in four dimensions. It contains (16+16) components and has been discussed in the literature [73]. This is a main observation which will allow us to place a cosmological constant and its supersymmetrization at the fixpoints of the orbifold.

Let us now focus on the construction of the supersymmetric Randall-Sundrum scenario, i.e. we have to find the supersymmetrization of cosmological constants on the branes. The supergravity theories from chapter 4 contain besides the minimal multiplet additional matter multiplets. A priori there are then two possible ways of constructing a cosmological constant on the boundary: Either one uses the minimal multiplet or one of the matter multiplets. The minimal multiplet is not suited: the lagrangian which is relevant for this discussion is the four dimensional $\mathcal{N} = 1$ variant of the lagrangian (4.5). It will be shown in the next sections, that its bosonic part is (cf. (5.7, 5.3))

$$\mathscr{L} = 2[C - \frac{1}{2}\widehat{\mathscr{D}}_{\dot{5}} t^3 - 6((t^1)^2 + (t^2)^2)].$$

One finds that integrating out the auxiliary fields leads to inconsistencies. We conclude that the minimal multiplet cannot be used to construct a cosmological constant at the boundaries. This is similar to the bulk, where the cosmological constant comes typically from the matter multiplets. So we have to turn to the matter multiplets.

We now discuss successively the behavior under the $\mathbb{Z}_2$ of the matter multiplets as they appear in the various supergravity theories and comment on possible cosmological constants at the fixpoints.

Version *I*

Version *I* contains three multiplets, the minimal, the nonlinear and the $SU(2)'$ vector multiplet. Each of these multiplets is a priori a candidate for the construction of a cosmological constant at the boundaries. As discussed before, the minimal multiplet is not

[2] A related work is [68], where similar problems are treated on-shell.

[3] Although it may cause some confusion we will call the $(16 + 16)$ component multiplet which is the projection of the five dimensional minimal multiplet by the same name. Let us stress that from a four dimensional $\mathcal{N} = 1$ point of view this terminology is misleading. Nevertheless it is the projection of the $D = 5, \mathcal{N} = 2$ minimal multiplet.

Field	χ	φ	V_a	$V_{\dot 5}$
Parity $\mathcal{P}$	-1	-1	1	-1

Table 5.1: Parities of the nonlinear multiplet.

suited. We now turn our attention to the nonlinear multiplet. The parities of this multiplet are collected in table 5.1. In addition, ${\phi^i}_\alpha$ transforms like

$$\phi \to \tau^3 \phi \tau^3$$

and the parity of the supersymmetry variation parameter ε is $\mathcal{P} = +1$.

The parity assignments of this multiplet are unique. That follows from the term

$$\delta\chi \sim \vec{\tau}\varepsilon\vec{t} + \dots$$

in the transformation law of the field χ: if the parities of the minimal multiplet are fixed, the ones of the nonlinear multiplet follow.

The problem of the $\mathbb{Z}_2$ projection appears in the transformation law of V^a:

$$\delta V^a = i\bar{\varepsilon}\gamma^{aB}\widehat{\mathscr{D}}_B\chi + \dots$$

which reduces at the fixpoints to

$$\delta V^a = i\bar{\varepsilon}\gamma^{a\dot 5}\partial_{\dot 5}\chi + \dots.$$

If we keep this term, a new fermion $\sim \partial_{\dot 5}\chi$ is introduced to the theory (this fermion is to be identified with a superposition of higher and therefore massive Kaluza-Klein modes of the χ-field in the case of compactification on an ordinary circle). Its supersymmetry variation contains terms of the form $\sim \partial_{\dot 5}\varphi$, so that a whole tower of new fields, which are higher and higher derivatives ∂_5 of the original fields, appear. We are not interested in such a theory. The hope is then, that the term $\sim \bar{\varepsilon}\gamma^{a\dot 5}\partial_{\dot 5}\chi$ in δV^a can be swallowed by a redefinition of fields, as will be performed frequently in the following discussion. However, detailed inspection of the transformation laws shows, that such a redefinition is not possible in the case of the nonlinear multiplet. In other words: in the case of the nonlinear multiplet the higher Kaluza-Klein modes do not decouple as they do in all other cases we have considered. This reflects the fact that in four dimensions the nonlinear multiplet is a genuine $\mathcal{N} = 2$ multiplet which cannot be restricted to $\mathcal{N} = 1$.

Therefore, since the minimal multiplet and the nonlinear multiplet are no good candidates for a cosmological constant at the boundary, the remaining possibility is the $SU(2)'$ super Yang-Mills multiplet. What complicates the discussion is that there are many different $\mathbb{Z}_2$ projections possible. The only constraint is that they have to be consistent with the restrictions (4.18). The easiest projection maps this multiplet to a vector multiplet on the boundary. Since this is in some sense the canonical choice, we have collected the

Field	$M^{\mathcal{A}}$	$B_m^{\mathcal{A}}$	$B_5^{\mathcal{A}}$	$\Omega^{\mathcal{A}}$	$X^{1\mathcal{A}}$	$X^{2\mathcal{A}}$	$X^{3\mathcal{A}}$
Parity $\mathcal{P}$	-1	$+1$	-1	$+1$	-1	-1	$+1$

Table 5.2: Parities of the super Yang-Mills multiplet.

parities of the super Yang-Mills multiplet in table 5.2. In the case at hand this projection is obviously incompatible with the gauge fixing (4.18) so that a twist of the gauge group has to be included. This can be done such that the $SU(2)'$ is broken to a $U(1)$ at the boundary. A solution to the problem is then probably to write down a Fayet-Iliopoulos term for this vector multiplet. Unfortunately, we don't know that D-term. One may also try to adjust the $\mathbb{Z}_2$ projections such that a chiral multiplet survives at the fixpoints. One could then use the F-term density (5.6) for a chiral multiplet to write down the required invariant. However, our attempts to choose the $\mathbb{Z}_2$ projection such that a chiral multiplet survives at the boundaries failed.

We then conclude that version *I* is no good candidate. Since the other two versions to be discussed below are much more promising, we stop the analysis of version *I* and consider these theories.

Version *II*

The next candidate is version *II*, which contains besides the minimal multiplet a hypermultiplet, see section 4.6. there is a little complication one has to deal with in this case: The hypermultiplet is charged under central charge transformations. Since the $\mathcal{N}=1$ algebra in four dimensions admits no central charge, this transformations have to be projected out. This is a first consistency test. The parity assignments of A_i^α and F_i^α are $\mathcal{P}(A) = -\mathcal{P}(F)$, so that we see from (3.14) that the parameter z of central charge transformations is odd under parity. Thus, these transformations are projected out at the boundary. Closer inspection of transformation laws shows that this version leads presumably to consistent results. The projection of the hypermultiplet to the fixpoints is a chiral multiplet for which we know an F-term density. This F-term should lead to additional cosmological constants and mass terms for the gravitino on the boundaries. We have not studied this theory in detail.

Version *III*

The last theory we have to discuss is version *III* from section 4.8, which contains beside the minimal multiplet the tensor multiplet. The $\mathbb{Z}_2$ projection of the tensor multiplet leads to a chiral multiplet of chiral weight $w = 2$ for which we can give an F-term density. This theory is then perfectly consistent and we will focus in what follows on this model. The reason for favoring this theory is that at each stage it contains never more $(48+48)$ components whereas version *II* contains at intermediate steps up to $(72+72)$ components. A disadvantage of version *III* is its high the degree of nonlinearity; the field $\vec{Y}$ appears

Field	e_m^a	$e_m^{\dot{5}}$	e_5^a	$e_5^{\dot{5}}$	ψ_m	ψ_5	A_m	A_5	V_m^1	V_m^2
Parity $\mathcal{P}$	$+1$	-1	-1	$+1$	$+1$	-1	-1	$+1$	-1	-1

Field	V_m^3	V_5^1	V_5^2	V_5^3	v_{ab}	$v_{a\dot{5}}$	λ	C	t^1	t^2	t^3
Parity $\mathcal{P}$	$+1$	$+1$	$+1$	-1	-1	$+1$	$+1$	$+1$	$+1$	$+1$	-1

Table 5.3: Parities of the minimal multiplet.

even non-polynomial in the lagrangian.

Before specializing to one special theory, i.e. version *III*, we have to perform some preliminary work and start with the detailed analysis of the minimal multiplet. Let us repeat that version *II* seems to be as promising as a candidate as version *III*.

5.3 The minimal multiplet

The parity assignments of the minimal multiplet are dictated from the physical requirement that the vielbein e_m^a is not projected out at the boundary. The remaining parities then follow uniquely from covariance of the supersymmetry transformation laws. We have collected the resulting parities in table 5.3.

The central result of this section is easily obtained by defining:

$$b_a = v_{a\dot{5}} \tag{5.3a}$$

$$a_m = -\frac{1}{2}(V_m^3 - \frac{2}{\sqrt{3}}\widehat{F}_{m5}e^5_{\dot{5}} + 4e_m^a v_{a\dot{5}}) \tag{5.3b}$$

$$S = C - \frac{1}{2}e^5_{\dot{5}}(\partial_5 t^3 - \bar{\lambda}\tau^3\psi_5 + V_5^1 t^2 - V_5^2 t^1) \tag{5.3c}$$

at the fixpoints. Then the fields

$$\left(e_m^a,\ \psi_m,\ b_a,\ a_m,\ \lambda,\ S,\ t^1,\ t^2\right) \tag{5.4}$$

form the $\mathcal{N} = 1$ supergravity multiplet in four dimensions which appeared first in [73]. We stress that it is not minimal; it contains (16+16) components, including an auxiliary spinor λ. Sometimes this multiplet is also called the intermediate multiplet of $\mathcal{N} = 1$ off-shell supergravity in four dimensions.

The $SU(2)_\mathcal{R}$ symmetry in the bulk is broken by the orbifold condition to a chiral $U(1)$ at the fixpoints. The gauge field is a_m, an auxiliary field. The fields transform as follows under this symmetry (with parameter α):

$$\delta\psi_m = \gamma^{\dot{5}}\psi_m\alpha, \qquad \delta\lambda = \gamma^{\dot{5}}\lambda\alpha$$
$$\delta a_m = \partial_m\alpha, \qquad \delta t^1 = -2t^2\alpha, \qquad \delta t^2 = 2t^1\alpha.$$

The supersymmetry transformation laws of the multiplet (5.4), valid of course only on the boundary, are

$$
\begin{aligned}
\delta e_m^a &= -i\bar{\varepsilon}\gamma^a\psi_m \\
\delta\psi_m &= \mathscr{D}_m\varepsilon - 3\gamma^{\dot{5}}\varepsilon b_m + \gamma_m(\gamma^a\gamma^{\dot{5}}b_a + 2i\tau^1 t^1 + 2i\tau^2 t^2)\varepsilon \\
\delta b_a &= -\frac{i}{4}\bar{\varepsilon}\gamma_a{}^{bc}\gamma^{\dot{5}}\widehat{\mathcal{R}}_{bc} - 2i\bar{\varepsilon}\gamma_a\gamma^{\dot{5}}\lambda \\
\delta a_m &= -\frac{i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma_m\lambda' \\
\delta\lambda &= -\frac{1}{4}\gamma^{\dot{5}}\varepsilon\widehat{\mathscr{D}}_a b^a + \varepsilon S - \frac{i}{2}\tau^1\gamma^a\varepsilon\widehat{\mathscr{D}}_a t^1 - \frac{i}{2}\tau^2\gamma^a\varepsilon\widehat{\mathscr{D}}_a t^2 \\
\delta S &= -\frac{i}{2}\bar{\varepsilon}(\gamma^m\widehat{\mathscr{D}}_m + \gamma^a\gamma^{\dot{5}}b_a)\lambda - \bar{\varepsilon}(\tau^1 t^1 + \tau^2 t^2)(\frac{1}{2}\gamma^{ab}\widehat{\mathcal{R}}_{ab} + 8\lambda) \\
\delta t^1 &= \bar{\varepsilon}\tau^1\lambda \\
\delta t^2 &= \bar{\varepsilon}\tau^2\lambda.
\end{aligned}
$$

Please note that fields like ψ_5 or A_5 do not appear. The appearing covariant derivatives are covariant w.r.t. four dimensional Lorentz transformations and chiral gauge transformations, e.g.

$$\mathscr{D}_m\varepsilon = \partial_m\varepsilon + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\varepsilon - \gamma^{\dot{5}}\varepsilon a_m.$$

The transformation law for the $U(1)$ gauge field a_m contains the useful field λ', defined by [73]:

$$\lambda' = 12\lambda + \gamma^{ab}\widehat{\mathcal{R}}_{ab}.$$

It transforms as follows under supersymmetry:

$$\delta\lambda' = 12\varepsilon S - \frac{1}{4}\varepsilon\widehat{R}(\widehat{\omega})^{ab}{}_{ab} + 48\varepsilon((t^1)^2 + (t^2)^2) - 6\varepsilon b_a b^a - \frac{1}{2}\gamma^{ab}\gamma^{\dot{5}}\varepsilon\widehat{f}_{ab}.$$

The supercovariant derivatives which appear in the transformation laws are explicitly

$$
\begin{aligned}
\widehat{\mathscr{D}}_m t^1 &= \partial_m t^1 - \bar{\lambda}\tau^1\psi_m + 2t^2 a_m \\
\widehat{\mathscr{D}}_m t^2 &= \partial_m t^2 - \bar{\lambda}\tau^2\psi_m - 2t^1 a_m \\
\widehat{\mathscr{D}}_m b_a &= \partial_m b_a + \widehat{\omega}_{ma}{}^b b_b + \frac{i}{4}\bar{\psi}_m\gamma_a{}^{bc}\gamma^{\dot{5}}\widehat{\mathcal{R}}_{bc} + 2i\bar{\psi}_m\gamma_a\gamma^{\dot{5}}\lambda \\
\widehat{\mathscr{D}}_m\lambda &= \partial_m\lambda + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\lambda - \gamma^{\dot{5}}\lambda a_m + \frac{1}{4}\gamma^{\dot{5}}\psi_m\widehat{\mathscr{D}}_a b^a \\
&- \psi_m S + \frac{i}{2}\tau^1\gamma^a\psi_m\widehat{\mathscr{D}}_a t^1 + \frac{i}{2}\tau^2\gamma^a\psi_m\widehat{\mathscr{D}}_a t^2
\end{aligned}
$$

and the supercovariant gravitino field strength is

$$\widehat{\mathcal{R}}_{mn} = \mathscr{D}_{[m}\psi_{n]} - 3\gamma^{\dot{5}}\psi_{[n}b_{m]} + \gamma_{[m}(\gamma^a\gamma^{\dot{5}}b_{|a|} + 2i\tau^1 t^1 + 2i\tau^2 t^2)\psi_{n]}.$$

The supercovariant field strength for the $U(1)$ gauge field is

$$\widehat{f}_{mn} = \partial_m a_n - \partial_n a_m + i\bar{\psi}_{[m}\gamma^{\dot{5}}(\gamma_{n]}\gamma^{ab}\widehat{\mathcal{R}}_{ab} + 12\gamma_{n]}\lambda)$$

and the gauge algebra at the fixpoints is the projection of the gauge algebra in the five dimensional bulk, eq. (2.15). It reads

$$\begin{aligned}[\delta_Q(\varepsilon), \delta_Q(\eta)] =& \delta_{g.c.}(i\bar{\eta}\gamma^m\varepsilon) + \delta_Q(i\bar{\varepsilon}\gamma^m\eta\psi_m) + \delta_{ch}(i\bar{\varepsilon}\gamma^m\eta a_m) \\ &+ \delta_{Lt}(i\bar{\eta}\gamma^m\varepsilon\widehat{\omega}_{mab} - 2i\bar{\varepsilon}\gamma_{abc}\gamma^{\dot{5}}\eta b^c + 4\bar{\varepsilon}\tau^1\gamma_{ab}\eta t^1 + 4\bar{\varepsilon}\tau^2\gamma_{ab}\eta t^2);\end{aligned} \tag{5.5}$$

where all transformations are to be understood in the four dimensional sense and $\delta_{ch}(\alpha)$ represents a chiral transformation with parameter α.

5.4 Matter multiplets at the fixpoints

The matter multiplets which couple to the intermediate supergravity multiplet found in the foregoing section have been discussed in detail in the work of Sohnius and West [73]. Since these authors use different conventions, in particular they use Majorana spinors whereas our spinors are Weyl spinors, the transcription is quite complicated. As a consequence, we have constructed the matter multiplets from scratch.

The appearing structures are quite similar to the ones which appear in the minimal models [37, 74, 11, 13, 72, 71, 70], so we will be a little brief. More explanatory details may be found in the literature and in [79].

5.4.1 The chiral multiplet

The chiral multiplet is well known. Its field content is

$$\mathbb{A} = (A,\ B,\ \psi,\ F,\ G).$$

For a chiral multiplet with arbitrary chiral weight w we find the following transformation laws:

$$\begin{aligned}
\delta A &= \bar{\varepsilon}\tau^2\psi + wB\alpha \\
\delta B &= \bar{\varepsilon}\tau^2\gamma^{\dot{5}}\psi - wA\alpha \\
\delta\psi &= -\frac{i}{2}\gamma^a\tau^2\varepsilon\widehat{\mathscr{D}}_a A - \frac{1}{2}\varepsilon F - \frac{i}{2}\gamma^a\gamma^{\dot{5}}\tau^2\varepsilon\widehat{\mathscr{D}}_a B - \frac{1}{2}\gamma^{\dot{5}}\varepsilon G + (w-1)\gamma^{\dot{5}}\psi\alpha \\
\delta F &= i\bar{\varepsilon}\gamma^a(\widehat{\mathscr{D}}_a\psi + \gamma^{\dot{5}}\psi b_a) + 4\bar{\varepsilon}(\tau^1\psi t^1 - \tau^1\gamma^{\dot{5}}\psi t^2) + \frac{w}{2}\bar{\varepsilon}\tau^2(A + \gamma^{\dot{5}}B)\lambda' + (2-w)G\alpha \\
\delta G &= i\bar{\varepsilon}\gamma^{\dot{5}}\gamma^a(\widehat{\mathscr{D}}_a\psi + \gamma^{\dot{5}}\psi b_a) - 4\bar{\varepsilon}\gamma^{\dot{5}}(\tau^1\psi t^1 - \tau^1\gamma^{\dot{5}}\psi t^2) + \frac{w}{2}\bar{\varepsilon}\tau^2\gamma^{\dot{5}}(A + \gamma^{\dot{5}}B)\lambda' + (w-2)F\alpha.
\end{aligned}$$

We have also indicated the transformation properties under chiral rotations. The supercovariant derivatives are

$$\begin{aligned}
\widehat{\mathscr{D}}_m A &= \partial_m A - \bar{\psi}_m \tau^2 \psi - wBa_m \\
\widehat{\mathscr{D}}_m B &= \partial_m B - \bar{\psi}_m \tau^2 \gamma^{\dot{5}} \psi + wAa_m \\
\widehat{\mathscr{D}}_m \psi &= (\partial_m \psi + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\psi + (1-w)\gamma^{\dot{5}}\psi a_m) \\
&+ \frac{i}{2}\gamma^a \tau^2 \psi_m \widehat{\mathscr{D}}_a A + \frac{1}{2}\psi_m F + \frac{i}{2}\gamma^a\gamma^{\dot{5}}\tau^2\psi_m \widehat{\mathscr{D}}_a B + \frac{1}{2}\gamma^{\dot{5}}\psi_m G \\
\widehat{\mathscr{D}}_m F &= \partial_m F + (w-2)a_m G - i\bar{\psi}_m \gamma^a (\widehat{\mathscr{D}}_a \psi + \gamma^{\dot{5}}\psi b_a) \\
&- 4\bar{\psi}_m(\tau^1 \psi t^1 - \tau^1 \gamma^{\dot{5}} \psi t^2) - \frac{w}{2}\bar{\psi}_m \tau^2 (A + \gamma^{\dot{5}} B)\lambda' \\
\widehat{\mathscr{D}}_m G &= \partial_m G + (2-w)a_m F - i\bar{\psi}_m \gamma^{\dot{5}} \gamma^a (\widehat{\mathscr{D}}_a \psi + \gamma^{\dot{5}}\psi b_a) \\
&+ 4\bar{\psi}_m \gamma^{\dot{5}} (\tau^1 \psi t^1 - \tau^1 \gamma^{\dot{5}} \psi t^2) - \frac{w}{2}\bar{\psi}_m \tau^2 \gamma^{\dot{5}} (A + \gamma^{\dot{5}} B)\lambda'.
\end{aligned}$$

The chirality of the fermion ψ is $\mathcal{P} = +1$.

We can give an invariant for a chiral multiplet with weight $w = 2$. This F-term density is:

$$\mathscr{L}_{ch} \equiv [\mathbb{A}]_F = F + i\bar{\psi}_m \gamma^m \psi + \frac{1}{2}\bar{\psi}_m \tau^2 \gamma^{mn} (A + \gamma^{\dot{5}} B)\psi_n - 12t^2 A - 12t^1 B. \tag{5.6}$$

From the minimal multiplet from section 5.3 we can form a chiral multiplet with chiral weight $w = 2$:

$$(A,\ B,\ \psi,\ F,\ G) = (t^2,\ t^1,\ \lambda,\ -2S,\ \frac{1}{2}\widehat{\mathscr{D}}_a b^a).$$

To get an action for the minimal multiplet we use this expression in the F-term density (5.6) to obtain

$$\mathscr{L} = 2S + i\bar{\psi}_m \gamma^m \lambda + \frac{1}{2}\bar{\psi}_m \tau^2 \gamma^{mn} (t^2 + \gamma^{\dot{5}} t^1)\psi_n - 12((t^1)^2 + (t^2)^2). \tag{5.7}$$

5.4.2 The general multiplet

The general multiplet is a reducible multiplet containing $(16+16)$ components. It is the basic building block for the coupling of matter to supergravity in the framework of the multiplet calculus for $\mathcal{N} = 1$ supersymmetry in four dimensions. The field content of this multiplet is

$$\mathbb{C} = (C,\ \chi,\ H,\ K,\ U_a,\ \varphi,\ D),$$

where χ and φ are spinors with the chirality $\mathcal{P} = +1$ and $\mathcal{P} = -1$, respectively. In principle one can distinguish between the complex and the real general multiplet. The complex one exists for arbitrary chiral weight whereas the real general multiplet requires vanishing chiral weight. We constrain ourselves to the real version of the general multiplet.

The transformation laws are

$$\begin{aligned}
\delta C &= \bar{\varepsilon}\tau^2\chi \\
\delta\chi &= \varepsilon H + \gamma^{\dot{5}}\varepsilon K - \frac{i}{2}\gamma^a\tau^2\varepsilon\widehat{\mathscr{D}}_a C - \frac{i}{2}\tau^2\gamma^a\gamma^{\dot{5}}\varepsilon U_a - \gamma^{\dot{5}}\chi\alpha \\
\delta H &= \bar{\varepsilon}\tau^2\varphi - \frac{i}{2}\bar{\varepsilon}\gamma^a\widehat{\mathscr{D}}_a\chi - 2\bar{\varepsilon}\tau^1\chi t^1 - 2\bar{\varepsilon}\tau^2\chi t^2 + \frac{i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma^a\chi b_a + 2K\alpha \\
\delta K &= \bar{\varepsilon}\gamma^{\dot{5}}\tau^2\varphi - \frac{i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma^a\widehat{\mathscr{D}}_a\chi + 2\bar{\varepsilon}\gamma^{\dot{5}}\tau^1\chi t^1 + 2\bar{\varepsilon}\gamma^{\dot{5}}\tau^2\chi t^2 - \frac{i}{2}\bar{\varepsilon}\gamma^a\chi b_a - 2H\alpha \\
\delta U_a &= -2i\bar{\varepsilon}\gamma_a\gamma^{\dot{5}}\varphi + \bar{\varepsilon}\gamma^{\dot{5}}\tau^2\widehat{\mathscr{D}}_a\chi + 2i\bar{\varepsilon}\gamma_a\chi t^1 + 2\bar{\varepsilon}\tau^3\gamma_a\chi t^2 + \bar{\varepsilon}\tau^2\gamma^b\gamma_a\chi b_b - 3\bar{\varepsilon}\tau^2\chi b_a \\
\delta\varphi &= \varepsilon D + \frac{1}{8}\gamma^{ab}\gamma^{\dot{5}}\varepsilon\widehat{u}_{ab} + \gamma^{\dot{5}}\varphi\alpha \\
\delta D &= -\frac{i}{2}\bar{\varepsilon}\gamma^a\widehat{\mathscr{D}}_a\varphi - \frac{3i}{2}\bar{\varepsilon}\gamma^{\dot{5}}\gamma^a\varphi b_a
\end{aligned}$$

with the supercovariant derivatives

$$\begin{aligned}
\widehat{\mathscr{D}}_m C &= \partial_m C - \bar{\psi}_m\tau^2\chi \\
\widehat{\mathscr{D}}_m \chi &= \partial_m\chi + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\chi + \gamma^{\dot{5}}\chi a_m - \psi_m H - \gamma^{\dot{5}}\psi_m K + \frac{i}{2}\gamma^a\tau^2\psi_m\widehat{\mathscr{D}}_a C + \frac{i}{2}\tau^2\gamma^a\gamma^{\dot{5}}\psi_m U_a \\
\widehat{\mathscr{D}}_m \varphi &= \partial_m\varphi + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\varphi - \gamma^{\dot{5}}\varphi a_m - \psi_m D - \frac{1}{8}\gamma^{ab}\gamma^{\dot{5}}\psi_m\widehat{u}_{ab} \\
\widehat{\mathscr{D}}_m H &= \partial_m H - 2a_m K - \bar{\psi}_m\tau^2\varphi + \frac{i}{2}\bar{\psi}_m\gamma^a\widehat{\mathscr{D}}_a\chi + 2\bar{\psi}_m\tau^1\chi t^1 + 2\bar{\psi}_m\tau^2\chi t^2 - \frac{i}{2}\bar{\psi}_m\gamma^{\dot{5}}\gamma^a\chi b_a \\
\widehat{\mathscr{D}}_m K &= \partial_m K + 2a_m H - \bar{\psi}_m\gamma^{\dot{5}}\tau^2\varphi + \frac{i}{2}\bar{\psi}_m\gamma^{\dot{5}}\gamma^a\widehat{\mathscr{D}}_a\chi \\
&\quad - 2\bar{\psi}_m\gamma^{\dot{5}}\tau^1\chi t^1 - 2\bar{\psi}_m\gamma^{\dot{5}}\tau^2\chi t^2 + \frac{i}{2}\bar{\psi}_m\gamma^a\chi b_a \\
\widehat{\mathscr{D}}_m U_a &= \partial_m U_a + \widehat{\omega}_{mab}U^b + 2i\bar{\psi}_m\gamma_a\gamma^{\dot{5}}\varphi - \bar{\psi}_m\gamma^{\dot{5}}\tau^2\widehat{\mathscr{D}}_a\chi \\
&\quad - 2i\bar{\psi}_m\gamma_a\chi t^1 - 2\bar{\psi}_m\tau^3\gamma_a\chi t^2 - \bar{\psi}_m\tau^2\gamma^b\gamma_a\chi b_b + 3\bar{\psi}_m\tau^2\chi b_a
\end{aligned}$$

and the supercovariant field strength

$$\widehat{u}_{ab} = 2\widehat{\mathscr{D}}_{[a}U_{b]} + 2\bar{\chi}\tau^1\widehat{\mathcal{R}}_{ab}$$

which transforms under supersymmetry like

$$\delta\widehat{u}_{ab} = 12i\bar{\varepsilon}\gamma_{[a}\varphi b_{b]} + 4i\bar{\varepsilon}\gamma_{abc}\varphi b^c - 8\bar{\varepsilon}\tau^2\gamma_{ab}\varphi t^1 + 8\bar{\varepsilon}\tau^1\gamma_{ab}\varphi t^2 + 4i\bar{\varepsilon}\gamma_{[a}\gamma^{\dot{5}}\widehat{\mathscr{D}}_{b]}\varphi.$$

The fields of the general multiplet can be combined to form a chiral multiplet with weight $w = 2$:

$$
\begin{aligned}
A &= H + 4t^2 C, \qquad B = K + 4t^1 C, \\
\psi &= \varphi - \frac{i}{2}\tau^2\gamma^a\widehat{\mathscr{D}}_a\chi + \frac{i}{2}\gamma^{\dot5}\gamma^a\tau^2\chi b_a + 2i\tau^3\chi t^1 + 2\chi t^2 + 4\lambda C \\
F &= -2D + 4t^2 H + 4t^1 K - 2b_a U^a - 8SC + \frac{1}{2}\widehat{\mathscr{D}}_a\widehat{\mathscr{D}}^a C - \frac{1}{2}\bar\chi\tau^1\gamma^{ab}\gamma^{\dot5}\widehat{\mathcal{R}}_{ab} - 8\bar\lambda\tau^2\chi \\
G &= 12t^1 H - 12t^2 K + 2b^a\widehat{\mathscr{D}}_a C + 2\widehat{\mathscr{D}}_a b^a C + \frac{1}{2}\widehat{\mathscr{D}}_a U^a + \frac{1}{2}\bar\chi\tau^1\gamma^{ab}\widehat{\mathcal{R}}_{ab}.
\end{aligned}
$$

In the literature [73] this multiplet is denoted by $\partial\mathbb{C}$ and forms a submultiplet of the so-called primed general multiplet. If one uses this multiplet in the invariant (5.6) one finds an invariant for the general multiplet, the D-term density:

$$
\begin{aligned}
[\mathbb{C}]_D =& 2D + 8t^2 H + 8t^1 K + 2b_a U^a + 8SC + 48(t^2)^2 C + 48(t^1)^2 C \\
&+ 2\bar\psi_a\tau^1\chi b^a + 4\bar\chi\tau^3\gamma^a\psi_a t^1 + 4i\bar\chi\gamma^a\psi_a t^2 + 8\bar\lambda\tau^2\chi - i\bar\psi_a\gamma^a\varphi - 4i\bar\psi_a\gamma^a\lambda C \\
&- 2\bar\psi_a\tau^2\gamma^{ab}(t^2 C + \gamma^{\dot5}t^1 C)\psi_b - \frac{i}{4}\psi_a\gamma^{abc}\gamma^{\dot5}\psi_b U_c - \frac{3i}{2}\bar\psi_{[a}\gamma^a\psi_b\bar\psi_{c]}\tau^2\gamma^{bc}\chi].
\end{aligned}
\tag{5.8}
$$

5.4.3 The super Yang-Mills multiplet

The super Yang-Mills multiplet with abelian gauge group may be obtained from the general multiplet [73]. However, for nonabelian gauge group there are some modifications necessary. Our conventions for the gauge group are the same as in the five dimensional case, cf. section 3.1. The field content of the super Yang-Mills multiplet is

$$\mathbb{Y} = (u_m^{\mathcal{A}},\ \chi^{\mathcal{A}},\ D^{\mathcal{A}}).$$

The transformation laws one finds are:

$$
\begin{aligned}
\delta u_m^{\mathcal{A}} &= i\bar\varepsilon\gamma_m\chi^{\mathcal{A}} \\
\delta\chi^{\mathcal{A}} &= -\gamma^{\dot5}\varepsilon D^{\mathcal{A}} + \frac{1}{4}\gamma^{ab}\varepsilon\widehat{u}_{ab}^{\mathcal{A}} \\
\delta D^{\mathcal{A}} &= \frac{i}{2}\bar\varepsilon\gamma^{\dot5}\gamma^m\widehat{\mathscr{D}}_m\chi^{\mathcal{A}} - \frac{3i}{2}\bar\varepsilon\gamma^a\chi^{\mathcal{A}}b_a,
\end{aligned}
$$

where the following supercovariant quantities appear:

$$
\begin{aligned}
\widehat{u}_{mn}^{\mathcal{A}} &= \partial_m u_n^{\mathcal{A}} - \partial_n u_m^{\mathcal{A}} + g f_{\mathcal{BC}}{}^{\mathcal{A}} u_m^{\mathcal{B}} u_n^{\mathcal{C}} - i\bar\psi_m\gamma_n\chi^{\mathcal{A}} + i\bar\psi_n\gamma_m\chi^{\mathcal{A}} \\
\widehat{\mathscr{D}}_m\chi^{\mathcal{A}} &= \partial_m\chi^{\mathcal{A}} + \frac{1}{4}\widehat{\omega}_{mab}\gamma^{ab}\chi^{\mathcal{A}} - \gamma^{\dot5}\chi^{\mathcal{A}}a_m + g f_{\mathcal{BC}}{}^{\mathcal{A}} u_m^{\mathcal{B}}\chi^{\mathcal{C}} + \gamma^{\dot5}\psi_m D^{\mathcal{A}} - \frac{1}{4}\gamma^{ab}\psi_m\widehat{u}_{ab}^{\mathcal{A}} \\
\widehat{\mathscr{D}}_m D^{\mathcal{A}} &= \partial_m D^{\mathcal{A}} + g f_{\mathcal{BC}}{}^{\mathcal{A}} u_m^{\mathcal{B}} D^{\mathcal{C}} - \frac{i}{2}\bar\psi_m\gamma^{\dot5}\gamma^a\widehat{\mathscr{D}}_a\chi^{\mathcal{A}} + \frac{3i}{2}\bar\psi_m\gamma^a\chi^{\mathcal{A}}b_a,
\end{aligned}
$$

with the gauge coupling constant g.

Field	Y^1	Y^2	Y^3	ρ	N	B_{mnp}	B_{mn5}
Parities $\mathcal{P}$	$+1$	$+1$	-1	$+1$	$+1$	$+1$	-1

Table 5.4: Parities of the tensor multiplet.

The supercovariant Bianchi identity is

$$\widehat{\mathscr{D}}_{[a}\widehat{u}^{\mathcal{A}}_{bc]} = 2i\bar{\chi}^{\mathcal{A}}\gamma_{[a}\widehat{\mathcal{R}}_{bc]}$$

and in the gauge algebra (5.5) an additional term appears:

$$[\delta_Q(\varepsilon), \delta_Q(\eta)] = \cdots + \delta_{gauge}(i\bar{\varepsilon}\gamma^m\eta u^{\mathcal{A}}_m).$$

In order to write down an action for the super Yang-Mills multiplet, we use the fact that the fields of this multiplet may be combined to a chiral multiplet with chiral weight $w = 2$:

$$\begin{aligned}
A = \frac{1}{2}\bar{\chi}^{\mathcal{A}}\tau^2\chi^{\mathcal{A}}, \qquad B = \frac{1}{2}\bar{\chi}^{\mathcal{A}}\tau^2\dot{\gamma}^5\chi^{\mathcal{A}}, \qquad \psi = -\dot{\gamma}^5\chi^{\mathcal{A}}D^{\mathcal{A}} - \frac{1}{4}\gamma^{ab}\chi^{\mathcal{A}}\widehat{u}^{\mathcal{A}}_{ab}, \\
F = 2D^{\mathcal{A}}D^{\mathcal{A}} - \frac{1}{4}\widehat{u}^{\mathcal{A}}_{ab}\widehat{u}^{ab\mathcal{A}} + 6\bar{\chi}^{\mathcal{A}}\tau^1\chi^{\mathcal{A}}t^1 \\
+6\bar{\chi}^{\mathcal{A}}\tau^2\chi^{\mathcal{A}}t^2 - 3i\bar{\chi}^{\mathcal{A}}\gamma^a\dot{\gamma}^5\chi^{\mathcal{A}}b_a + i\bar{\chi}^{\mathcal{A}}\gamma^a\widehat{\mathscr{D}}_a\chi^{\mathcal{A}}, \\
G = -\frac{1}{8}\varepsilon^{abcd}\widehat{u}^{\mathcal{A}}_{ab}\widehat{u}^{\mathcal{A}}_{cd} + 6\bar{\chi}^{\mathcal{A}}\tau^2\chi^{\mathcal{A}}t^1 - 6\bar{\chi}^{\mathcal{A}}\tau^1\chi^{\mathcal{A}}t^2 + \bar{\chi}^{\mathcal{A}}\tau^3\gamma^a\widehat{\mathscr{D}}_a\chi^{\mathcal{A}}.
\end{aligned}$$

Using this expression in the F-term density (5.6) leads to a reasonable action formula for the super Yang-Mills multiplet:

$$\begin{aligned}
\mathscr{L}_{sym} = [2D^{\mathcal{A}}D^{\mathcal{A}} - \frac{1}{4}\widehat{u}^{\mathcal{A}}_{ab}\widehat{u}^{ab\mathcal{A}} - 3i\bar{\chi}^{\mathcal{A}}\gamma^a\dot{\gamma}^5\chi^{\mathcal{A}}b_a \\
+i\bar{\chi}^{\mathcal{A}}\gamma^a\widehat{\mathscr{D}}_a\chi^{\mathcal{A}} - i\bar{\psi}_m\gamma^m(\dot{\gamma}^5\chi^{\mathcal{A}}D^{\mathcal{A}} + \frac{1}{4}\gamma^{ab}\chi^{\mathcal{A}}\widehat{u}^{\mathcal{A}}_{ab}) \\
+\frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\psi_n\bar{\chi}^{\mathcal{A}}\tau^2\chi^{\mathcal{A}} + \frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\dot{\gamma}^5\psi_n\bar{\chi}^{\mathcal{A}}\tau^2\dot{\gamma}^5\chi^{\mathcal{A}}].
\end{aligned} \tag{5.9}$$

5.5 Supersymmetric Randall-Sundrum scenario

We now move to the construction of the supersymmetric Randall-Sundrum lagrangian. The non-supersymmetric configuration [66] contains five dimensional gravity with a cosmological constant in the bulk of the orbifold $S^1/\mathbb{Z}_2$. At the fixpoints there are additional cosmological constants located. The goal is to supersymmetrize this configuration.

As already indicated various times, we use version *III* of gauged off-shell supergravity in the bulk, with action given in eq. (4.36). In addition to the minimal multiplet, this theory contains the tensor multiplet, whose parities are collected in table 5.4. The fields of this multiplet form a chiral multiplet with chiral weight $w = 2$ at the fixpoints. The exact correspondence is

$$\big(A,\ B,\ \psi,\ F,\ G\big) = \big(Y^2,\ Y^1,\ \rho,\ -2N + \widehat{\mathscr{D}}_{\dot{5}} Y^3,\ +2W^{\dot{5}} + 12(t^1 Y^2 - Y^1 t^2)\big).$$

Using this and the gauge fixing condition (4.35) in (5.6), one finds

$$\mathscr{L}_{\partial M} = -N + e^u V^1_{\dot{5}} - \bar{\rho}\tau^3 \psi_{\dot{5}} + i\bar{\psi}_m \gamma^m \rho + \frac{1}{2} e^u \bar{\psi}_a \tau^2 \gamma^{ab} \psi_b - 12 e^u t^2. \tag{5.10}$$

Our action is then

$$S = \int d^5x\ e\left(\mathscr{L}_{VIII} + \Lambda_1 \delta(x^5) \mathscr{L}_{\partial M} + \Lambda_2 \delta(x^5 - \ell) \mathscr{L}_{\partial M}\right), \tag{5.11}$$

where Λ_1 and Λ_2 are constants. The two lagrangians which appear here are given explicitly in eqs. (4.36) and (5.10). Contrary to common usage, our δ-functions are tensors. By replacing

$$\delta(x^5) \to e_{\dot{5}}^{5}\ \delta(x^5),$$

one could obtain the standard delta-function. Note, that the gauge fixing condition (4.35) fixes the $SU(2)_{\mathcal{R}}$ in the bulk. In addition, the chiral $U(1)$, gauged by the auxiliary field on the boundary, is also fixed. There is still a $U(1)$ subgroup of the former $SU(2)_{\mathcal{R}}$ gauged (and there is of course the $U(1)$ for which the graviphoton is the gauge field) and unfixed. This means that after gauge fixing there is a residual $U(1)$ in the bulk and no gauge symmetry on the boundary. The $U(1)$ drops out by elimination of the auxiliary fields and is replaced by the $U(1)$ which is gauged by the graviphoton.

After the gauge fixing, the minimal multiplet on the boundary is extended to $(20+20)$ components, probably the theory in [39].

Let us return to the action (5.11). Eliminating the auxiliary fields leads to the action

$$\mathscr{L} = \mathscr{L}_{gauged} - \big(\Lambda_1 \delta(x^5) + \Lambda_2 \delta(x^5 - \ell)\big)\big(\sqrt{3} g' - \frac{1}{2} \bar{\psi}_a \tau^2 \gamma^{ab} \psi_b\big), \tag{5.12}$$

with the on-shell lagrangian of gauged supergravity $\mathscr{L}_{gauged}$ in the bulk, eq. (4.21). In addition, there are cosmological constants and four dimensional mass terms for the gravitino at the fixpoints.

Let us note that in the course of the elimination of the auxiliary fields by their field equations terms of the form $\delta(x^5)^2$ appear. At the end these terms cancel.

A comment on the appearance of the constants Λ_i in the lagrangian (5.12) is in order. One drawback of the Randall-Sundrum scenario is that it contains a fine tuning. In the original setup, the cosmological constants on the branes have to be in a very precise ratio to the cosmological constant in the bulk. A distortion δ of this relation leads to a cosmological constant in the effective four dimensional theory, whose magnitude is dictated by the magnitude of the distortion δ. Experimentally, the effective four dimensional cosmological constant Λ_{eff} is strongly constrained,

$$M_{Pl}^2 \Lambda_{eff} < 10^{-120} M_{Pl}^4.$$

Thus, δ has to be extremely small. One may have hoped that this fine tuning is dictated by supersymmetry, but the appearance of the Λ_i shows the opposite, at least for the lagrangian. A less ambitious hope is that the Λ_i are fixed by requiring the ground state, which we want to identify with the Randall-Sundrum solution, to be supersymmetric. We will discuss that topic in the next section.

To do that, we need the on-shell supersymmetry transformation laws, which are easily found:

$$\begin{aligned}
\delta e_M^A =& - i\bar{\varepsilon}\gamma^A\psi_M \\
\delta\psi_M =& \mathscr{D}_M\varepsilon + \frac{1}{4\sqrt{3}}\gamma_{MAB}\varepsilon\widehat{F}^{AB} - \frac{1}{\sqrt{3}}\gamma^N\varepsilon\widehat{F}_{MN} + \frac{ig'}{2\sqrt{3}}\gamma_M\tau^2\varepsilon \\
&+ i\Lambda_1\delta_M^5\delta(x^5)\tau^2\gamma^{\dot{5}}\varepsilon e_5^{\dot{5}} + i\Lambda_2\delta_M^5\delta(x^5-\ell)\tau^2\gamma^{\dot{5}}\varepsilon e_5^{\dot{5}} \\
\delta A_M =& - \frac{\sqrt{3}i}{2}\bar{\varepsilon}\psi_M.
\end{aligned} \tag{5.13}$$

The only modification to standard gauged supergravity is the appearance of additional terms in $\delta\psi_5$. The covariant derivatives which appear in the transformation laws are covariant under local Lorentz transformations and $U(1)$ gauge transformations, cf. eq. (4.20):

$$\mathscr{D}_M\varepsilon = \partial_M\varepsilon + \frac{1}{4}\widehat{\omega}_{MAB}\gamma^{AB}\varepsilon - ig'\tau^2\varepsilon A_M.$$

5.6 Solution of the field equations and Killing spinors

We wish to solve the field equations which follow from the lagrangian (5.12) using the Randall-Sundrum ansatz[4] [66]

$$ds^2 = e^{-2\sigma(x^5)}\eta_{mn}dx^m dx^n - (dx^5)^2. \tag{5.14}$$

η_{mn} ist of course the four dimensional Minkowski metric and $\sigma(x^5)$ a function which has to be determined. Furthermore, we would like to analyze the preserved supersymmetries of this configuration. This means that we have to solve the Killing spinor equations $\delta\psi_M = 0$.

We start with the Killing spinor equations. These are for the metric (5.14) and vanishing graviphoton and gravitino (assuming that ε depends only on x^5):

$$\tau^2\gamma^{\dot{5}}\varepsilon\sigma' = \frac{ig'}{\sqrt{3}}\varepsilon \tag{5.15a}$$

$$\partial_5\varepsilon = \left(\frac{ig'}{2\sqrt{3}} - i\Lambda_1\delta(x^5) - i\Lambda_2\delta(x^5-\ell)\right)\gamma^{\dot{5}}\tau^2\varepsilon. \tag{5.15b}$$

Here we use the shorthand notation $\sigma' = \partial_5\sigma$. To continue, we have to decompose the spinors in two component objects:

$$\varepsilon^i = \begin{pmatrix}\varepsilon_+^i \\ \varepsilon_-^i\end{pmatrix} \tag{5.16}$$

[4] We are grateful to Jan Conrad for extended discussions on the topic of this section.

and analogously for all other spinors. In the basis (A.1, A.4), the symplectic Majorana condition (A.3) becomes

$$\varepsilon_-^i = i\varepsilon^{ij}\tau^2\varepsilon_+^{j*} \tag{5.17}$$

and the $\mathbb{Z}_2$ projection (5.1) is

$$\varepsilon_+^1 \to \varepsilon_+^1, \qquad \varepsilon_+^2 \to -\varepsilon_+^2, \qquad \varepsilon_-^1 \to -\varepsilon_-^1, \qquad \varepsilon_-^2 \to \varepsilon_-^2,$$

which is obviously consistent with the symplectic Majorana condition (5.17).

Using the decomposition (5.16), the Randall-Sundrum ansatz

$$\sigma(x^5) = \frac{g'}{\sqrt{3}}|x^5| \tag{5.18}$$

solves together with [2]

$$\varepsilon_+^1 = e^{-\frac{1}{2}\sigma}\eta, \qquad \varepsilon_+^2 = -i\theta(x^5)e^{-\frac{1}{2}\sigma}\eta \tag{5.19}$$

the Killing spinor equations (5.15), if

$$\Lambda_1 = -\Lambda_2 = 2. \tag{5.20}$$

η is a constant complex two component spinor which generates the supersymmetry of the ground state. $\theta(x^5)$ is the stepfunction[5]

$$\theta(x) = \begin{cases} +1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases} .$$

We now turn to the equations of motion. The Einstein equations are

$$\begin{aligned} R^{MB}{}_{AB} - \frac{1}{2}e_A^M R^{BC}{}_{BC} &+ 2g'^2 e_A^M \\ &-2\sqrt{3}g'\delta_m^M\delta_A^a e_a^m\Lambda_1\delta(x^5) - 2\sqrt{3}g'\delta_m^M\delta_A^a e_a^m\Lambda_2\delta(x^5-\ell) = 0. \end{aligned} \tag{5.21}$$

By using the Randall-Sundrum ansatz (5.14) one finds two equations:

$$3\sigma'^2 = g'^2$$

$$\sigma'' = \frac{2}{\sqrt{3}}g'\left(\Lambda_1\delta(x^5) + \Lambda_2\delta(x^5-\ell)\right).$$

These equations are solved by (5.18), which has already been used to solve the Killing spinor equations (5.15). However, the coefficients Λ_1 and Λ_2 have to be given by

$$\Lambda_1 = -\Lambda_2 = 1$$

[5]In the course of the calculation one has to use repeatedly $\theta(x^5)^2 = 1$ everywhere, cf. the comments in [2] and [4]. A nice counterexample to this statement is easily constructed (this is motivated from [9]): Computing the derivative of this equation gives $\theta(x^5)\delta(x^5) = 0$. Multiplying this equation with $\theta(x^5)$ and using $\theta(x^5)^2 = 1$ leads to $\delta(x^5) = 0$. An equivalently statement is: $\big(\theta(x^5)\theta(x^5)\big)\delta(x^5) \neq \theta(x^5)\big(\theta(x^5)\delta(x^5)\big)$.

which is in contradiction with (5.20). Therefore we have to conclude that the Randall-Sundrum metric (5.14) represents no supersymmetric ground state of the lagrangian (5.12). However, we cannot exclude the possibility that there are other solutions than (5.19), although we have tried hard to find such a solution.

We should note that our result does not agree with the result of Altendorfer et al. [2][6].

5.7 Matter at the fixpoints

In this section we describe the coupling of gauged five dimensional supergravity to a chiral and a super Yang-Mills multiplet at the boundary. We keep the cosmological constant and its supersymmetrization, which we have discussed in the last sections, where they are, namely on the boundaries.

5.7.1 Chiral matter

First we have to find a lagrangian for the chiral multiplet. To achieve that, one embeds the chiral multiplet in a general multiplet

$$\mathbb{C} = \mathbb{A} \times \mathbb{A}.$$

As usual, this embedding is defined by its lowest component. One finds

$$\begin{gathered}
C = A^2 + B^2, \qquad \chi = 2\psi A + 2\dot{\gamma}^5\psi B, \qquad H = -AF + BG, \\
K = -AG - BF, \qquad U_a = 2A\widehat{\mathscr{D}}_a B - 2B\widehat{\mathscr{D}}_a A - 2\bar{\psi}\tau^3\gamma_a\psi, \\
\varphi = i\tau^2\gamma^a\psi\widehat{\mathscr{D}}_a A + i\tau^2\gamma^a\dot{\gamma}^5\psi\widehat{\mathscr{D}}_a B - \psi F + \dot{\gamma}^5\psi G - \frac{w}{2}\lambda'(A^2+B^2) \\
D = \frac{1}{2}\widehat{\mathscr{D}}_a A\widehat{\mathscr{D}}^a A + \frac{1}{2}\widehat{\mathscr{D}}_a B\widehat{\mathscr{D}}^a B + \frac{1}{2}F^2 + \frac{1}{2}G^2 + i\bar{\psi}\gamma^a\widehat{\mathscr{D}}_a\psi + 3\bar{\psi}\tau^3\gamma^a\psi b_a \\
+\frac{w}{2}\bar{\chi}\tau^1\lambda' + \frac{w}{2}\bar{\chi}\tau^2\lambda' + w[\frac{1}{8}\widehat{R}_{ab}{}^{ab} - 6S - 24((t^1)^2+(t^2)^2) + 3b_a b^a](A^2+B^2).
\end{gathered}$$

Using the D-term density (5.8), one can now give an action for the chiral multiplet. The bosonic part is

$$\begin{aligned}
\mathscr{L}_{kin} = &- \mathscr{D}_a B\mathscr{D}^a B - \mathscr{D}_a A\mathscr{D}^a A - 8t^2 GB - F^2 - G^2 \\
&+8t^2 FA + 8t^1 FB + 8t^1 GA - 4\mathscr{D}_a BAb^a + 4\mathscr{D}_a ABb^a \\
&+(A^2+B^2)(12wS - \frac{w}{4}R_{ab}{}^{ab} + 48(w-1)((t^1)^2+(t^2)^2) - 6wb_a b^a - 8S).
\end{aligned} \tag{5.22}$$

[6] It seems to us that in equation (13) of their work, the supersymmetry transformation law of the gravitino ψ_m^+ contains a term $\sim i\Lambda e_m{}^a\sigma_a\bar{\eta}_+$ which is missing in eq. (20). When this term is included, eq. (21) has to be changed such that eq. (22) remains unchanged and one recovers a certain limit of our work (namely the case $\Lambda_1 = 2$, eq. (5.20)).

Next, we eliminate the composite fields of the minimal multiplet on the boundary via (5.3). One finds

$$\begin{aligned}\mathscr{L}_{kin} =& -\partial_a A \partial^a A - \partial_a B \partial^a B + (wV_m^3 - \frac{2w}{\sqrt{3}} F_{m\dot{5}} + 4(w-1)v_{m\dot{5}})(A\partial^m B \\ & - B\partial^m A) - 8t^2 GB + 8t^2 FA + 8t^1 FB + 8t^1 GA - F^2 - G^2 \\ & - (A^2 + B^2)\big[-12wC + 8C + (6w-4)(\partial_{\dot{5}} t^3 + V_{\dot{5}}^1 t^2 - V_{\dot{5}}^2 t^1) \\ & + \frac{w}{4} R_{ab}{}^{ab} - 48(w-1)((t^1)^2 + (t^2)^2) - 6wv_{a\dot{5}} v^{a\dot{5}} + \frac{w^2}{4}(V_m^3 - \frac{2}{\sqrt{3}} F_{m\dot{5}} \\ & + 4v_{m\dot{5}})(V^{m3} + \frac{2}{\sqrt{3}} F^{m\dot{5}} - 4v^{m\dot{5}}) + 2wv^{m\dot{5}}(V_m^3 - \frac{2}{\sqrt{3}} F_{m\dot{5}} + 4v_{m\dot{5}})\big]\end{aligned} \quad . \tag{5.23}$$

The lagrangian we consider is the following. In the bulk, there is again gauged supergravity, with $\mathscr{L}_{VIII}$ given in (4.36). On the boundaries, there are cosmological constants positioned. The relevant lagrangian is $\mathscr{L}_{\partial M}$ whose explicit form may be found in (5.10). In addition, there is a chiral multiplet on each boundary, given by $\mathscr{L}_{kin}$ and $\mathscr{L}'_{kin}$. The total action is then

$$S = \int d^5x\, e\, \Big(\mathscr{L}_{VIII} + \Lambda_1 \delta(x^5) \mathscr{L}_{\partial M} + \tilde{\Lambda}_1 \delta(x^5) \mathscr{L}_{kin} + \Lambda_2 \delta(x^5 - \ell) \mathscr{L}_{\partial M} + \tilde{\Lambda}_2 \delta(x^5 - \ell) \mathscr{L}'_{kin} \Big) \, .$$

$\tilde{\Lambda}_1$ and $\tilde{\Lambda}_2$ are additional constants. The next step would be to eliminate the auxiliary fields. This turns out to be a remarkably complicated exercise which we have not completed.

5.7.2 Gauge fields

The other possibility is to consider a super Yang-Mills multiplet on the boundary. This is conceptually similar to the model of Hořava and Witten [51, 52]. However, there are also some differences. First, their model contains no cosmological constant, neither in the bulk nor on the boundaries (before compactification). Next, the anomaly constraints in their theory do not appear in five dimensions. And finally, the anomalous Bianchi identity which is present turns their theory to a higher derivative theory. This may be the reason for the appearance of $\delta(x^{11})^2$ terms in their theory. In our case this does not happen.

We consider an action with gauged supergravity in the bulk. On the boundaries there are cosmological constants plus super Yang-Mills multiplets. Our action is

$$S = \int d^5x\, e\, \Big(\mathscr{L}_{VIII} + \Lambda_1 \delta(x^5) \mathscr{L}_{\partial M} + \tilde{\Lambda}_1 \delta(x^5) \mathscr{L}_{sym} \Big)$$

with $\tilde{\Lambda}_1$ a constant and $\mathscr{L}_{sym}$ is given in (5.9). For notational convenience we consider only one brane at $x^5 = 0$.

In the case at hand it is easy to eliminate the auxiliary fields from the lagrangian. After replacing the composite auxiliary fields of the four dimensional minimal multiplet through the equations (5.3) and fixing the gauge via (4.35) one can eliminate the auxiliary fields.

As in the case of the pure supersymmetric Randall-Sundrum scenario, there appear $\delta(x^5)^2$ terms which cancel at the end. The result is

$$\begin{aligned}\mathscr{L} &= \mathscr{L}_{gauged} - \Lambda_1\delta(x^5)(\sqrt{3}g' - \frac{1}{2}\bar{\psi}_a\tau^2\gamma^{ab}\psi_b)\\ &+\tilde{\Lambda}_1\delta(x^5)Tr[-\frac{1}{4}\widehat{u}_{ab}\widehat{u}^{ab} - \frac{i\sqrt{3}}{4}\bar{\chi}\gamma^a\gamma^{\dot{5}}\chi\widehat{F}_{a\dot{5}} + i\bar{\chi}\gamma^m\widehat{D}_m\chi - \frac{i}{2}\bar{\chi}\gamma^{mab}\psi_m\widehat{u}_{ab}\\ &+\frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\psi_n\bar{\chi}\tau^2\chi + \frac{1}{4}\bar{\psi}_m\tau^2\gamma^{mn}\gamma^{\dot{5}}\psi_n\bar{\chi}\tau^2\gamma^{\dot{5}}\chi].\end{aligned} \tag{5.24}$$

The first line is the lagrangian of the supersymmetric Randall-Sundrum model from section 5.5, the rest is the coupling to supersymmetric Yang-Mills theory at the fixpoints.

It would be of great interest to add a Fayet-Iliopoulos term [33] at one of the fixpoints and study the transmission of supersymmetry breaking to the other wall. Since this supersymmetry breaking is mediated via the supergravity multiplet in the bulk, one should expect a strong suppression. This would be the first geometric realization of the hidden sector supergravity models we discussed in the introduction. One should note that there is a little difference between this model and the standard hidden sector supergravity models, namely the supersymmetry breaking mediator is in the brane world case a higher dimensional supergravity theory, whereas the standard hidden sector models assume the transmission of supersymmetry breaking through four dimensional supergravity.

The first step to study these models in detail would be to find a Fayet-Iliopoulos term within the context of the non-minimal $(16+16)$ theory.

We close this section with the on-shell supersymmetry transformation laws under which the lagrangian (5.24) is invariant. One finds the same transformation laws as in (5.13) with an additional term $\delta'\psi_m$ in $\delta\psi_m$:

$$\delta'\psi_m = \tilde{\Lambda}_1\delta(x^5)(\frac{i}{2}\gamma^{\dot{5}}\varepsilon\bar{\chi}\gamma_m\gamma^{\dot{5}}\chi - \frac{i}{4}\gamma_{mn}\gamma^{\dot{5}}\varepsilon\bar{\chi}\gamma^n\gamma^{\dot{5}}\chi).$$

For the fields on the boundaries one finds the usual expressions:

$$\delta u_m = i\bar{\varepsilon}\gamma_m\chi \qquad \text{and} \qquad \delta\chi = \frac{1}{4}\gamma^{ab}\varepsilon\widehat{u}_{ab}.$$

Chapter 6

Conclusions and Outlook

In this thesis we have presented the complete local tensor calculus for $\mathcal{N} = 2$ supersymmetry in five dimensions. The main building block of our construction is the minimal multiplet, which contains as propagating fields the vielbein, the gravitino and the graviphoton. In addition, there are several auxiliary fields, which complete this multiplet. Based on this multiplet the transformation laws for all matter multiplets with $(8 + 8)$ components which exist in five dimensions were found. Using these results we derived action formulas which are useful tools for the construction of supersymmetric invariants. When these formulas are applied to matter multiplets one finds actions for these multiplets. We were able to derive three different off-shell formulations of supergravity and extended these results to gauged supergravity.

Inspired by the work of Mirabelli and Peskin [57] we used the developed tensor calculus to construct actions for gauged five dimensional supergravity on the orbifold $S^1/\mathbb{Z}_2$. We were able to implement the $\mathbb{Z}_2$ mapping in a supersymmetrically covariant way on various multiplets. A major observation is that the minimal multiplet, projected to the fixpoints forms a non-minimal $\mathcal{N} = 1$ supergravity multiplet in four dimensions. With this result in hands, the supersymmetric generalization of the Randall-Sundrum scenario was found and subsequently extended to more general configurations on the boundary: first, we coupled chiral multiplets located on the boundaries. It turned out, that the auxiliary field structure is very complicated so that the elimination of these fields is a tedious task which we have not completed. Next, we placed super Yang-Mills multiplets at the fixpoints. In this case the auxiliary field structure is simple enough and the elimination of the non-dynamical degrees of freedom is feasable.

Let us now comment on possible generalizations of our work. Concerning the work on five dimensional off-shell supergravity there are two possible continuations. Our action for super Yang-Mills theory coupled to supergravity is the canonical one, resulting from the embedding of the super Yang-Mills multiplet in the linear multiplet. Kugo and Ohashi showed in a recent paper how this embedding may be generalized [56]. Using their result, it would be instructive to derive the most general coupling of super Yang-Mills theory to supergravity from an off-shell point of view and gain in that way a deeper understanding of the appearing geometric structures. This could be easily extended to include tensor and hypermultiplets.

Another extension would be to construct higher derivative invariants for the supergravity multiplet. This very tedious work would be useful for the analysis of anomalies in the *AdS / CFT* correspondence (see e.g. [1]).

There are various ways to extend the work on the brane world scenario. A very interesting topic is the discussion of supersymmetry breaking in this picture. In the spirit of Mirabelli and Peskin, one could try to construct a Fayet-Iliopoulos D-term within the non-minimal $\mathcal{N} = 1$ tensor calculus at the fixpoints. There are then two possible ways to proceed. Either one takes this D-term to be coupled to the bulk only through supergravity or, and this would be a straightforward generalization of the work of Mirabelli and Peskin, one adds super Yang-Mills multiplets to the bulk whose projection on the boundaries gives $\mathcal{N} = 1$ super Yang-Mills multiplets for which the supersymmetry breaking D-term is written down. It should be clear that these two cases should lead to different suppressions of supersymmetry breaking.

An alternative way of breaking supersymmetry would be to introduce hypermultiplets to the bulk. They constitute chiral multiplets on the boundary for which one can easily write down a superpotential. By choosing this superpotential to be of the O' Raifeartaigh type (F-term breaking) [64] this would provide an alternative mechanism of spontaneous supersymmetry breaking.

Let us repeat that both methods of supersymmetry breaking would be geometric realizations of the hidden sector supergravity scenarios.

The model of Randall and Sundrum offers fascinating possibilities for solving certain fundamental problems in theoretical physics. However, as has been alluded to above, there are still many open questions. The off-shell techniques developed in this work are particularly well suited to investigate these problems.

Appendix A

A collection of our notations and conventions

In agreement with common usage in high energy physics we use natural units, i.e.

$$\hbar = c = 1$$

and mostly we set the gravitational coupling constant $\kappa = 1$. It is an elementary exercise to restore this quantity by dimensional analysis.

Our metric is $\eta_{AB} = diag(+----)$ and $A, B = 0, 1, 2, 3, 5$ are Lorentz indices whereas $M, N = 0, 1, 2, 3, 5$ are world indices. spinor indices are denoted by underlined greek letters, for example $\underline{\alpha} = 1, \ldots 4$. These and the remaining index conventions are collected in table A.1.

The Dirac matrices satisfy the Clifford algebra

$$\{\gamma^A, \gamma^B\} = 2\eta^{AB}.$$

The reality properties of these matrices are described by

$$\gamma^{A^\dagger} = \gamma^0 \gamma^A \gamma^0.$$

Local Lorentz indices, $D = 5$	$A, B = 0, 1, 2, 3, 5$
World indices, $D = 5$	$M, N = 0, 1, 2, 3, 5$
Local Lorentz indices, $D = 4$	$a, b = 0, \ldots 3$
World indices, $D = 4$	$m, n = 0, \ldots 3$
Spinor indices	$\underline{\alpha}, \underline{\beta} = 1, \ldots 4$
$SU(2)_{\mathcal{R}}$ $\mathcal{R}$-symmetry	$i, j = 1, 2$
Adjoint of gauge group	$\mathcal{A}, \mathcal{B} = 1, \ldots \dim \mathfrak{G}$
Representation of $\mathfrak{G}$	$\alpha, \beta = 1, \ldots 2r$

Table A.1: The index conventions of this work.

A useful representation which we use in section 5.6 is [57]

$$\gamma^a = \begin{pmatrix} 0 & \sigma^a \\ \bar{\sigma}^a & 0 \end{pmatrix}, \qquad \gamma^{\dot{5}} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \tag{A.1}$$

where $\sigma^a = (\mathbb{1}, \vec{\tau})$ and $\bar{\sigma}^a = (\mathbb{1}, -\vec{\tau})$ are the standard Pauli matrices. The dot on the index "5" denotes a flat Lorentz index.

We antisymmetrize with strength one, i.e.

$$\gamma^{(n)} \equiv \gamma^{A_1 \ldots A_n} = \gamma^{[A_1}\gamma^{A_2} \ldots \gamma^{A_n]} = \frac{1}{n!} \sum_{perm's} (-1)^p \gamma^{A_1} \ldots \gamma^{A_n}$$

and the sum is over all permutations of the indices. The gamma matrices are chosen such that they satisfy

$$\gamma^{A_1 \ldots A_5} = \mathbb{1}\varepsilon^{A_1 \ldots A_5},$$

with the totally antisymmetric Levi-Civita symbol, using the conventions that (with flat indices)

$$\varepsilon^{01234} = \varepsilon_{01234} = +1.$$

Occasionally we use a Levi-Civita symbol with curved indices. It is defined by

$$\varepsilon^{MNPQR} = \varepsilon^{ABCDE} e_A^M e_B^N e_C^P e_D^Q e_E^R,$$

and satisfies

$$\partial_M \left(e\varepsilon^{MNPQR} \right) = 0.$$

it is now straightforward to derive the Poincaré duality relations for the gamma matrices:

$$\gamma^{A_1 \ldots A_p} = \frac{\alpha_p}{(5-p)!} \varepsilon^{A_1 \ldots A_p B_1 \ldots B_{5-p}} \gamma_{B_1 \ldots B_{5-p}}, \qquad \alpha_p = \begin{cases} -1 & \text{for } p = 2,3 \\ +1 & \text{for } p = 0,1,4,5 \end{cases}.$$

As explained in section 2.1, spinors carry an $SU(2)_{\mathcal{R}}$ index $i = 1,2$. The Dirac conjugate is defined by

$$\bar{\psi}_i \equiv (\psi^i)^\dagger \gamma^0. \tag{A.2}$$

One can impose a symplectic Majorana condition,

$$\bar{\psi}_i = \varepsilon_{ij} (\psi^j)^T C, \tag{A.3}$$

where the charge conjugation matrix C fulfills the following relations:

$$C = -C^T = -C^{-1}, \qquad \gamma_A^T = C\gamma_A C^{-1}.$$

In the basis given in eq. (A.1), the charge conjugation matrix is

$$C = \begin{pmatrix} -i\tau^2 & 0 \\ 0 & -i\tau^2 \end{pmatrix}. \tag{A.4}$$

Whenever possible we suppress the $SU(2)_{\mathcal{R}}$ indices. In this case the summation convention is from southwest to northeast, for example

$$\bar{\psi}\gamma^A\lambda \equiv \bar{\psi}_i\gamma^A\lambda^i, \qquad \bar{\psi}\vec{\tau}\gamma^A\lambda \equiv \bar{\psi}_i\gamma^A\lambda^j(\vec{\tau})^i{}_j, \tag{A.5}$$

with $(\vec{\tau})^i{}_j$ the standard Pauli matrices and the arrows denotes generically the vector representation of $SU(2)_{\mathcal{R}}$.

A fundamental and rather useful identity is

$$\bar{\psi}_i\gamma^{A_1}\dots\gamma^{A_n}\lambda^j = -\varepsilon_{ik}\varepsilon^{jl}\bar{\lambda}_l\gamma^{A_n}\dots\gamma^{A_1}\psi^k,$$

from which one easily deduces the symmetry properties of fermionic bilinears:

$$\bar{\lambda}\gamma^{(n)}\psi = \alpha_n\bar{\psi}\gamma^{(n)}\lambda, \qquad \alpha_n = \left\{ \begin{array}{ll} -1 & \text{for } n = 0,1,4,5 \\ +1 & \text{for } n = 2,3 \end{array} \right. .$$

In the case of an additional Pauli matrix, as for example in the second formula of (A.5), an additional sign has to be included.

We use also a different type of reality constraint. For fermions like the hyperino which satisfy a reality constraint (3.12), there are different symmetry rules for bilinears:

$$\bar{\lambda}_\alpha\gamma^{(n)}\psi^\beta = \alpha_n\rho_{\alpha\gamma}\rho^{\beta\delta}\bar{\psi}_\delta\gamma^{(n)}\lambda^\gamma, \qquad \alpha_n = \left\{ \begin{array}{ll} -1 & \text{for } n = 0,1,4,5 \\ +1 & \text{for } n = 2,3 \end{array} \right. \tag{A.6}$$

and

$$\bar{\zeta}_\alpha\gamma^{(n)}\varepsilon^i = \alpha_n\rho_{\alpha\beta}\varepsilon^{ij}\bar{\varepsilon}_j\gamma^{(n)}\zeta^\beta, \qquad \alpha_n = \left\{ \begin{array}{ll} -1 & \text{for } n = 0,1,4,5 \\ +1 & \text{for } n = 2,3 \end{array} \right. . \tag{A.7}$$

For four spinors ψ, λ, χ, ξ the Fierz identity is

$$\begin{aligned}(\bar{\psi}_iM\lambda^j)(\bar{\chi}_kN\xi^l) = & -\frac{1}{4}(\bar{\chi}_k\lambda^j)(\bar{\psi}_iMN\xi^l) \\ & -\frac{1}{4}(\bar{\chi}_k\gamma_A\lambda^j)(\bar{\psi}_iM\gamma^AN\xi^l) + \frac{1}{8}(\bar{\chi}_k\gamma_{AB}\lambda^j)(\bar{\psi}_iM\gamma^{AB}N\xi^l),\end{aligned}$$

where M and N are arbitrary (combinations of) γ-matrices.

Our definition of the spin connection reads

$$\omega_{MAB} = \Omega_{MBA} + \Omega_{ABM} - \Omega_{MAB}, \tag{A.8}$$

using the coefficients of anholonomy

$$\Omega_{MN}{}^A = \partial_{[N}e^A_{M]}.$$

The definition of supercovariance is standard: the supersymmetry variation of a supercovariant object contains no derivatives of the variation parameter. This can be achieved by addition of an appropriate combination of gravitinos, cf. eq. (2.11). We indicate supercovariance by a hat on the object in question. For example, the supercovariant graviphoton field strength is

$$\widehat{F}_{MN} = F_{MN} + \frac{\sqrt{3}i\kappa}{2}\bar{\psi}_M\psi_N \qquad \text{with} \qquad F_{MN} = \partial_M A_N - \partial_N A_M. \tag{A.9}$$

Another example is the supercovariant spin connection,

$$\widehat{\omega}_{MAB} = \omega_{MAB} + \frac{i\kappa^2}{2}(\bar{\psi}_M\gamma_B\psi_A - \bar{\psi}_A\gamma_M\psi_B + \bar{\psi}_B\gamma_A\psi_M). \tag{A.10}$$

In section 4 we use a modified spin connection, defined by

$$\widetilde{\omega}_{MAB} = \widehat{\omega}_{MAB} - \frac{i\kappa^2}{4}\bar{\psi}_C\gamma^{CD}{}_{MAB}\psi_D. \tag{A.11}$$

Our Riemann tensor is

$$R(\omega)^{AB}{}_{MN} = 2\partial_{[M}\omega_{N]}{}^{AB} + 2\omega_{[M}{}^{AC}\omega_{N]C}{}^{B}. \tag{A.12}$$

Finally, we collect the supercovariant derivatives of the fields as they appear in equation (4.13):

$$\begin{aligned}
\widehat{\mathscr{D}}_M\chi &= \mathscr{D}_M\chi + \frac{\kappa}{2}\psi_M\varphi - 3i\kappa\vec{\tau}\psi_M\vec{t} - \frac{\kappa}{2}\gamma^A\psi_M V_A \\
&+ \frac{i\kappa}{4}\gamma^N\vec{\tau}\psi_M\vec{V}'_N - \frac{\kappa}{2}\gamma^{AB}\psi_M v'_{AB} - i\kappa^2\chi\bar{\chi}\psi_M
\end{aligned} \tag{A.13a}$$

$$\begin{aligned}
\widehat{\mathscr{D}}_M V_A &= \mathscr{D}_M V_A - i\kappa\bar{\psi}_M\gamma_A{}^B\widehat{\mathscr{D}}_B\chi + \frac{\kappa^2}{2}\bar{\psi}_M\vec{\tau}\chi\vec{V}'_A + 2i\kappa\bar{\psi}_M\gamma_A\lambda' \\
&- i\kappa^2\bar{\psi}_M\gamma^B\chi v'_{AB} - \frac{i\kappa^2}{2\sqrt{3}}\bar{\psi}_M\gamma^B\chi\widehat{F}_{AB} - 2\kappa^2\bar{\psi}_M\vec{\tau}\gamma_A\chi\vec{t} \\
&- \frac{i\kappa^2}{2\sqrt{3}}\bar{\psi}_M\gamma_{ABC}\chi\widehat{F}^{BC} + i\kappa^2\bar{\psi}_M\gamma_A\gamma^B\chi V_B + i\kappa^2\bar{\psi}_M\gamma_A\chi\varphi
\end{aligned} \tag{A.13b}$$

$$\begin{aligned}
\widehat{\mathscr{D}}_M v'^{AB} &= \mathscr{D}_M v'^{AB} + \frac{i\kappa}{4}\bar{\psi}_M\gamma^{[A}\gamma^{B]CD}\widehat{\mathcal{R}}_{CD} \\
&+ \frac{3i\kappa}{2}\bar{\psi}_M\gamma^{[A}\gamma_C\widehat{\mathcal{R}}^{B]C} - 2i\kappa\bar{\psi}_M\gamma^{AB}\lambda'
\end{aligned} \tag{A.13c}$$

$$\begin{aligned}
\widehat{\mathscr{D}}_M\vec{V}'_A &= \mathscr{D}_M\vec{V}'_A + 4\kappa\bar{\psi}_M\vec{\tau}\gamma_A\lambda' - \kappa\bar{\psi}_M\vec{\tau}\gamma_A{}^{BC}\widehat{\mathcal{R}}_{BC} - 2\kappa\bar{\psi}_M\vec{\tau}\widehat{\mathscr{D}}_A\chi \\
&- i\kappa^2\bar{\chi}\psi_M\vec{V}'_A + \kappa^2\bar{\chi}\vec{\tau}\gamma_{ABC}\psi_M v'^{BC} + \frac{\kappa^2}{2\sqrt{3}}\bar{\chi}\vec{\tau}\gamma_{ABC}\psi_M\widehat{F}^{BC} \\
&+ 4i\kappa^2\bar{\chi}\vec{\tau}\tilde{\tau}\gamma_A\psi_M\tilde{t} - \frac{2\kappa^2}{\sqrt{3}}\bar{\chi}\vec{\tau}\gamma^B\psi_M\widehat{F}_{AB} + \kappa^2\bar{\psi}_M\vec{\tau}\chi\times\vec{V}'_A
\end{aligned} \tag{A.13d}$$

$$\widehat{\mathscr{D}}_M\vec{t} = \partial_M\vec{t} - \kappa\bar{\psi}_M\vec{\tau}\lambda' + \frac{\kappa}{4}\bar{\psi}_M\vec{\tau}\gamma^{AB}\widehat{\mathcal{R}}_{AB} - 2\kappa^2\vec{t}\times(\bar{\psi}_M\vec{\tau}\chi). \tag{A.13e}$$

We remind the reader that the derivatives appearing here are only covariant w.r.t. local Lorentz transformations.

The conventions which we use in four dimensions always follow from the orbifold projection of the ones chosen in five dimensions. However, some quantities are not uniquely defined in this way.

One is the four dimensional Levi-Civita symbol which we take to be

$$\varepsilon^{abcd} = \varepsilon^{abcd\dot{5}}.$$

Appendix B

The $N = 2$ superconformal algebra in five dimensions

In this appendix we present the complete $\mathcal{N} = 2$ superconformal algebra in five dimensions. The generators of this algebra are collected in table B.1.

The bosonic part of the superconformal algebra, i.e. the conformal algebra is

$$[M_{AB}, P_C] = iP_{[A}\eta_{B]C}, \qquad [M_{AB}, M^{CD}] = -2i\delta_{[A}^{[C} M_{B]}{}^{D]}$$

$$[M_{AB}, K_C] = iK_{[A}\eta_{B]C}, \qquad [D, P_A] = iP_A$$

$$[D, K_A] = -iK_A, \qquad [P_A, K_B] = -2i(\eta_{AB}D + 2M_{AB}).$$

This algebra closes independently of the dimension of spacetime. The first two relations together with the following relations form the super Poincaré algebra[1]:

$$\{Q^i_{\underline{\alpha}}, Q^j_{\underline{\beta}}\} = \varepsilon^{ij}(\gamma^A C)_{\underline{\alpha\beta}} P_A,$$

$$[Q^i_{\underline{\alpha}}, M^{AB}] = \frac{i}{4}(\gamma^{AB} Q^i)_{\underline{\alpha}}, \qquad [Q^i_{\underline{\alpha}}, P_A] = 0.$$

The algebra of automorphisms, i.e. of $\mathcal{R}$-symmetry is

$$[T^{\underline{I}}, T^{\underline{J}}] = i\varepsilon^{\underline{IJK}} T^{\underline{K}}, \qquad [\vec{T}, Q^i_{\underline{\alpha}}] = -\frac{1}{2}(\vec{\tau})^i{}_j Q^j_{\underline{\alpha}}.$$

By adding special supersymmetry one arrives at the superconformal algebra:

$$\{S^i_{\underline{\alpha}}, S^j_{\underline{\beta}}\} = \varepsilon^{ij}(\gamma^A C)_{\underline{\alpha\beta}} K_A$$

$$[S^i_{\underline{\alpha}}, M^{AB}] = \frac{i}{4}(\gamma^{AB} S^i)_{\underline{\alpha}}, \qquad [\vec{T}, S^i_{\underline{\alpha}}] = -\frac{1}{2}(\vec{\tau})^i{}_j S^j_{\underline{\alpha}}$$

$$[P_A, S^i_{\underline{\alpha}}] = i(\gamma_A Q^i)_{\underline{\alpha}}, \qquad [K_A, Q^i_{\underline{\alpha}}] = -i(\gamma_A S^i)_{\underline{\alpha}}$$

[1] In the framework of Poincaré Supersymmetry there is a central charge possible, cf. eq.(2.3).

Type	Generator	$SU(2)_{\mathcal{R}}$	Properties
Translations	P_A	$\mathbf{1}$	
Lorentz Transformations	M_{AB}	$\mathbf{1}$	$M_{AB} = -M_{BA}$
Dilatations	D	$\mathbf{1}$	$D^* = D$
Special Conformal Transformations	K_A	$\mathbf{1}$	
Supersymmetry	$Q^i_{\underline{\alpha}}$	$\mathbf{2}$	$\bar{Q}_i = \varepsilon_{ij}(Q^j)^T C$
Special Supersymmetry	$S^i_{\underline{\alpha}}$	$\mathbf{2}$	$\bar{S}_i = \varepsilon_{ij}(S^j)^T C$
$\mathcal{R}$-Symmetry	$T^{\underline{I}}$	$\mathbf{3}$	$T^{\underline{I}\dagger} = T^{\underline{I}}$

Table B.1: The generators of the $\mathcal{N} = 2$ superconformal group in $D = 5$.

$$[D, Q^i_{\underline{\alpha}}] = \frac{i}{2} Q^i_{\underline{\alpha}}, \qquad [D, S^i_{\underline{\alpha}}] = -\frac{i}{2} S^i_{\underline{\alpha}}$$

$$\{Q^i_{\underline{\alpha}}, S^j_{\underline{\beta}}\} = -\varepsilon^{ij} C_{\underline{\alpha\beta}} D - \varepsilon^{ij} (\gamma^{AB} C)_{\underline{\alpha\beta}} M_{AB} - 3i C_{\underline{\alpha\beta}} \vec{T} (\vec{\tau}\varepsilon)^{ij}.$$

S and Q are as indicated in the table, symplectic Majorana spinors. To check the consistency of this algebra one has to compute the graded Jacobi identities. This is a trivial calculation except for the $(Q,\ Q,\ S)$ identity. Technical details for the computation of this identity may be found in appendix B of [3].

Let us close with the remark that the superconformal algebra, in contradistinction to the super Poincaré algebra, does not allow central charges.

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